ÓBUDA UNIVERSITY
Donát Bánki Faculty of Mechanical and Safety Engineering

Dr. Ferenc Szlivka

Thermo- and Fluid Dynamics

ÓE-BGK-3074

Budapest, 2019.

Lektor: Dr. Molnár Ildikó

ISBN 978-963-449-141-5

Contents

Introduction

This note is primarily intended for mechanical engineer students and the learning and application of Thermodínamics and Fluid Mechanics. Of course, for others it is very useful, to those who wish to get acquainted with the basics of fluid mechanics, and I would like to apply their knowledge of industrial practice. The knowledge is based on the lectures of fluid mechanics and thermo dynamics held at the Technical University of Budapest, the Szent István University, the University of Dunaújváros and Óbudai University. I tried to shed light on the mathematical background when I presented the basic equations of fluid mechanics and thermo dynamics without going into details.

Not only technical but also other behaviours, for example, atmospheric, biological phenomena related explanations are also incorporated into the note. Application examples are also pointed out with unmistakable icons. Hopfully these marks allow for faster and more accurate orientation for the reader in the material.

A special thanks to my colleague Dr. Ildikó Molnár, who helped the precious comments during peer review for correcting errors.

2019 Budapest Author

1. Properties of liquids and gases

1.1 Ideal liquid

In physics, we distinguish three different states of matter: the solid, the liquid and the gaseous state. The solid bodies have a defined shape while the liquid and airborne materials take up the shape of the container containing them. The liquid material forms a free surface in the vessel while the air is filled to the space available to it. A very important difference between liquid and gaseous substances is that, while the liquids are almost unbreakable, i.e. their density also differs little in the case of very high pressure changes (e.g. only about 5% of the ocean's deepest point changes) the density of the airborne bodies by the pressure - at a constant temperature - varies proportionally.
In fluid mechanics, gases are also referred to as liquids, but the use of the name is not consistent. Only a part of the properties of liquids and gases are taken into account when we are prescribe laws, and we idealize properties.
For us, in this case, the ideal liquid is homogeneous, frictionless and uncompressed.
In the case of non-compressibility in liquids, we rarely have to give up -ld. Allievi theory - in the case of gases, it is permissible to neglect the sound speed approach.
Homogeneous is meant to exclude the molecular structure and the liquid, and imagine the space uniformly filling everywhere. The density of the homogeneous or continuum is the same as the actual liquid density.
The term continual refers to the fact that the material continuously and uniformly forms a free surface (excluding gases) or completely (gases), and therefore does not have its own shape. In addition to the term continual, especially for liquids and gases, the commonly used Hungarian term is frequently used.

When defining the density of a liquid, only average density is used:

$$\rho = \lim \frac{\Delta m}{\Delta V}$$

where "Δm" is the mass in the "ΔV" volume. Limes are meant to reduce the size of a small, but "ΔV" size to more than a thousand magnitudes. Continuum approximation is practically always sustainable, with the exception of only very rare gases being investigated.
The condition of friction-free condition must be dispensed with as soon as possible when examining the flow of real liquids. However, basic laws are phrased by neglecting friction, because - as we will see - taking friction often leads to untold hardships.

1.2 The most important physical characteristics of liquids and gases

Since the mechanics of heat and fluids deal with a small part of a continuous medium, it is advisable to introduce the concept of weight per unit volume and weight. In addition to the physical quantities, they are also given units of measurement in the International System of Units (SI) in which the basic units are the following (units required for us):

length = meters (m)
time = seconds (s)
weight = kg (kg)
temperature = Kelvin (K)
and the

Derived quantity is
Force = newton (N) or
Density is the volume of the volume contained in the unit volume, in Greek with ρ.

$$\rho = \lim \frac{\Delta m}{\Delta V}$$ where "ΔV" is the volume of the designated "Δm" mass. 1.1

Specific volume is the volume occupied by the unit mass, "v" (it is not used in the flow case because the velocity is also indicated by "v"). The specific volume is the reciprocal of density:

$$v = \frac{V}{m} \left[\frac{m^3}{kg}\right] \quad 1.2$$

Specific weight *is the weight of the unit volume, in Greek* γ.

$$\gamma = \frac{G}{V} \left[\frac{N}{m^3}\right] \quad 1.3$$

The temperature marked with "T" or "t" and the degree of internal energy level of the fluid. The absolute temperature is Kelvin "K", whose relationship with the Celsius degree we use is as follows:

$$K = {}^0 C + 273.16 \quad 1.4$$

Pressure per unit of surface pressure perpendicular to the surface:

$$p = \frac{F}{A} \left[\frac{N}{m^2} = Pa\right] \quad 1.5$$

(As pressure is one of the most important physical quantities in heat and flow, we will return to a more precise definition.)

Heat (unit of mass) is heat reported or extracted from the system, not a status indicator.

$$q = \frac{Q}{m} \left[\frac{J}{kg}\right] \quad 1.6$$

Work (per unit of mass) is a status indicator that is reported or removed from the system. (W_F physical work, closed system, W_T technical work, open system)

$$w = \frac{W}{m} \left[\frac{J}{kg}\right] \qquad w_F = -\int p \cdot dv \; ; \; w_T = -\int v \cdot dp \quad 1.7$$

Internal energy (per unit of mass) is the internal energy of the closed system. Ideal for gas

$$u = c_v \cdot T$$

$$u = \frac{U}{m} \left[\frac{J}{kg}\right] \quad 1.8$$

Enthalpy (per unit of mass) is the energy of the open system. Ideal for gas $u = c_p \cdot T$.

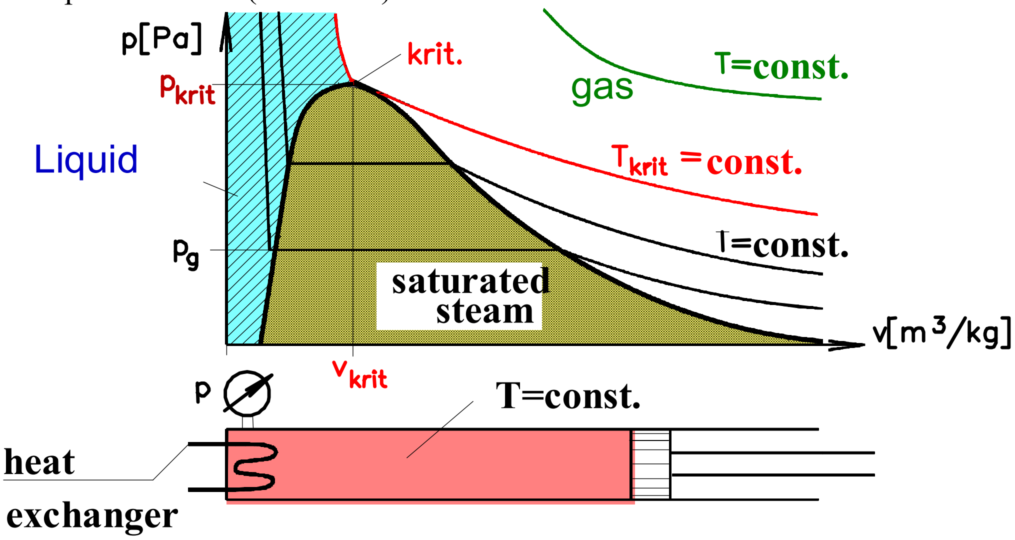
$$h(i) = \frac{H}{m} \left[\frac{J}{kg}\right] \qquad h = u + p \cdot v \qquad 1.9$$

Entropy (unit mass) is the degree of disorder of the system.

$$s = \frac{S}{m} \left[\frac{J}{kg \cdot K}\right] \qquad ds = \frac{dq}{T} \qquad 1.10$$

1.3 Properties of liquids and gases

The most important characteristic of fluids and gases is the correlation between pressure and volume, the well-known "p-v" diagram shown in **Figure 1.1** for water. The diagram shows constant temperature lines (isotherms).



Phase diagram
Figure 1.1

Table 1.1 Critical values for gases

	t_{krit} °C	p_{krit} bar	v_{krit} m³/kg	ρ kg/m³
water	374	225	3.1 10^{-3}	322
oxygen	-119	51	2.33 10^{-3}	429
nitrogen	-147	34.6	3.20 10^{-3}	312
air	-141	38	2.98 10^{-3}	335
carbon dioxide	31.1	73	2.15 10^{-3}	464

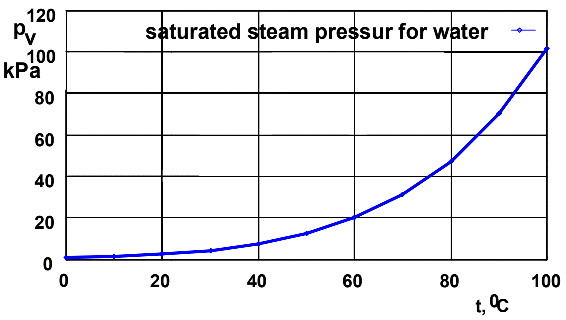
Steam with a movable plunger is sealed in the cylinder. By moving the piston, the volume of the steam can be varied while measuring its "p" pressure. The temperature "T" is maintained at a constant value by means of a heat exchanger with heat introduction or withdrawal.

Move the plunger while maintaining a constant temperature, and measure the volumetric mass "v" and "p" in the diagram.

At a given constant temperature, reducing the volume of the steam, the pressure begins to increase and then becomes constant. In the constant pressure section there is a liquid and vapor phase in the cylinder. Through the heat exchanger, heat is always extracted from the system, which is the hidden or latent heat generated by condensation. After all vapors are condensed, a liquid phase remains in the cylinder.

Here a small volume reduction is responded with very high pressure increase, while constant temperature curves are virtually vertical. The upper right part of the figure is the airway, the left-hand side section of the figure shows the liquid state. The thick line below the curve is a field corresponding to the mixture of boiling water and saturated steam. No liquid state (water) can occur at temperatures above the upper limit curve. At critical temperature condensation or evaporation is the release of hidden heat or evaporation without being tied.

1.3.1 Pressure of saturated vapour

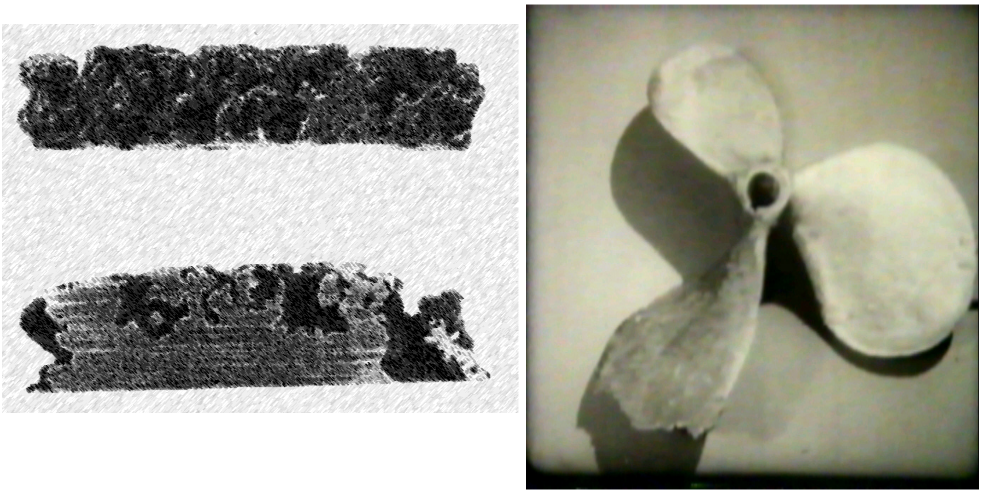


Pressurized steam pressure
Figure 1.2

The transformation from fluid to airborne state is evaporation, which occurs at all temperatures and pressure where fluid can exist.

