International Mechatronical Students micro-Conference
IMSμC'2013

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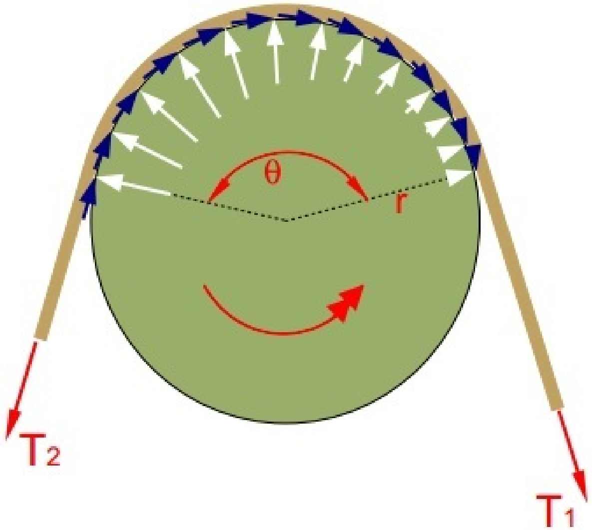
LOCATION: **Óbuda University, Bánki Donát Faculty of Mechanical and Safety Engineering**

PREZENTER: **Hérics Attila**

TOPIC: **Belt Fricton**

BELT FRICTION

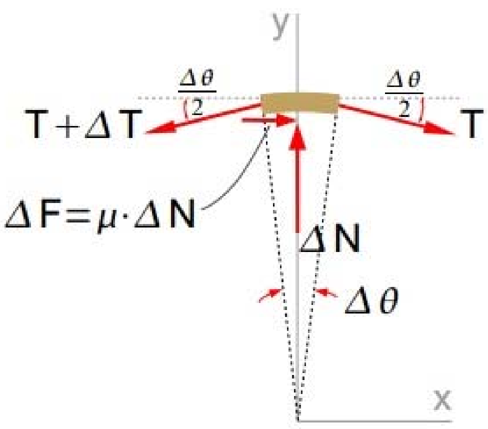
- When flat belts, V-belts, band brakes, and line-wrap around capstans are used in any form of product, the frictional forces developed must be determined. All forms of wrap friction are analyzed similarly. Let's first look at flat belts



- A flat belt has a wrap angle around a pulley of θ radians. Normal forces develop at each finite point of contact with a resulting frictional force acting tangent to the pulley and opposite the direction of rotation. This reduces belt tension around the pully such that tension $T_1 < T_2$
- Furthermore, since belt tension changes continuously, so do the incremental normal and frictional forces

BELT FRICTION

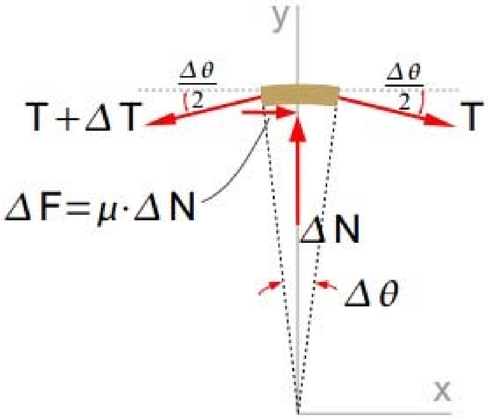
- To help determine how frictional forces affect belt tension, consider a finite element over angle Δθ. Acting on this element is slackside tension T and tightside tension T + ΔT



- Incremental forces ΔN and ΔF develop as a result of the applied incremental tensions. By performing an equilibrium analysis:

$$\Sigma F_x = 0$$
$$= T\cos\left(\frac{\Delta\theta}{2}\right) + \mu\cdot\Delta N - (T + \Delta T)\cdot\cos\left(\frac{\Delta\theta}{2}\right)$$
$$\Sigma F_y = 0$$
$$= \Delta N - (T + \Delta T)\cdot\sin\left(\frac{\Delta\theta}{2}\right) - T\cdot\sin\left(\frac{\Delta\theta}{2}\right)$$

BELT FRICTION



For small $\Delta\theta$, $\sin\left(\frac{\Delta\theta}{2}\right) = \frac{\Delta\theta}{2}$ and $\cos\left(\frac{\Delta\theta}{2}\right) = 1$