However, at certain pressures and temperatures, steam is also formed inside the liquid. This pressure is called saturated vapour pressure. In **Figure 1.1**, a given constant pressure is associated with a given temperature within the boundary curve. Its values for water are shown in **Figure 1.2.**

The diagram shows that water can even boil at zero degree if the pressure is sufficiently low. In liquid-flow flow machines, especially in pumps for impellers and propellers, the pressure in the high-speed flow decreases to such an extent that it reaches the saturated vapour pressure. Then steam bubbles are generated. When these bubbles get to a higher pressure, the steam condenses, the bubbles crash. If the collapse occurs near a solid wall, it causes significant destruction. The formation and collapse of steam bubbles are called **cavitation**. The destruction of the solid wall is called cavitation erosion**.** **Figure 1.3** shows a piece of the propeller blade. It can be seen the homogeneous steel structure of the shovel becomes spongy-shaped due to erosion.



Cavitation erosion on a piece of a propeller blade

Figure 1.3

1.3.2 General gas law

From the evaporator curve in **Fig. 1.1**, the vapor is overheated to the right at the points near it, and away from the boundary curve, e.g. $T >> T_{krit}$ if it is not water, we are talking about gas. Ideal gas is the term used in physics, a simplified model of gases whose thermodynamic behaviour can be described by simple mathematical means. Realistic gases are more or less aligned with the ideal state (the most ideal gas in our current knowledge is helium).

In the case of air (O_2 and N_2 mixtures), $t_{krit} = -141^{\circ}C$ $T_{krit} = 132K$ at normal temperatures $T >> T_{krit}$, such as air gas, which is approached in good approximation with the **ideal gas law**:

$$\frac{p}{\rho} = R \cdot T \qquad 1.11$$

to which additional equations should be added to complete the mathematical model describing the ideal gases.

$$c_p - c_v = R \qquad 1.12$$

$$\frac{c_p}{c_v} = \kappa \qquad 1.13$$

isentropic or adiabatic exponent,

where "p" pressure of gas;
"ρ" density of gas;
"T" absolute temperature of gas;
"R" specific gas constant;

" c_p " specific heat capacity at constant pressure;

" c_v " specific heat capacity at constant volume.

The constants in the gas law for air:

$$R_{lev} = 287\frac{J}{kg \cdot K} = 287\frac{N \cdot m}{kg \cdot K} = 287\frac{m^2}{s^2 \cdot K}, \qquad 1.14$$

$$c_p = 1005\frac{N \cdot m}{kg \cdot K}, \qquad 1.15$$

$$c_v = 718\frac{N \cdot m}{kg \cdot K}, \qquad 1.16$$

$$\kappa = \frac{c_p}{c_v} = 1.4 \qquad 1.17$$

1.4 Simple state changes are ideal in gases

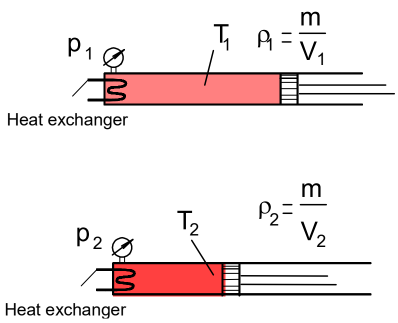


Figure 1.4

Close air or other gas in the cylinder shown in **Figure 1.1** and examine the relationship between the status indicators for different state changes. Monitoring the change of status indicators is possible in the knowledge of the general gas law. There are some simple state changes where each status indicator is kept constant and the other status indicators are examined. Indicators for a given gas are: pressure "p", density "ρ" and temperature "T". ("R" is not a status indicator.)

Take a given mass of gas as shown in **Figure 1.4**. During the state change, the state from "1" to "2" is placed in the gas while its mass remains constant and does not mix with the gas in its vicinity. It can be heated or cooled through a heat exchanger.

1.4.1 Isothermal state change

If the temperature is constant $T_1 = T_2 = T = \text{const.}$ then it is an isothermal state change. If $p_2 > p_1$ the gas is compressed to keep it at a constant temperature, it must be cooled by the heat exchanger. If $p_2 < p_1$ the gas expands to keep its temperature constant, it should be heated. This experiment was used to draw the isotherm lines of **Figure 1.1** well above the $T_{krit.}$ value. Replaced in the general gas law:

$$\frac{p_1}{\rho_1} = R \cdot T \qquad \frac{p_2}{\rho_2} = R \cdot T \qquad 1.18$$

$$\frac{p_1}{\rho_1} = \frac{p_2}{\rho_2}$$

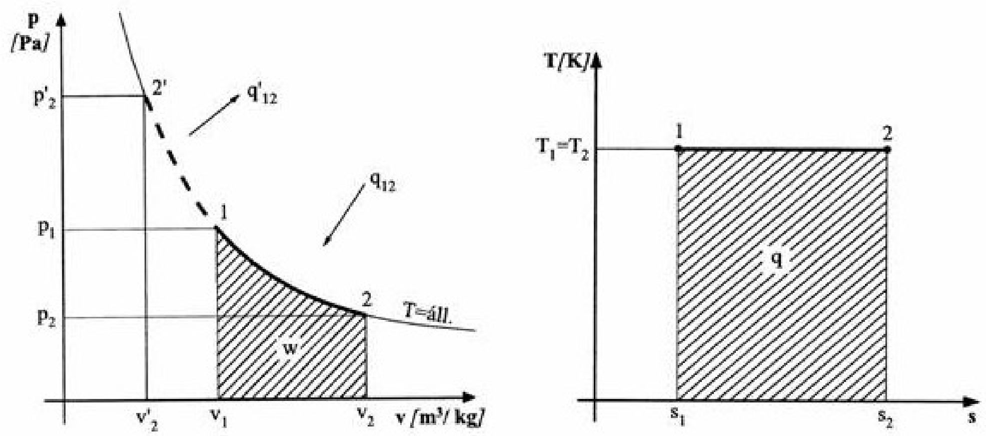
If we replace $\rho = \frac{m}{V}$ the expression, we get the following:

$$p_1 \cdot V_1 = p_2 \cdot V_2 \qquad 1.19$$

which is known as the **Boyle-Mariotte law**. (Robert Boyle, English physicist 1627-1691 and Edme Mariotte, 1620-1684, were discovered independently by the French physicist, who also discovered the blind spot of the eye.) The law states that the volume is inversely proportional to the pressure or the density is directly proportional to pressure. Figure 1.5 shows the state change in p-v (pressure-type volume) and T-s (temperature entropy) diagrams.

1.4.2 State change in constant volume

If the volume is $V_1 = V_2 = V = \text{const.}$ constant, we are talking about an isochronous state change. This can be accomplished by fixing it at a given location in the cylinder. If the gas is



Isothermal state change

Figure 1.5

heated at a constant volume, its pressure increases $p_2 > p_1$, if it is cooled, its pressure decreases $p_2 < p_1$. Replace the conditions in the general gas law:

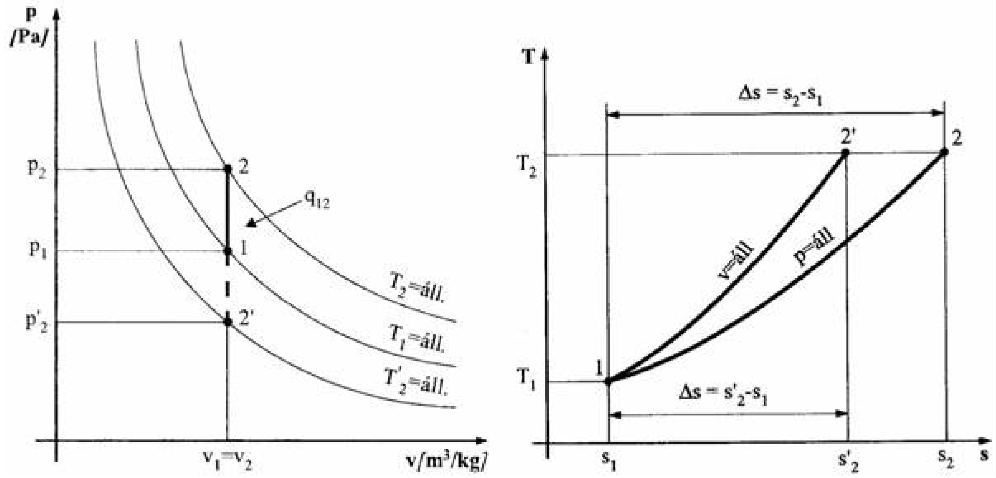
$$\frac{p_1}{m/V} = \text{const. and} \quad \frac{p_2}{m/V} = \text{const.}, \text{ from which}$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad 1.20$$

results are obtained.

This is the **Law of Gay Lussac I. law**. (John Louis Gay-Lussac, French chemist and physicist from 1778 to 1850. He also invented the lead chamber tower used in sulphuric acid, which he named it.) This is also a simple relationship between the status indicators because temperature and pressure vary in a straight proportion.

During the change of state, a single degree of cooling from the unit weight gas, or the heat injected or introduced during heating, is given by the specific heat „c_v" measured on the constant volume.



Isochoric state change
Figure 1.6

1.4.3 Isobar state change

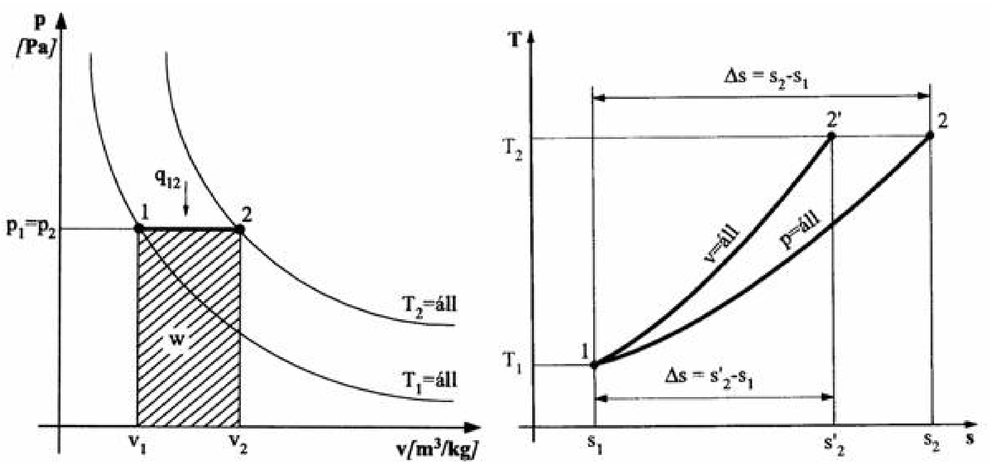
If the pressure is constant $p_1 = p_2 = p = constant$, then we are talking about an isobaric state change. During the experiment, the piston is allowed to move (or push in) against a constant force. The gas is heated at constant pressure, the volume is increased $V_2 > V_1$, when cooled, its volume decreases $V_2 < V_1$. Replace the conditions in the general gas law:

$$\frac{p}{m/V_1} = R \cdot T_1 \text{ and } \frac{p}{m/V_2} = R \cdot T_2 \text{, from which}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad 1.21$$

results are obtained..

This is **Gay-Lussac II. law** according to which the temperature and the volume are directly proportional to the change of state. During the change of state, a single degree of cooling from



Isobar state change
Figure 1.7

the unit weight gas, or heat extracted or introduced during heating, is given by the specific heat at constant pressure "c_p".

1.4.4 Adiabatic and frictionless state change

If both the pressure and the volume are changed but the system is isolated from the environment then an adiabatic change of state occurs in the gas. By doing so, move the piston fast in or out to create a change of state. We do not give time to heat transfer. This is also called isentropic (adiabatic and friction-free) state change because entropy does not change during the state change if there is no internal friction in the medium. Each status indicator changes, but there is the following connection between status indicators:

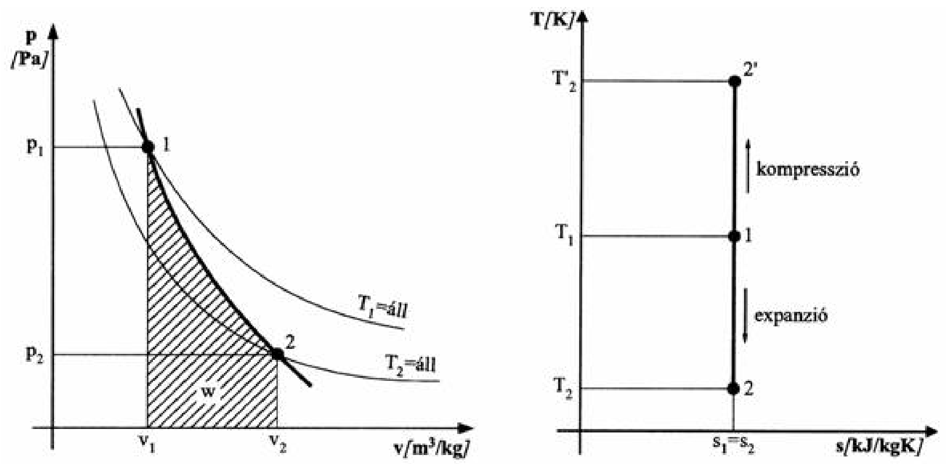
$$\frac{p_1}{\rho_1^{\kappa}} = \frac{p_2}{\rho_2^{\kappa}} \qquad 1.22$$

Replaced in the gas law and simplifying the

$$\frac{p_1}{T_1^{\frac{\kappa}{\kappa-1}}} = \frac{p_2}{T_2^{\frac{\kappa}{\kappa-1}}} \qquad 1.23$$

expression is obtained,

where $\kappa = \frac{c_p}{c_v}$ the adiabatic or isentropic exponent. (for two-atomic gases $\kappa = \frac{c_p}{c_v} = 1{,}4$)



Izentropic state change
Figure 1.8

1.4.5 Polytrophic change of state

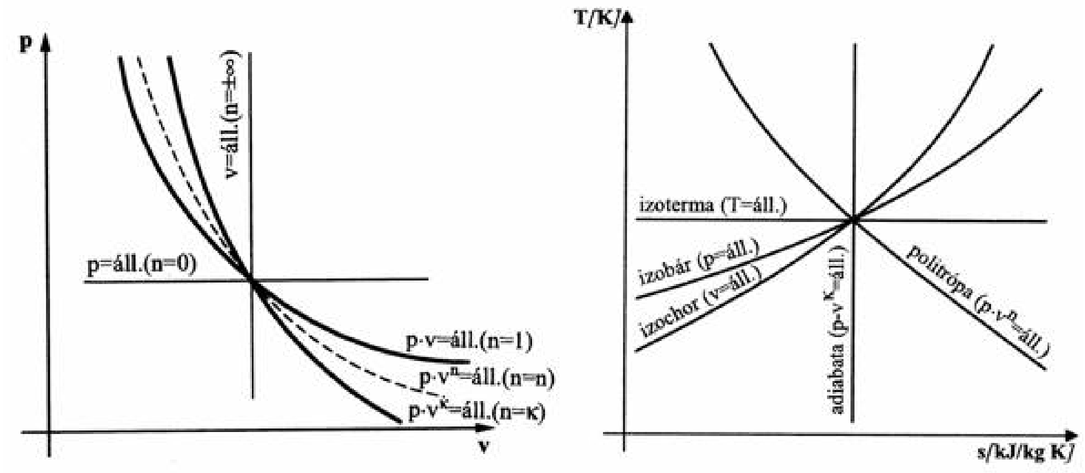
In fact, this can be a completely normal state change because all three status indicators may vary, even heat can be released into the environment, or heat can be taken from there. In such a case, the relationship between the status indicators is given by the following relationships:

$$\frac{p_1}{\rho_1^{n}} = \frac{p_2}{\rho_2^{n}}, \qquad 1.24$$

substituting the gas stream, the connection between the pressures and the temperatures is also obtained here

$$\frac{p_1}{T_1^{\frac{n}{n-1}}} = \frac{p_2}{T_2^{\frac{n}{n-1}}}, \qquad 1.25$$

where "n" is the polytrophic exponent. If the gas is compressed and cooled, then $n < \kappa$. If there is no cooling or even heating the gas, then $n \geq \kappa$. Similar considerations may be given for the expansion of "n" and "κ".
The terms of adiabatic and polytrophic changes are the same except for the exponent.
If n = 1, then the polytrophic state change is the same with the isothermal state change.



Polytrophic change of state
Figure 1.9

1.5 Compressibility of liquids

Table 1.2

Liquid	Flexibility modulus "E"
	Mpa
Ethyl alcohol	896
Petrol	1 062
Machine oil	1 303
Water	2 179
Glycerol	4 509
Mercury	24 750
Steel	200 000

Fluids can almost always be regarded as unbreakable in flowing terms. In the "p-v" diagram, the isotherms in the liquid area are approximately v = const, vertical lines. The compressibility or elastic modulus can be given by the following relationship:

$$E = \frac{-\Delta p}{\Delta V / V} \qquad 1.26$$

The elasticity modulus of some liquids are summarized in **Table 1.2.** (In the strength class, the "pressure" has a negative sign to make the result positive, so a negative sign is needed.) The elasticity modulus of the steel is also indicated for the sake of comparability.
For example, compression of the water volume by 0.5% requires a pressure of 10.89 MPa, which is about 100 times the normal atmospheric pressure. In the deepest place on Earth, in the Marianna Ditch in the Pacific Ocean, where the sea is the deepest, the pressure at 1 bar ressure atmospheric is about 1100 times. Relative compression of water is only 5% here.
Other important properties of liquids such as surface tension and viscosity are discussed in separate chapters.

2. Basic of thermodynamics

2.1 The thermodynamic system

Thermodynamic tests refer to the thermodynamic system. The thermodynamic system is a separated part of space we have chosen. The demarcation can take place with a real wall or an imaginary demarcation surface. The part within the boundary wall is the thermodynamic system. The part outside the thermodynamic system is called environment.

Depending on the properties of the boundary walls, there may be several interactions between the system and the environment. The most commonly known is the **mechanical interaction**. The system can perform **mechanical work** (W) on the environment or the environment on the system. During **thermal interaction**, heat (Q) flows from the system to the environment or from the environment to the system. During **mass transfer** (m), the system and the environment can be exchanged. Other interactions can occur between the system and its environment, eg. electric, magnetic, etc. processes, which are ultimately analogous to mechanical interaction. Hereinafter, the emphasis is placed on examining the three interactions listed above.

The walls separating the thermodynamics system from the environment are listed in the following groups based on the listed interactions:

1. **A rigid wall** that prevents any mechanical interaction, a deformable wall which allows any mechanical interaction.
2. **A non-permeable** or **semi-permeable wall**, only prevents the penetration of certain substances.
3. **A diathermic wall** that allows heat (thermal) interaction or an adiatermic wall that prevents it.

Adiatermic and non-permeable walls, which allow only mechanical interaction, are adiabatic walls, and the system bounded with such walls is adiabatic, and the processes involved are called adiabatic processes.

A **closed system** with no mass flow on its wall (leaving no interaction left)

An open system with mass flow on its wall.

A homogeneous system is where intense variables are not dependent on place and time coordinates.

2.2 Thermodynamic and Calorie Quantities

Clear (one-valued) functions of the state of the system depend only on the current status of the system and are independent of the previous state of the system and of the state change through which the system got to that state.
Status indicators may be:
- scalar,
- vector,
- tensor quantities.

Types of status indicators are: - extensive, - intensive volumes.

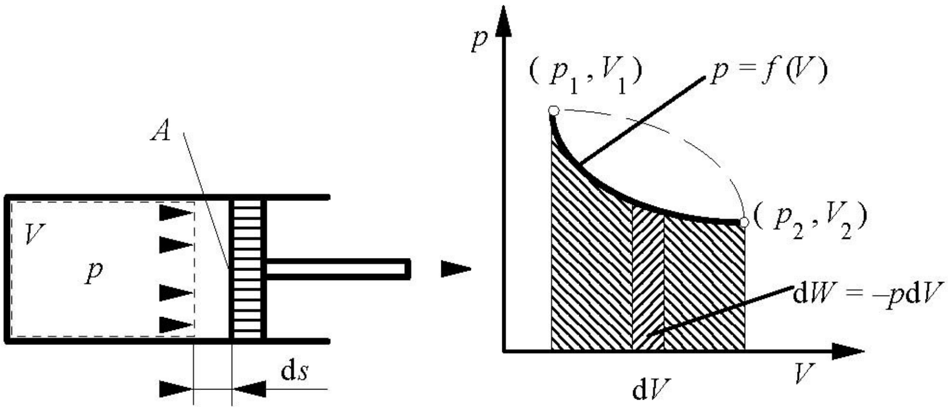
Extensive status indicators: state of the thermodynamic system proportional to the mass and volume. The sums of quantities measured in parts of the thermodynamic system are characteristic of the entire system. Such as weight, entropy, energy, etc.
Intensive Status Indicators: Balancing Status Indicators. In some parts of the thermodynamic system, the typical quantity of the whole system can be measured. Such as pressure, temperature, etc.
Specified status indicators: two extensible status ratios. Such as density, bulk volume, specific weight, and so on.
Factual Quantities: Material Characteristics, e.g. specific heat, coefficient of thermal expansion, thermal conductivity, dynamic viscosity.

Non-status but very important caloric characteristic is the specific heat quantity (q, [J/kg] and the specific work (w, [J/kg] because the size of the changes depends on the state of change of state.

2.3. Specific work and specific heat

Internal energy (U): Compared to the center of gravity, the microscopic elements of the system move at varying speeds and have the potential energy from each other's interactions. This movement does not cease in the absence of the macroscopic system. This energy, which is the sum of the kinetic and potential energy of the microscopic building blocks of the system relative to the center of gravity, is called **internal energy**. For ideal gases, the specific internal energy can be calculated by the expression.

Work (W): scalar product of force and displacement vector:
Work is the amount of energy transport on the system interface created by mechanical interaction or the inhomogeneity of temperature difference or other intense status indicators.