Substituting:

ΣF_x => $\Delta T = \mu \cdot \Delta N$

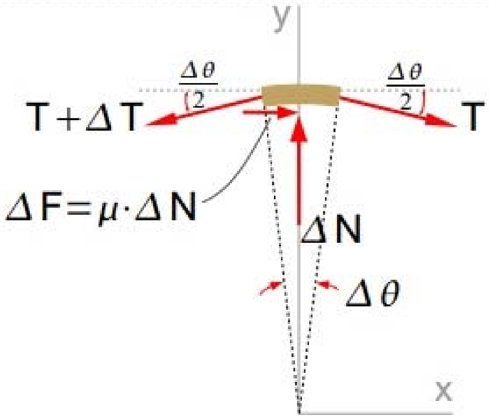
ΣF_y => $\Delta N = T \cdot \Delta\theta$

Substituting ΔN => $\Delta T = \mu \cdot T \cdot \Delta\theta$

- Since this expression is developed over a very small angle, this would have to be summed around the wrap angle

$$\sum_i \Delta T_i = \sum_i \left(\mu \cdot T \cdot \Delta\theta_i\right)$$

BELT FRICTION

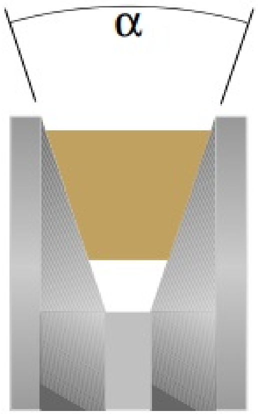


- Since this expression is developed over a very small angle, applying such a summation would be impractical
- However, if we replaced the finite quantities of ΔT and Δθ with infinitely small values of dT and dθ, we could integrate over θ as follows:

$$\int_{T_1}^{T_2} \frac{dT}{T} = \mu \int_0^{\theta} d\theta$$

- This evaluates to: $T_2 = T_1 e^{\mu\theta}$
- It's important to remember T_2 is always larger than T_1 and θ is in radians

V-BELT FRICTION



- The friction developed by a V-belt can be developed in a similar fashion. The relationship between tightside and slackside tension for a V-belt is:

$$T_2 = T_1 e^{\mu \theta / \sin(\alpha/2)}$$

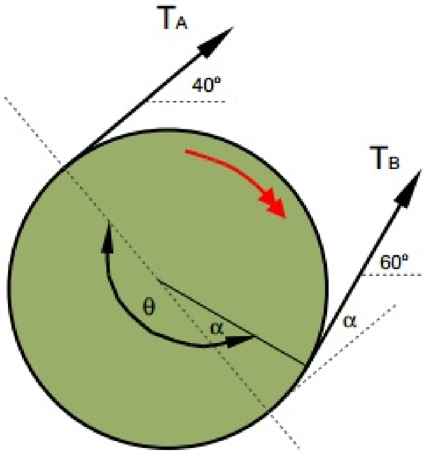
- Angle α must be expressed in radians

BELT FRICTION

- The accessory package for the engine shown below requires a torque of 30 ft-lbf. The pulley in contact with the belt has a diameter of 8” and a coefficient of static friction of 0.30. Determine the tension in each part of the belt if the belt is not to slip and:

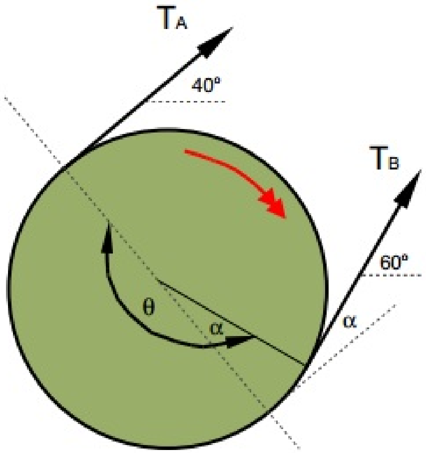
- the system uses a flat belt
- the system uses a V-belt with a 38° V and the same coefficient of friction