Physical work
Figure 2.1

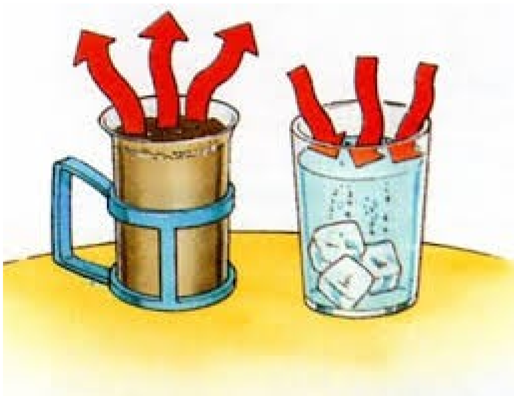
$$dW = -p \cdot A \cdot ds \qquad dW = -p \cdot dV \qquad 2.1$$

Physical work (W)
Physical work involves all sorts of work on the system or and by the system. This can be chemical, electric, magnetic, etc. work and can be bulk work:

$$dW = -p \cdot A \cdot ds \qquad 2.2$$

We look at the sign of work from the perspective of the system. If it is positive, the system gets a job if it is negative, the system gives work to the environment. If the gas expansis the system gives work to the environment.

Heat (Q): the amount of energy transport without mass interaction on the system interface that is induced by the inhomogeneity of the temperature distribution. It is not a status indicator and can not be identified with the energy stored in the system. For example, **Figure 2.2.** heat from the hot coffee cup to the environment, the coffee slowly cools, but its amount does not change (little evaporation is negligible). Or the cold iced drink slowly warms because the heat from the environment flows into the glass, but the mass of the mass does not change here either.



Heat
Figure 2.2

By crossing the heat, the system boundary increases the potential and / or kinetic energy of the elementary elements (atoms, molecules, subatomic particles) that form the system, or the decrease in those energies is the source of the heat that exits the system. The sign of heat is considered positive that flows to the given system and is a negative deriving from the system. Q is used to denote the total heat pertaining to the given system, the quantity specified for the mass unit is q, in accordance with the usual upper case capitalization.

The common properties of work and heat:
-The work and heat is a characteristic of interaction between the system and the environment in the interface of the system.
Work and heat also characterize the transition between the two states of the thermodynamic system (transition) and not the system.
- Both are characteristic of the transition process, i.e., process characteristics and **not status indicators** for the system.
- Work and heat also depend on the mode of change of state, that is, way depending, and consequently are not status indicators for the system.

2.2 The energy content of the thermodynamic system, the basic laws of thermodynamics

2.2.1 The null general theorem of thermodynamics

The thermodynamic system (which is perfectly insulated against each interaction) is balanced when there is no macroscopic change in it, in which case the intense status indicators are homogeneously distributed throughout the system. If two or more equilibrium thermodynamic systems interact with one another, that is, they are not isolated from each other in isolation, then the systems tested have as many intensive properties of the same value in each system as the boundary walls can interact. The main subject can be said in another way: The necessary and sufficient condition of the equilibrium of interacting systems is the equality of the intensity indicators of the possible interactions - the empirical intensity parameters. For example, if there are two systems that are separated by a heat-transmitting (diathermic) but rigid wall, their temperature will be the same for a certain time, but it will withstand pressure, so the pressure will not be equalized. In another example, if the wall is deformable but is a heat-insulating (adiatermic) wall then the pressure levels out, but the temperature does not.

The thermodynamic balance is transitive, which means that if system A is in equilibrium with system B, and B with system C then system A is equilibrated with system C as well. The thermodynamic equilibrium is symmetric, i.e if system A is in equilibrium with system B, system B equilibrates with system A. This transitivity is the basis for temperature measurement. For example, if fever is measured, our body is system A, the thermometer is system B. When the thermometer was verified, the system B, the thermometer, was compared with an etalon, which means C. During validation, the systems B and C were in thermodynamic equilibrium. When thermometers were measured, the systems A and B were in a thermodynamic equilibrium. That's how we know how much fever is. Our body and credible standards are equilibrated with the 0 grade.

Temperature measurement requires a material that has a simple and easy-to-measure property that clearly changes with temperature. All material properties that are a clear function of temperature are suitable for temperature measurement. Preferably, an easily measurable property is used. Such properties can be e.g. the length of the liquid column in the capillary tube, at constant pressure, the volume, at constant volume, pressure, electrical resistance, thermal voltage, crystal (e.g., quartz) vibration frequency, etc. By choosing any effect, there is a need for well-reproducible bases to which the measurements of the given property under given conditions are assigned. Let "t" be the temperature property, and a simple function determines the temperature scale.
Characteristics of different design thermometers:

Thermometer Typical property (s)
Liquid glass tube Length, l
Gases kept at constant pressure Volume, V
Gas Constant Volume Pressure, p
Electrical Resistance, R
Thermocouple Electrical voltage, U

A calibrated curve should be chosen for a quantity that can be determined using an easily reproducible state. This selected state is considered as the reference point of the temperature scale. For this purpose, the triple point of water was chosen, the state in which the three phases of the water are in thermodynamic equilibrium. The other point on the Celsius scale is the boiling point of water at 1 bar. On the Celsius scale it is 100^0 C. In addition, there are several other temperature scales. In the Anglo-Saxon countries Farenheit, at the freezing point of water 32 F at the boiling point of water 212 F. So its null point and scale deviate from the Celsius scale. The first term of thermodynamics states on energy thermodynamics, that is, that energy can be transformed during thermodynamic processes, but it can not be formed or lost. This is often expressed as follows: The change in the internal energy of a system is equal to the amount of heat delivered by the system and the amount of work on the system, i.e.

$$\Delta U = Q + W \tag{2.3}$$

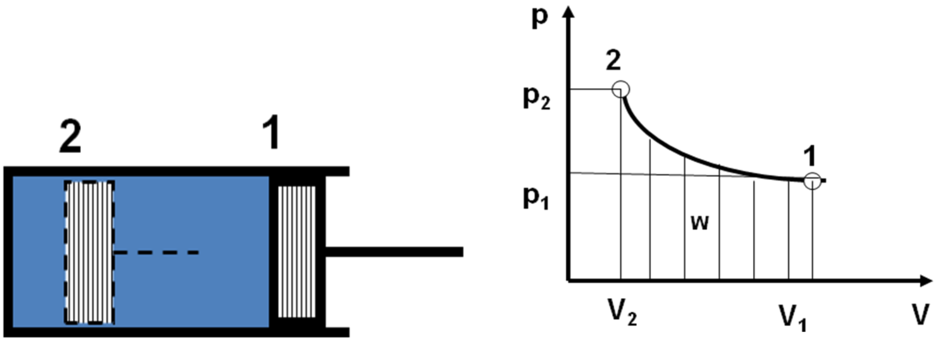
This is often described for elemental changes, at this time $dU = \delta Q + \delta W$, or frequent writing of the unit weight, so we use small letters:

$$du = \delta q + \delta w \tag{2.4}$$

2.2.2 The first general theorem is in a closed system

The first general rule is in a closed system, that is, when there is no mass exchange with the environment, the change of internal energy between two states is equal to the declared heat plus the physical work on the system. Formulated with the following formulas:

2.5



Physical work in a closed system $W_{12} = -\int_{V_1}^{V_2} p \cdot dV$

Figure 2.3

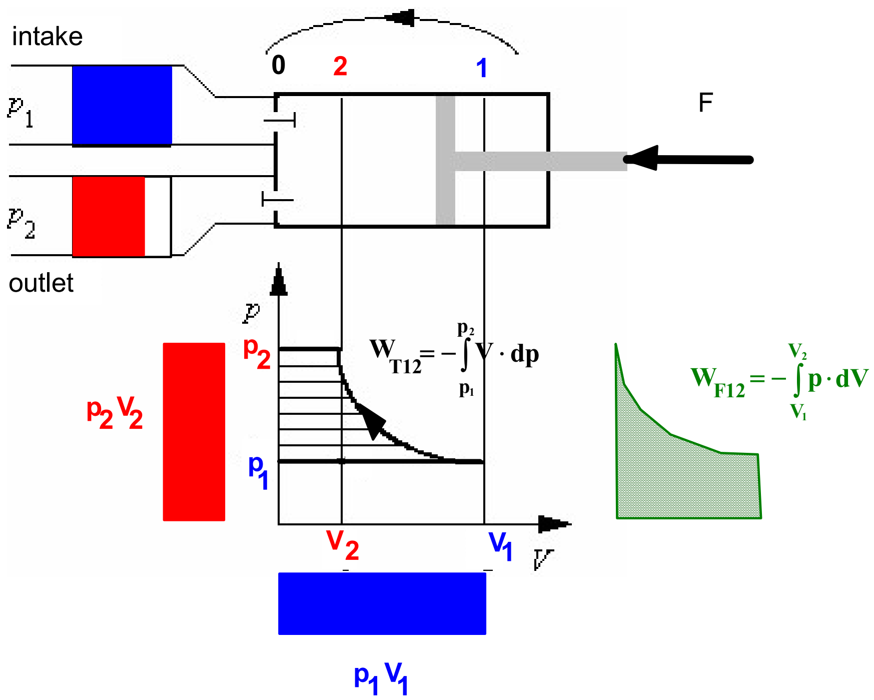
$$\Delta U_{12} = Q_{12} + W_{12} = Q_{12} - \int_{V_1}^{V_2} p \cdot dV \tag{2.6}$$

2.2.3 The first main theorem in an open system

In technical life, we often use open systems when there is mass exchange with the environment. In such cases it is advisable to introduce two new concepts, enthalpy and technical work.

Technical work

Mass transport can be created on the open system boundary surfaces. Due to mass transport, the mass of the open system can vary in time, and we are talking about an instacioner open system. If the values of incoming and outgoing streams are the same regardless of time, we are talking about a stationary or stationary open system. We are investigating the work in such a system. The examination of the open systems is examined on the periodically operating piston compressor shown in Figure 2.4. The compressor vacuums air from place p_1, at a lower pressure point and pushes the medium to a higher pressure p_2. Suppose the plunger can completely remove the working fluid from the cylinder, there is no harmful space.



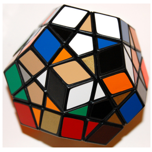
Technical work in a stationary open system
Figure 2.4

In the first phase, the piston moves outward through the 0-1 section, the volume increases, the system gives work to the environment, the size of the work $W_1 = p_1 \cdot V_1$, which is negative. During compression, the piston moves between points 1 and 2, the environment works on the system, this physical work $W_{F12} = -\int_{V_1}^{V_2} p \cdot dV$ is positive. In the 2-0 section, the plunger moves inward, the environment is working on the system, this is a positive sign. Let us summon the three types of work in a provisional way. The amount signed is technical work.

$$W_{T12} = p_2 \cdot V_2 - p_1 \cdot V_1 - \int_{V_1}^{V_2} p \cdot dV = \int_{p_1}^{p_2} V \cdot dp$$

The technical work, therefore, represents the area under the vertical pressure-curve in the figure. Since p_1 is smaller than p_2, dp is positive, i.e., a pressure increase occurs in the system between the two states, so the system gets technical work.

Compressor Testing



With the ideal compressor in **Figure 2.4**, determine the physical, technical work, the entering job and the existing job. Assume that the compressor has no harmful space and there is no heat transfer to the environment, so adiabatic is the change of state.