BELT FRICTION



- First determine angle of wrap. Draw a construction line at the base of vector T_B and parallel to vector T_A. Angle α is the difference between angles of the two vectors and is equal to 20°. This results in a wrap angle of 200° or 1.11π radians
- Since there are two unknowns, we must develop two mathematical relationships to solve for the unknown tensions. These are the moment about the center of the pulley and the friction equation for a flat belt. Since moment is applied clockwise, tension T_B is tightside tension; we will let $T_2 = T_B$ and $T_1 = T_A$

BELT FRICTION



$$\Sigma M_o = T_B \cdot 4\,in - T_A \cdot 4\,in - (30\,ft \cdot lb_f) \cdot 12\,in/ft$$

$$\therefore \quad T_B = T_A + 90 \qquad \text{(Moments)}$$

-and-

$$T_B = T_A \cdot e^{0.3 \cdot (1.11\pi)} \qquad \text{(Friction)}$$

Substituting:

$$T_A + 90 = T_A \cdot e^{0.3 \cdot (1.11\pi)} = 2.85 \cdot T_A$$

$$\therefore \quad T_A = 48.6\,lb_f \quad \text{and} \quad T_B = 139\,lb_f$$

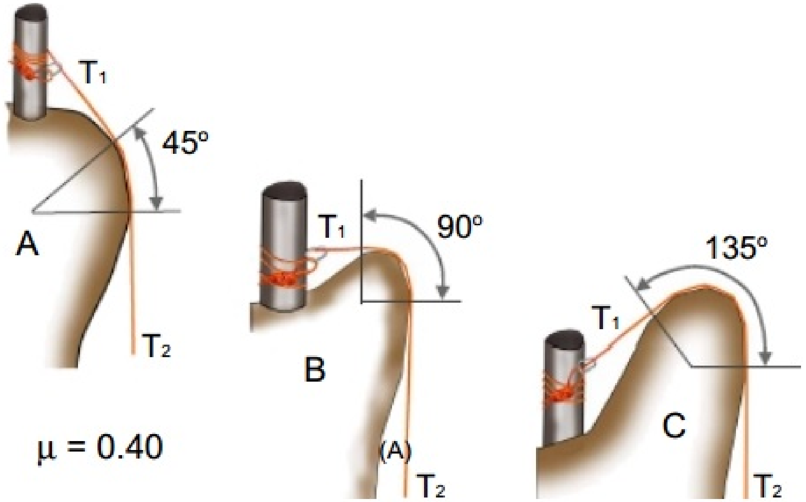
The application of a V-belt changes only the friction equation. The 38^o V is 0.211π radians. Modifying the friction equation changes the solution to:

$$T_A + 90 = T_A \cdot e^{0.3 \cdot (1.11\pi)/\sin(0.106\pi)} = 24.5 \cdot T_A$$

$$\therefore \quad T_A = 3.82\,lb_f \quad \text{and} \quad T_B = 98.4\,lb_f$$

Notice the efficiency increase of a V-belt over that of a flat belt. The reduced tensions help increase bearing life

BELT FRICTION



- Since area does not affect the developed friction for a belt, the formula for flat belts is applicable to round line. Only angle of wrap and coefficient of friction govern developed tension

(A) $\theta = 45^o = \dfrac{45 \cdot \pi}{180} = 0.25\pi\ rad$

$T_2 = T_1 \cdot e^{0.40 \cdot (0.25\pi)} = 1.37 \cdot T_1$

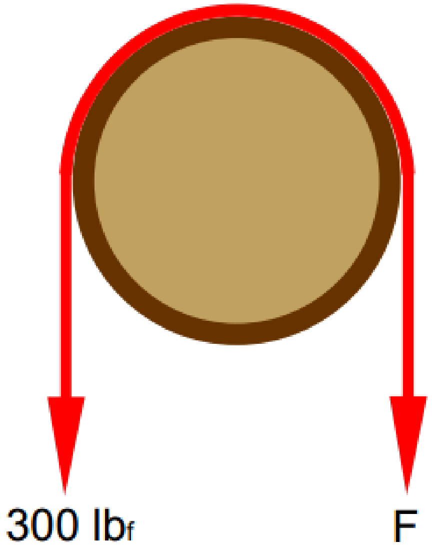
(B) $\theta = 90^o = \dfrac{90 \cdot \pi}{180} = 0.50\pi\ rad$