Data: $p_1 = 1\,\text{bar}\,(\text{abs})$; $p_2 = 2\,\text{bar}\,(\text{abs})$; $V_1 = 1000\,\text{cm}^3$; $R = 287 \frac{J}{kg \cdot K}$; $c_p = 1005 \frac{J}{kg \cdot K}$; $\kappa = 1{,}4$; $t_1 = 20^0\,C$

Solution: The compressor compresses the intake air to the point until the pressure reaches the compression pressure $p_2 = 2\,\text{bar}$. At this time, the discharge valve is released and the compressed air is released into the trap.

The first task is to calculate the volume. Using that adiabatic state change, we can write that

$p_1 \cdot V_1^\kappa = p_2 \cdot V_2^\kappa$, from which the V_2 volume is expressed

$$V_2 = V_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1}{\kappa}} = 1000 \cdot \left(\frac{1}{2}\right)^{\frac{1}{1{,}4}} = 609{,}5\,\text{cm}^3$$

After that, **the entry work**

$$W_1 = p_1 \cdot V_1 = 10^5\,\text{Pa} \cdot 0{,}001\,\text{m}^3 = 100\,\text{Nm}$$

The exit work

$$W_2 = p_2 \cdot V_2 = 2 \cdot 10^5\,\text{Pa} \cdot 0{,}000609\,\text{m}^3 = 121{,}9\,\text{Nm}$$

Physical work is at a stroke $W_{F12} = -\int_{V_1}^{V_2} p \cdot dV$

Let us assume that it is adiabatic during the change of state, $p_1 \cdot V_1^\kappa = p \cdot V^\kappa$ i.e., during compression. From this we express the pressure $p = \frac{p_1 \cdot V_1^\kappa}{V^\kappa}$, which we substitute for the expression of physical work.

$$W_{F12} = -\int_{V_1}^{V_2} p \cdot dV = -\int_{V_1}^{V_2} \frac{p_1 \cdot V_1^{\kappa}}{V^{\kappa}} \cdot dV = -p_1 \cdot V_1^{\kappa} \cdot \left[\frac{V^{1-\kappa}}{1-\kappa}\right]_{V_1}^{V_2} = -\frac{p_1 \cdot V_1^{\kappa}}{1-\kappa} \cdot \left[V_2^{1-\kappa} - V_1^{1-\kappa}\right] =$$

$$= -\frac{10^5 \cdot 0{,}001^{1{,}4}}{1-1{,}4} \cdot \left[0{,}000609^{1-1{,}4} - 0{,}001^{1-1{,}4}\right] = 54{,}85\,\text{Nm}$$

A positive sign indicates that the system is gaining work and an environment conducts work. In this case the piston.

The technical work is at in an attact $W_{T12} = \int_{p_1}^{p_2} V \cdot dp$

Let us assume that it is adiabatic during the change of state, i.e., $p_1 \cdot V_1^{\kappa} = p \cdot V^{\kappa}$ during compression. Express this volume $V = V_1 \cdot \left(\frac{p_1}{p}\right)^{\frac{1}{\kappa}}$, which is replaced by the expression of technical work

$$W_{T12} = \int_{p_1}^{p_2} V \cdot dp = \int_{p_1}^{p_2} \frac{V_1 \cdot p_1^{\frac{1}{\kappa}}}{p^{\frac{1}{\kappa}}} \cdot dp = V_1 \cdot p_1^{\frac{1}{\kappa}} \cdot \left[\frac{p^{1-\frac{1}{\kappa}}}{1-\frac{1}{\kappa}}\right]_{p_1}^{p_2} = -\frac{\kappa \cdot V_1 \cdot p_1^{\frac{1}{\kappa}}}{\kappa - 1} \cdot \left[p_2^{1-\frac{1}{\kappa}} - p_1^{1-\frac{1}{\kappa}}\right] =$$

$$= \frac{1{,}4 \cdot 0{,}001 \cdot \left(10^5\right)^{\frac{1}{1{,}4}}}{1{,}4-1} \cdot \left[\left(2 \cdot 10^5\right)^{\left(1-\frac{1}{1{,}4}\right)} - \left(10^5\right)^{\left(1-\frac{1}{1{,}4}\right)}\right] = 76{,}65\,\text{Nm}$$

A positive sign indicates that the system is gaining work and an environment conducts work. In this case the piston. Verify that the result is consistent with the relationship between physical and technical work!

$$W_{T12} = W_2 - W_1 + W_{F12}$$

$$76{,}65 = 121{,}9 - 100 + 54{,}85$$

$$76{,}65 \cong 76{,}75\,\text{Nm}$$

The difference is due to rounding during the calculation.

Enthalpy change

In the next chapter we give you the exact definition of enthalpy, here we calculate the compressor in advance and compare it with the technical work. The enthalpy change in the air passing through the compressor can be most easily calculated by calculating the temperature rise and mass.
First, calculate the mass of air entering the compressor, which will require the density of the inlet air. This general gas law provides:

$\frac{p}{\rho} = RT$. Applied to state 1 $\rho_1 = \frac{p_1}{R \cdot T_1} = \frac{10^5}{287 \cdot (273+20)} = 1{,}189 \frac{kg}{m^3}$

So we can calculate the mass of inlet air. $m_1 = \rho_1 \cdot V_1 = 1{,}189 \frac{kg}{m^3} \cdot 0{,}001\, m^3 = 0{,}00189\, kg$

In the second step, we must calculate the exit temperature T_2.

For adiabatic change of state, the relationship between temperature and pressure has already been calculated: $\frac{p_1}{T_1^{\frac{\kappa}{\kappa-1}}} = \frac{p_2}{T_2^{\frac{\kappa}{\kappa-1}}}$

From this, the outlet temperature is set: $T_2 = T_1 \cdot \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} = 293 \cdot \left(\frac{2}{1}\right)^{\frac{1{,}4-1}{1{,}4}} = 357{,}1\, K$

Changes in enthalpy:

$H_{12} = m \cdot c_p \cdot (T_2 - T_1) = 0{,}001189\, kg \cdot 1006 \frac{J}{kg \cdot K} \cdot (357{,}1 - 293) = 76{,}68\, J$

According to our results, the enthalpy change is the same as the technical work.

Enthalpy

Examine a continuously flowing, stationary open system, a steam turbine, as shown in **Figure 2.5**. Write the internal energy change between the initial (1) and the end state (2). In the state 1, the pressure and temperature of the steam are higher, in the state 2 the pressure and temperature of the steam is reduced. The steam turbine has the same amount of mass flowing in and out. The steam turbine generates mechanical work from the steam energy. That is in the system, the steam constantly loses technical work. It does this at the expense of the energy of the steam.

$$U_2 - U_1 = Q_{12} + W_{F12}$$

Using the foregoing, we express physical work with technical work:

$$W_{F12} = W_{T12} + W_2 - W_1$$

Write this into the internal energy change:

$$U_2 - U_1 = Q_{12} + W_{T12} + W_2 - W_1 = Q_{12} + W_{T12} - p_2 \cdot V_2 + p_1 \cdot V_1 \qquad 2.8$$

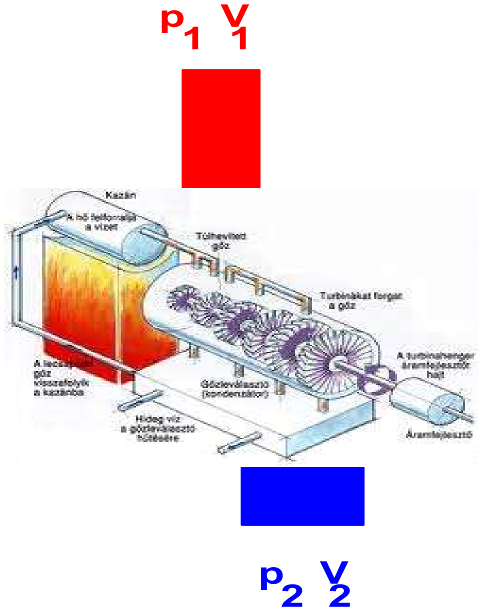
Sort the same index on one page.

Instead of the parentheses on the left side of the equation we introduce a new quantity of enthalpy. The enthalpy (H) is a state variable, since the attributes in it are all state variables defined by the following equation:

$$H = U + p \cdot V \qquad 2.9$$

This is the shape of the first law of thermodynamics for an open system. We can also determine the technical and physical work of state changes. Which of the two jobs will be equal to the work of a particular system will determine what the system is like. The work of a closed system: the physical work of the state change; open system work: technical work of state change. For ideal gases, the specific enthalpy can be calculated by the expression $h = c_p \cdot T$.

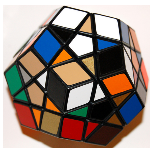
$$(U_2 + p_2 \cdot V_2) - (U_1 + p_1 \cdot V_1) = Q_{12} + W_{T12} \qquad 2.10$$



Steam turbine in a steady open system
Figure 2.5

$$H_{12} = Q_{12} + W_{T12}$$

Gas turbine inspection



For the ideal gas turbine in **Figure 2.6**, determine the physical and technical work of the work and the change of enthalpy! Assume there is no thermal communication with the environment, so the state is adiabatic. Examine the energy content of gas in and out of a given time unit.

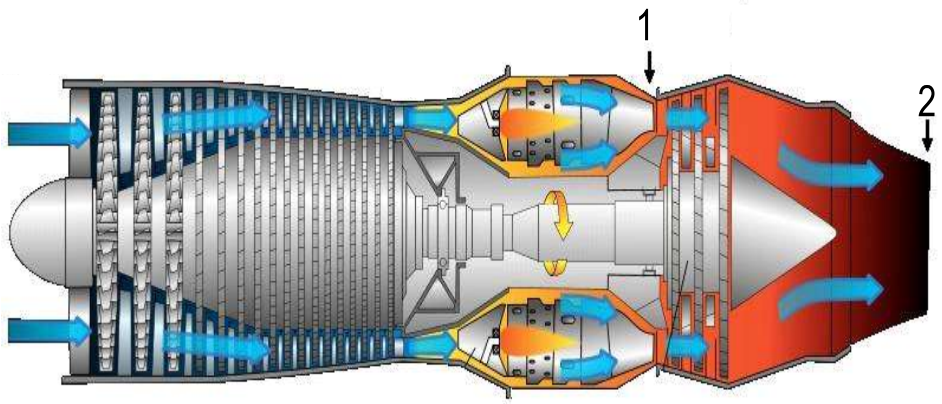
Data: $p_1 = 6\,\text{bar(abs)}$; $p_2 = 1\,\text{bar(abs)}$; $V_1 = 1000\,\text{cm}^3$; $R = 287 \dfrac{J}{kg \cdot K}$; $c_p = 1005 \dfrac{J}{kg \cdot K}$; $\kappa = 1{,}4$; $t_1 = 400^0\,C$

Solution: Compressed high-temperature air (flue gas) flows into the gas turbine. The flow through the turbine drives the turbine shaft and cools down and its pressure decreases and then flows out.
The first task is to calculate the volume. Using adiabatic state change, we can write that $p_1 \cdot V_1^\kappa = p_2 \cdot V_2^\kappa$, of which V_2 the volume is expressed

$$V_2 = V_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1}{\kappa}} = 1000 \cdot \left(\frac{6}{1}\right)^{\frac{1}{1,4}} = 3596\ cm^3$$

After that, **entry work**

$$W_1 = p_1 \cdot V_1 = 6 \cdot 10^5\ Pa \cdot 0{,}001 m^3 = 600\ Nm$$



Gas turbine in a stationary open system
Figure 2.6

Exit work

$$W_2 = p_2 \cdot V_2 = 10^5\ Pa \cdot 0{,}00359 m^3 = 359{,}69\ Nm$$

Physical work is at a stroke $W_{F12} = -\int_{V_1}^{V_2} p \cdot dV$

Let us assume that it is adiabatic during the change of state, $p_1 \cdot V_1^{\kappa} = p \cdot V^{\kappa}$ i.e., during compression. From this we express the pressure $p = \frac{p_1 \cdot V_1^{\kappa}}{V^{\kappa}}$, which we substitute for the expression of physical work.

$$W_{F12} = -\int_{V_1}^{V_2} p \cdot dV = -\int_{V_1}^{V_2} \frac{p_1 \cdot V_1^{\kappa}}{V^{\kappa}} \cdot dV = -p_1 \cdot V_1^{\kappa} \cdot \left[\frac{V^{1-\kappa}}{1-\kappa}\right]_{V_1}^{V_2} = -\frac{p_1 \cdot V_1^{\kappa}}{1-\kappa} \cdot \left[V_2^{1-\kappa} - V_1^{1-\kappa}\right] =$$

$$= -\frac{6 \cdot 10^5 \cdot 0{,}001^{1,4}}{1-1{,}4} \cdot \left[0{,}003596^{1-1,4} - 0{,}001^{1-1,4}\right] = -600{,}9\ Nm$$

A negative sign means that the system is losing work, so the process is produced for the environment.

The technical work is in one stroke $W_{T12} = \int_{p_1}^{p_2} V \cdot dp$.

Let us assume that it is adiabatic during the change of state, $p_1 \cdot V_1^{\kappa} = p \cdot V^{\kappa}$ during compression. Express this volume $V = V_1 \cdot \left(\frac{p_1}{p}\right)^{\frac{1}{\kappa}}$, which is replaced by the expression of technical work

$$W_{T12} = \int_{p_1}^{p_2} V \cdot dp = \int_{p_1}^{p_2} \frac{V_1 \cdot p_1^{\frac{1}{\kappa}}}{p^{\frac{1}{\kappa}}} \cdot dp = V_1 \cdot p_1^{\frac{1}{\kappa}} \cdot \left[\frac{p^{1-\frac{1}{\kappa}}}{1-\frac{1}{\kappa}}\right]_{p_1}^{p_2} = -\frac{\kappa \cdot V_1 \cdot p_1^{\frac{1}{\kappa}}}{\kappa - 1} \cdot \left[p_2^{1-\frac{1}{\kappa}} - p_1^{1-\frac{1}{\kappa}}\right] =$$

$$= \frac{1{,}4 \cdot 0{,}001 \cdot \left(6 \cdot 10^5\right)^{\frac{1}{1{,}4}}}{1{,}4 - 1} \cdot \left[\left(10^5\right)^{\left(1-\frac{1}{1{,}4}\right)} - \left(6 \cdot 10^5\right)^{\left(1-\frac{1}{1{,}4}\right)}\right] = -\,841{,}39\,\mathrm{Nm}$$

A negative sign means that the system is losing work, so the process is produced for the environment.

Verify that the result is consistent with the relationship between physical and technical work!

$$W_{T12} = W_2 - W_1 + W_{F12}$$

$$-841{,}29\,\mathrm{J} \equiv 359{,}69 - 600 - 600{,}9 = -841{,}39\,\mathrm{J}$$

The difference is due to rounding during the calculation.

The enthalpy change

The enthalpy changes in the gas turbine can easily be calculated by calculating the temperature rise and mass.
First, calculate the mass of air into the turbine, which will require the density of the inlet air. This general gas law provides:

$\frac{p}{\rho} = RT$. Applied to state 1 $\rho_1 = \frac{p_1}{R \cdot T_1} = \frac{6 \cdot 10^5}{287 \cdot (273 + 400)} = 3{,}106 \frac{kg}{m^3}$

So we can calculate the mass of inlet air.

$$m_1 = \rho_1 \cdot V_1 = 3{,}106 \frac{kg}{m^3} \cdot 0{,}001 m^3 = 0{,}003106\,kg$$

In the second step, we must calculate the exit temperature T_2.
For adiabatic change of state, the relationship between temperature and pressure has already been calculated:

$$\frac{p_1}{T_1^{\frac{\kappa}{\kappa-1}}} = \frac{p_2}{T_2^{\frac{\kappa}{\kappa-1}}}$$

From this, the outlet temperature is set: $T_2 = T_1 \cdot \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} = 693 \cdot \left(\frac{1}{6}\right)^{\frac{1,4-1}{1,4}} = 403,35\,K$

Changes in enthalpy:

$$H_{12} = m \cdot c_p \cdot (T_2 - T_1) = 0,003106\,kg \cdot 1006 \frac{J}{kg \cdot K} \cdot (403,35 - 693) = -841,8\,J$$

Our result shows that the enthalpy change is the same way the technical work. According to the primary system I prescribed for the open system

$$H_{12} = Q_{12} + W_{T12}$$

this must be met if there is no thermal communication with the environment.

2.3 Second theorem of thermodynamics

The second primary law sets the direction of spontaneous heat processes. There are several seemingly distinct wording.
Clausius's Formulation (1850): There is no process in nature that heat would voluntarily go from a colder body to a warmer body without external work. Only reverse processes are possible.
Kelvin-Planck's formulation (1851, 1903): In nature, there is no process in which a body loses heat and this transforms itself into heat. It could be a ship that would attract heat from the water of the sea and drive itself with the extracted heat energy. This does not contradict the fact that energy does not get wasted yet it is not feasible.

Such a machine is called the second perpetuum mobile, so there is no second perpetuum mobile. The two terms are mutually consistent, but their derivation is not quite simple.
These and similar terms of the Second Basic Law are embarrassing because they deny the existence of something other than the laws of physics that establish connections. For a better formulation a new concept has been introduced: entropy. The second basic law of thermodynamics can be formulated using entropy as follows: for spontaneous processes, the entropy of the systems left alone can not be reduced.

2.3.1 Entropy

Entropy is quite complex and relatively difficult to understand.
In the following I will quote some wording:
"Entropy is an important concept of science (primarily heat and information technology), characterized by the degree of disorganization of a system. The concept of entropy was introduced by Rudolph Clausius (1822-1888), and in this way characterized the molecular disorder of the financial systems and their degree of thermodynamic probability in thermodynamics. From this it can be inferred from the process of self-perceived events: more and more likely states occur in nature. For example, the heat flows from the warmer body to the colder body. So a certain amount of work is lost in any spontaneous processes, transforming into heat. For this reason spontaneous processes are irreversible in nature. Work, but any kind of energy can be completely converted to heat, while heat can only partially be transformed into another kind of energy (which is why it is considered to be lower energy). The equivalence of entropy and disorder is, in principle, still apparent in thermodynamics, but Erwin Schrödinger has finally made it clear in terms of life affinities. Later, according to

formal similarity, János Neumann suggested to Shannon that his formula be called entropy. But since there was a negative sign before the formula, its name (formerly antientropic) has become negentropic, which expresses the degree of orderlyness of the systems.
"In order to minimize possible misunderstanding, I summarize what I mean by Clausius thermodynamics. Clausius's thermodynamics in Clausius's original formulation is the Thermodynamics II. The term "heat never goes from the cooler place to the warmer place", ie the temperature difference never increases, it always decreases. In the Clausius structure, heat is the distinct concept, and in the mathematical formulation of thermodynamics entropy (S)

$$dS = \frac{dQ}{T} \qquad 2.11$$

introduced into equation.

That is, the task of building theory is to prove the existence and uniqueness of the integrating divider. An important result of thermodynamics is the existence of the absolute entropy and the absolute temperature scale, that is, T is the integrator of heat.

"The second primary law in this formulation is very complicated, but in a simple form it becomes understandable: The disappearance of temperature differences is a very ordinary experience. Why do you have to say that? His colleague, Gustav Zeuner, Clausius also blamed him by saying hat the proposed II. mainstream was banality. On this basis, as a natural law, we can say that the river always flows down. Gustav Zeuner was partly right. It's natural that hot coffee cools down in the room, and ice melts. Kids usually understand the Jean joke:

- How many degrees are inside here?

- 16

- and outside?

- 6

- Get that 6 too!

An item similar to the second term of thermodynamics could in fact be formulated in mechanics. For example, every body (weaker than air) falls down on Earth. If we want to raise it, then we have to work.

But let's try to understand how the entropy can be used to reformulate temperature equalization.

Another formulation of the second major article is that the totality of a closed system can not be reduced.

Or in mathematical writing

$$\sum \frac{dQ}{T} \geq 0 \qquad 2.12$$

To better illuminate, I quote the wording of the Pallas lexicon for "The Thermodynamics II. the entropy of a closed system should not be reduced. If the process in the closed system is reversible (reversible), entropy remains constant. If the process can not be translated (irreversible) and processes that are naturally occurring, the entropy of the system increases.

Entropy is a degree of disorder. If entropy increases, disorder increases too. When the maximum of entropy is achieved, a total resting state, total equilibration, total dispersion occur. The degree of disorder is as great as possible, there is no grouping or organization in the system. According to the heat-death theory, the universe as a whole is inevitably in the state of maximum entropy, that is, of full equilibrium. However, based on our present knowledge in 1996: it is not possible to force when When the maximum of entropy is achieved, a total resting state, total equilibration, total dispersion occur.

The entropy of open systems interacting with the environment can be constant, "v" may even decrease if the entropy of the environment is adequately increased. For example, their living being, growth and reproduction are accompanied by entropy reduction processes, but at the same time, processes with greater entropy, which cause decay in their environment, are taking place. So the entropy of the system of living and its surroundings is growing. - Entropy as the degree of disorder and the concept of probability can be very close to each other. Roughly speaking, the entropy is the probability that the system is in a given state, and therefore the smallest entropy is the least likely state. The transformation towards the most probable conditions of entropy growth corresponds. The probabilistic interpretation makes it possible to use the term 'concept in information theory'. [Pallas VI: 186. - KL II: 21. - LThK III: 904.]

The concept of entropy introduced in thermodinamics has practical significance. This entropy is a status indicator and the flow of the thermal processes can very well be illustrated in T-diagrams. The following chapters will apply these.

Note: Such a concept could be introduced during the discussion about mechanics but we are we can do without it quite easily. Instead, we say "neglect the friction". Indeed, it can be introduced in other sciences eg. also used in computer science.

2.4 The third theorem of thermodynamics

We only mention thermodynamics III. main theorem. Nernst Heat: Any entropy change that accompanies any physical or chemical transformation will be zero if the temperature is zero: if $\Delta S \rightarrow 0$, if $\Delta T \rightarrow 0$

If the entropy of all elements is taken to zero $T = 0$ in the stable state, each substance has a positive entropy, $T = 0$ which may become zero and will certainly be zero for all the perfect crystalline substances, including compounds.

Summarizing thermodynamics

I. The internal energy of the closed system is constant until it is changed by work or heat exchange.

II. An entropy of an isolated system increases in a voluntary process.

III. a. / The entropy of all elements is at zero in steady state, so the entropy of each material is a positive value. The entropy of the compounds will also be zero if they form a perfect crystal.