$T_2 = T_1 \cdot e^{0.40 \cdot (0.50\pi)} = 1.87 \cdot T_1$

(C) $\theta = 135^o = \dfrac{135 \cdot \pi}{180} = 0.75\pi\ rad$

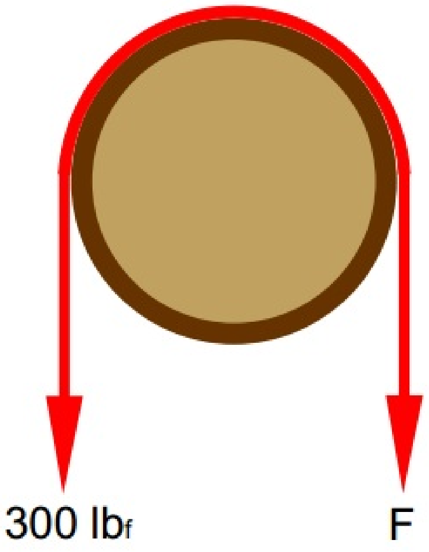
$T_2 = T_1 \cdot e^{0.40 \cdot (0.75\pi)} = 2.56 \cdot T_1$

BELT FRICTION



- You are lifting an engine out of your friends car. In your front yard is a sturdy oak tree. You wrap a rope around a tree branch to lift the engine. The engine weighs 300 lb_f, the rope has a 180° wrap, and $\mu = 0.1$
  - How much force does it take to lift the engine from the vehicle?
  - How much force must you apply to lower the engine back into place?

BELT FRICTION



- Line wrap is 180° or π radians. When lifting, force F is the tightside of the line (T_2). To solve for T_2:

$$T_2 = T_1 e^{\mu\theta}$$
$$T_2 = (300)\cdot e^{0.1\cdot\pi} = 411\, lb_f$$

- To lower the engine, force F becomes the slack side of the line (T_1). To solve for T_1:

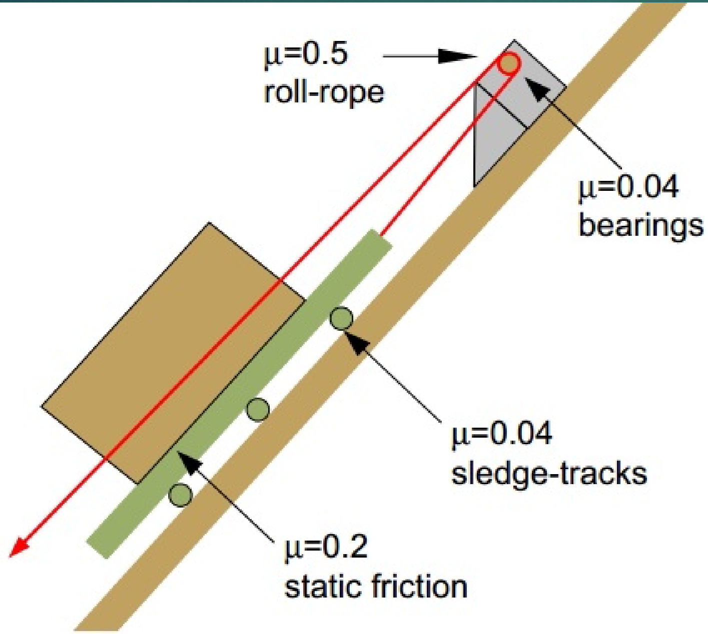
$$T_1 = T_2 e^{-\mu\theta}$$
$$T_1 = (300)\cdot e^{-0.1\cdot\pi} = 219\, lb_f$$

BELT FRICTION IN REAL LIFE MECHANISMS

- Mechanisms using wrap friction are often critical life-safety devices. Such devices are used in rappelling, rock climbing, sailing, and rigging of equipment

rappell rack; the gyb`easy to control the boom of a sailboat during gybing.

BUILDING THE PYRAMIDS AND FRICTION



- The Egyptions and Mayans responsible for building the pyramids on their respective continents were able to reduce friction where it wasn't wanted and maximize friction where it was useful and desired. We still do this today in equipment design

THANK YOU FOR YOUR ATTENTION!