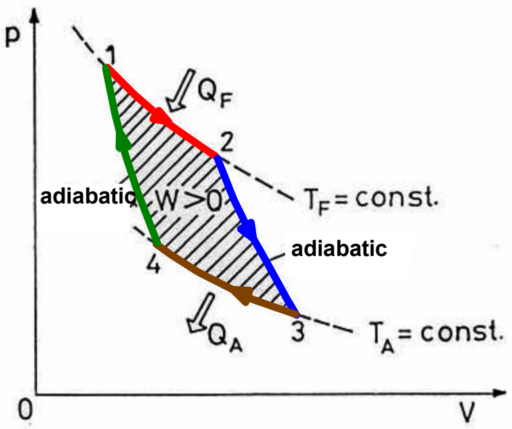
b. $T = 0$ absolute zero degree is arbitrarily accessible but never reachable.

3. Carnot cycles

3.1. Carnot-cycles

The Carnot cycle is a special thermodynamic cycle carried out by the theoretical (idealized) Carnot heat engine. The fundamental cycle of thermodynamics, written by French engineer Sadi Carnot in 1824, while trying to explain the principle of the operation of the steam engine. It consists of four stages: isothermal, then adiabatic expansion, followed by isothermal and then adiabatic compression. The Carnot machine is the largest possible thermal power engine (Carnot's law). This physical model was proposed by Nicolas Léonard Sadi Carnot in 1824 and later elaborated by Émile Clapeyron in the 1830s and 1840s.
The status of a thermodynamic system is described by status indicators (pressure, temperature, volume, enthalpy, entropy). The thermodynamic cycle is generated when the system returns to its initial state after a series of changes in state. During the round process, the system can work in the environment to operate as a heat engine. During the operation of a heat engine, energy from a warmer region of its environment can be transferred to a cooler region and can convert part of the energy into mechanical work. The cycle can be translated. By introducing an external work, the system can transfer heat energy from a colder place to a warmer location, so it can operate as a heat pump.

The Carnot cycle is the best efficient cycle that transforms a given amount of thermal energy into mechanical work or transforms a certain amount of mechanical work into heat energy for cooling purposes.



Carnot cyclus
Figure 3.1

Let's try to determine how much heat and how much work is done on each stage of the process and how much work is gained and what is the result. The fluid involved in the process should be the ideal gas.

3.1.1 Isothermal expansion is 1-2 stages

Apply the first theorem of the thermodynamics to the closed system in section 1-2. On the 1-2 stage there isothermal expansion (internal energy does not change).

$$\Delta U_{12} = Q_{12} + W_{12} = Q_{12} - \int_{V_1}^{V_2} p \cdot dV = 0$$

$$Q_{12} = Q_F = -W_{12} \qquad 3.1$$

Apply the Clapeiron equation of the ideal gases:

$$p \cdot V = m \cdot R \cdot T$$

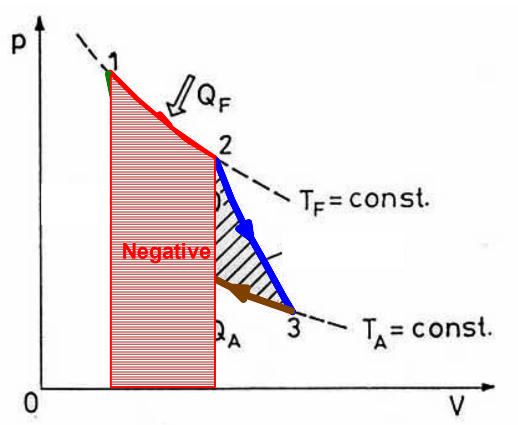
For the isothermal state change the temperature does not change, so you can write that $p \cdot V = \text{const.}$ Expressing this pressure

$$p = \frac{\text{const.}}{V}$$

Calculate the work done in section 1-2.

$$W_{12} = -\int_{V_1}^{V_2} pdV = -\int_{V_1}^{V_2} \frac{\text{const.}}{V} dV = -[\text{const.} \cdot \ln V]_{V_1}^{V_2} = -p_1 V_1 \cdot \ln \frac{V_2}{V_1} = -m \cdot R \cdot T_1 \cdot \ln \frac{V_2}{V_1} \qquad 3.2$$

Here the minus sign means that the closed system gives you work, that is, the system is lost, the environment, that is, the user gets work!



Work on sections 1-2
Figure 3.2

Since the heat absorbed by the system is the same as that of the isothermal state change, the heat

$$Q_{12} = Q_F = -W_{12} = m \cdot R \cdot T_1 \cdot \ln \frac{V_2}{V_1} \qquad 3.3$$

Let's look at the next section of the cycle.

3.1.2 Adiabatic expansion is 2-3 stages

Apply the first theorem of the thermodynamics to the closed system in section 2-3. In section 2-3, adiabatic expansion occurs (no reported or abstract heat $Q_{23} = 0$).

$$\Delta U_{23} = Q_{23} + W_{23} = -\int_{V_2}^{V_3} p \cdot dV$$

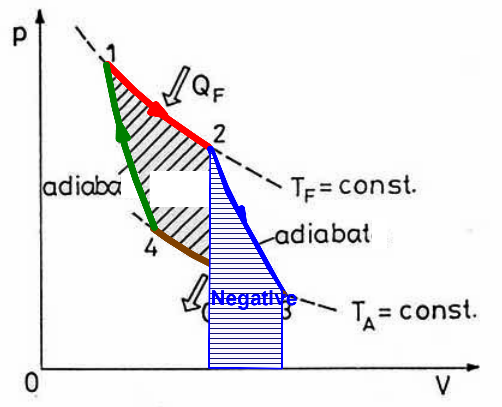
$$\Delta U_{23} = W_{23} \qquad 3.4$$

It is easier to get the result if the internal energy change of the ideal gases varies proportionally with temperature change, so

$$\Delta U_{23} = m \cdot c_v \cdot \Delta T_{23}$$

$$W_{23} = m \cdot c_v \cdot (T_3 - T_2) \qquad 3.5$$

Here, too, the system loses work (temperature 2 is higher than 3 so the result will be negative) environment, ie. the user gets work!



Work on stage 2-3
Figure 3.3

3.1.3 Isothermal compression in 3-4 stages

Let us also apply the first theorem of thermodynamics to the closed system in section 3-4. On the 3-4 segment isothermal compression takes place (the internal energy does not change.) The task is analogous to the 1-2 stage, only the signs are to be watched!

$$\Delta U_{34} = Q_{34} + W_{34} = Q_{34} - \int_{V_3}^{V_4} p \cdot dV = 0 \qquad 3.6$$

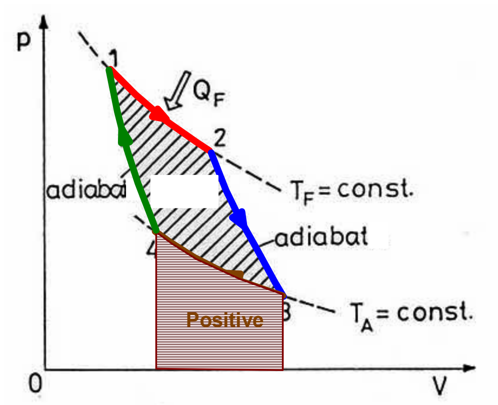
$$Q_{34} = Q_A = -W_{34} \qquad 3.7$$

Q_A the heat delivered at the lower temperature.

$$W_{34} = -\int_{V_3}^{V_4} p dV = -\int_{V_3}^{V_4} \frac{const.}{V} dV = -[const. \ln V]_{V_3}^{V_4} = -p_3 V_4 \cdot \ln \frac{V_4}{V_3} =$$

$$= -m \cdot R \cdot T_3 \cdot \ln \frac{V_4}{V_3} = m \cdot R \cdot T_3 \cdot \ln \frac{V_3}{V_4}$$

Because in volume 4 the volume is smaller than in state 3, its $\ln \frac{V_4}{V_3}$ value is negative. So on stage 3-4, the system gains the same amount of heat!



Work included in section 3-4
Figure 3.4

3.1.4 Adiabatic compression in section 4-1

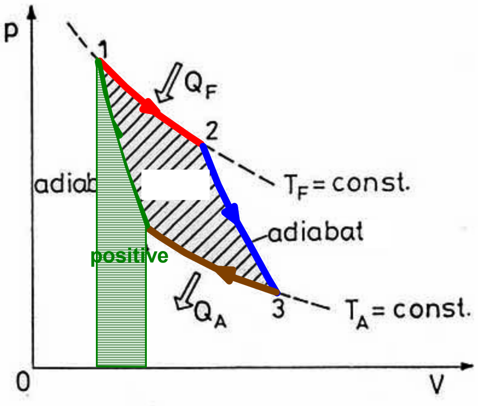
Apply the first theorem in section 4-1 similar way like in section 2-3, adiabatic compression takes place (no reported or abstract heat $Q_{41} = 0$).

$$\Delta U_{41} = Q_{41} + W_{41} = -\int_{V_4}^{V_1} p \cdot dV$$

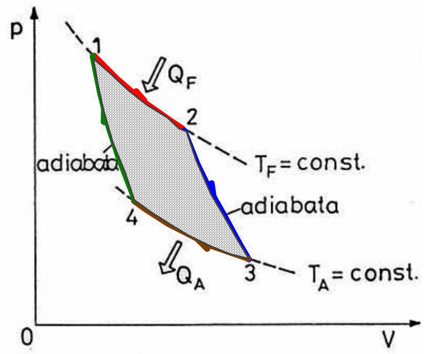
$$\Delta U_{41} = W_{41} \qquad 3.8$$

$$W_{41} = m \cdot c_v \cdot (T_1 - T_4) \qquad 3.9$$

Here, too, the system gets work (the temperature 1 is higher than the 4, so the result will be positive) the environment, that is, the user will do the work!



Work included in Section 4-1
Figure 3.5



The result work of the cycle
figure 3.6

Summarize the physical work that is gained or delivered by the system to the entire cycle.

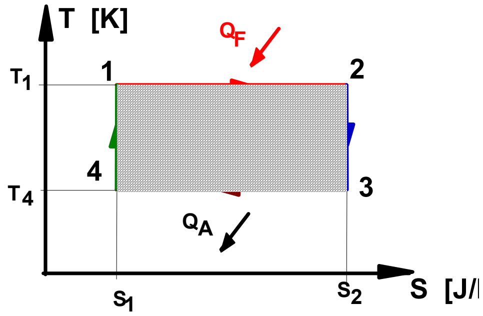
$$\sum W_{12} + W_{23} + W_{34} + W_{41} = -m \cdot R \cdot T_1 \cdot \ln\frac{V_2}{V_1} - m \cdot c_v \cdot (T_2 - T_3) + m \cdot R \cdot T_3 \cdot \ln\frac{V_3}{V_4} + m \cdot c_v \cdot (T_1 - T_4)$$

Perform the operation graphically:

Overall, the system gives work to the environment. That is, work can be extracted from the cycle by heat input. In other words, the heat engine can produce mechanical work from heat energy.

3.2. Carnot cycle efficiency

In the previous chapter, we introduced the concept of entropy, which can be shown to be a status indicator.
The Carnot cycle is much simpler to represent in a T-diagram, with an absolute temperature, an entropy diagram. Because of the two constant temperature and two adiabatic state changes, in this diagram, a rectangle is the result of the round process.



The work of the Carnot cycle is in T-s diagram
Figure 3.7

Definition of entropy, more precisely its elemental change $dS = \frac{dQ}{T} \left[\frac{J}{K}\right]$. This means that if there is no heat exchange, the entropy will not change. Therefore adiabatic state changes are vertical lines in the diagram.
Negotiating the Carnot Circuit is much more spectacular on the T-S chart. Two processes are isothermal, i.e their image is two horizontal lines; the two processes are adiabatic ($Q = 0 \rightarrow \Delta S = 0$), i.e their picture is two vertical lines. The full circle on the T-S diagram is a rectangle. During this cycle, the area of this rectangle is the useful work done by the gas.

Express the heat to each section. The amount of heat introduced at the upper temperature $Q_F = (S_2 - S_1) \cdot T_1$ during isothermal expansion, as illustrated by the upper T_1 horizontal rectangle.
The amount of heat delivered at the lower temperature during isothermal compression, as illustrated by the lower rectangle $Q_A = (S_2 - S_1) \cdot T_4$ under the lower T4.
The difference between the two rectangles is the work done, or, in other words, the area around is proportional to the work done.

$$W = (S_2 - S_1) \cdot (T_1 - T_4)$$

The area of the rectangle will be bigger, the higher the heat input and the lower the heat withdrawal temperature. The efficiency of the cycle is typically characterized by the proportion of the work carried out relative to the amount of heat input. Also known as Carnot efficiency.

$$\eta = \frac{W}{Q_F} = \frac{(S_2 - S_1) \cdot (T_1 - T_4)}{(S_2 - S_1) \cdot T_1} = \frac{T_1 - T_4}{T_1} = 1 - \frac{T_4}{T_1} \qquad 3.10$$

More Generic Expression:

$$\eta = 1 - \frac{T_{lowerr}}{T_{lhigher}} \qquad 3.11$$

3.2.1. The Carnot cycle (reversible) cycle

According to the second theorem of thermodynamics, if only reversible (reversible) processes occur in the system, the totality of the system does not increase. In the above Carnot cycle we try to prove this. In section 1-2, heat is communicated to the system, basically on an infinite high heat exchanger surface, with a zero temperature stage, which will cause reversible state change. The amount of heat delivered is Q_1. According to equation 3.3

$$Q_{12} = Q_F = -W_{12} = m \cdot R \cdot T_1 \cdot \ln \frac{V_2}{V_1}$$

Stages 2-3 and 4-1 are adiabatic for state changes, so there is no heat transfer and no entropy change. On the 3-4 section isothermal compression is performed and the heat delivered is based on equation 3.7

$$Q_{34} = Q_A = -W_{34} = -m \cdot R \cdot T_3 \cdot \ln \frac{V_3}{V_4}$$

Calculate the change in the totality of the cycle during the cycle

$$\sum \Delta S_{1234} = \frac{Q_F}{T_1} + 0 + \frac{Q_A}{T_4} + 0 = \frac{m \cdot R \cdot T_1 \cdot \ln \frac{V_2}{V_1}}{T_1} + 0 + \frac{-m \cdot R \cdot T_3 \cdot \ln \frac{V_3}{V_4}}{T_3} + 0 = m \cdot R \left[\ln \frac{V_2}{V_1} - \ln \frac{V_3}{V_4} \right] \qquad 3.12$$

We need to see expression obtained that is actually zero.
To do this, we must use that the state changes in 2-3 and 4-1 are adiabatic changes. So $p_2 \cdot V_2^\kappa = p_3 \cdot V_3^\kappa$ and $p_4 \cdot V_4^\kappa = p_1 \cdot V_1^\kappa$ as well.
Express V_3 volume with V_2 and V_4 volume with V_1 from the above two equations.

$$V_3 = V_2 \left(\frac{p_2}{p_3} \cdot \right)^{\frac{1}{\kappa}} \qquad V_4 = V_1 \left(\frac{p_1}{p_4} \cdot \right)^{\frac{1}{\kappa}}$$

Replace this in equation 3.12

$$\sum \Delta S_{1234} = m \cdot R \left[\ln \frac{V_2}{V_1} - \ln \frac{V_3}{V_4} \right] = m \cdot R \left[\ln \frac{V_2}{V_1} - \ln \left\{ \frac{V_2}{V_1} \cdot \left(\frac{p_2}{p_3} \cdot \right)^{\frac{1}{\kappa}} \cdot \left(\frac{p_4}{p_1} \cdot \right)^{\frac{1}{\kappa}} \right\} \right]$$

Now you just have to see that $\left(\frac{p_2}{p_3} \cdot \right)^{\frac{1}{\kappa}} \cdot \left(\frac{p_4}{p_1} \cdot \right)^{\frac{1}{\kappa}} = 1$.

Raise the equation for κ power at that time $\frac{p_2}{p_3} \cdot \frac{p_4}{p_1} = 1$ we got this formula.

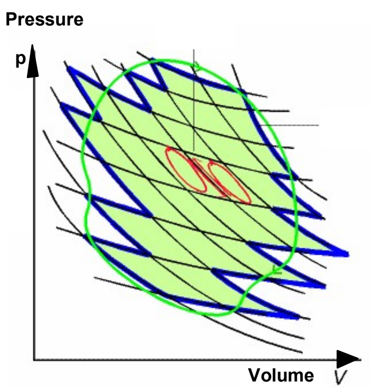
Use Equation 1.23 between 2-3 points $\frac{p_2}{T_2^{\frac{\kappa}{\kappa-1}}} = \frac{p_3}{T_3^{\frac{\kappa}{\kappa-1}}}$ and 4-1 $\frac{p_4}{T_4^{\frac{\kappa}{\kappa-1}}} = \frac{p_1}{T_1^{\frac{\kappa}{\kappa-1}}}$.

Replaced and sorted: $\frac{T_2^{\frac{\kappa}{\kappa-1}}}{T_3^{\frac{\kappa}{\kappa-1}}} \cdot \frac{T_4^{\frac{\kappa}{\kappa-1}}}{T_1^{\frac{\kappa}{\kappa-1}}} = 1$. This term exists because $T_1 = T_2$ és $T_3 = T_4$.

So we proved that at the end of the closed reversible cycle, the totality change is zero.

3.3 General Circuits

The caloric machines work with different circuits. The clear Carnot cycle is just approaching. But it can be proven that all major cycles can be divided into smaller Carnot cycles.



General cycle resolution for Carnot cycles
Figure 3.8

The roads within the curve are equalizing, so only the circumference should be considered. The more subtle cycles there are, the better the approximation of the big circle is.
Since the entropy change is zero in all the individual Carnot cycles, so integrating the entropy around the curve is zero.

3.3 Reverse Carnot cycles

The ideal cooling cycle
If the cycle takes place in the opposite direction (indirect or reverse cycle), then W is the work that the external forces perform on the medium. As a result, Q_L takes heat from the lower-temperature heat reservoir and Q_A releases heat to the higher temperature heat reservoir.
The refrigerator takes heat from the cooler space to the warmer environment, therefore its goodness factor, its degree of goodness is more than one. For refrigerators, the ratio that is interpreted as the quotient of the extracted heat (Q_{Lower}) and the input labor (W) is a goodness factor.

$$\varepsilon = \frac{Q_{Lower}}{W} = \frac{(S_2 - S_1)\cdot T_4}{(S_2 - S_1)\cdot (T_1 - T_4)} = \frac{T_4}{T_1 - T_4} = \frac{T_{lower}}{T_{Upper} - T_{Lower}} \quad 3.13$$

The ideal heat pump cycle
The heat pump transfers heat from a colder space to a warmer location, so it is a well-known factor or well-known COP (Coefficient of Performance):

$$COP = \frac{Q_{Upper}}{W} = \frac{(S_2 - S_1)\cdot T_1}{(S_2 - S_1)\cdot (T_1 - T_4)} = \frac{T_1}{T_1 - T_4} = \frac{T_{Upper}}{T_{Upper} - T_{Lower}} \quad 3.14$$

(Here is the heat delivered, which we use is more than one number)

The expression of the heat pumps COP and the degree of cooling of the refrigerators are not independent. Their contact is as follows:

3.15

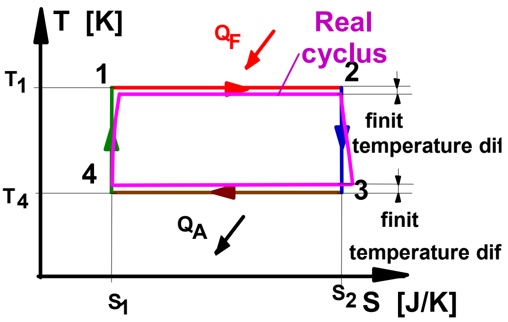
$$COP = \frac{T_{Upper} - T_{Lower} + T_{Lower}}{T_{Upper} - T_{Lower}} = 1 + \frac{T_{Lower}}{T_{Upper} - T_{Lower}} = 1 + \varepsilon$$

For more information on refrigerators and heat pumps, see **Chapter 5**.

3.4 Reversible and irreversible cycles

The process is **reversible** if, after the process is completed, the system moves in the opposite direction to the system through the same intermediate states without its other environmental changes.
The process is **irreversible** if it is not reversible. If the system can not reverse trouhg the same intermediate states without any change in its environment.
Notes
Every real process is irreversible.
The idealized processes mentioned above may be reversible.
Internal comvistion engines are irreversible.

The heat exchange takes place between the environment and the gas in the system with a finite difference in temperature. If the environment gets heat, the gas in the system must be warmer than the environment. If the heat exchange is to be carried out backwards, that is, if the environment emits heat to the gas in the system, then the environment must be warmer than the gas.
Another common irreversibility is adiabatic compression or expansion. The adiabatic, heat-exchange-free process can usually be achieved by a good approximation, but the frictional heating can not be eliminated systematically. Therefore friction warms the medium and thus increases entropy. This means that both compression and expansion do not slip in the vertical direction, but bend to the right, causing the growth of entropy. Thus, the entire cycle will not be reversible. We can not prove it here. But like the reversibility you can see the statement. Real cycle processes are always irreversible.

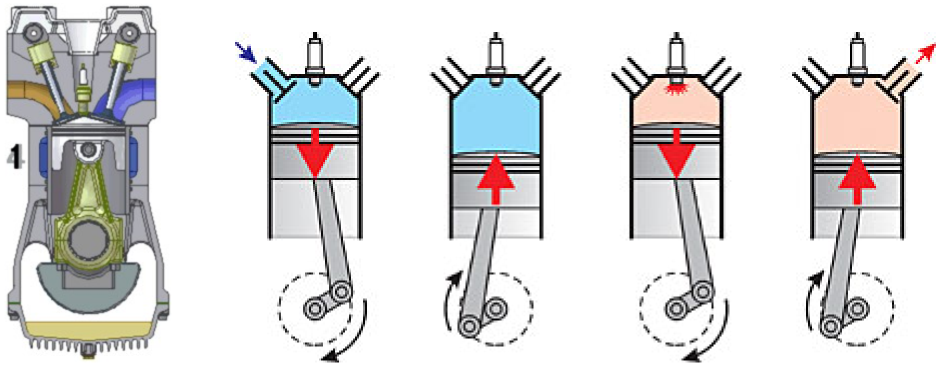


Irreversibles in a realistic circle
Figure 3.8

4. Different cycles

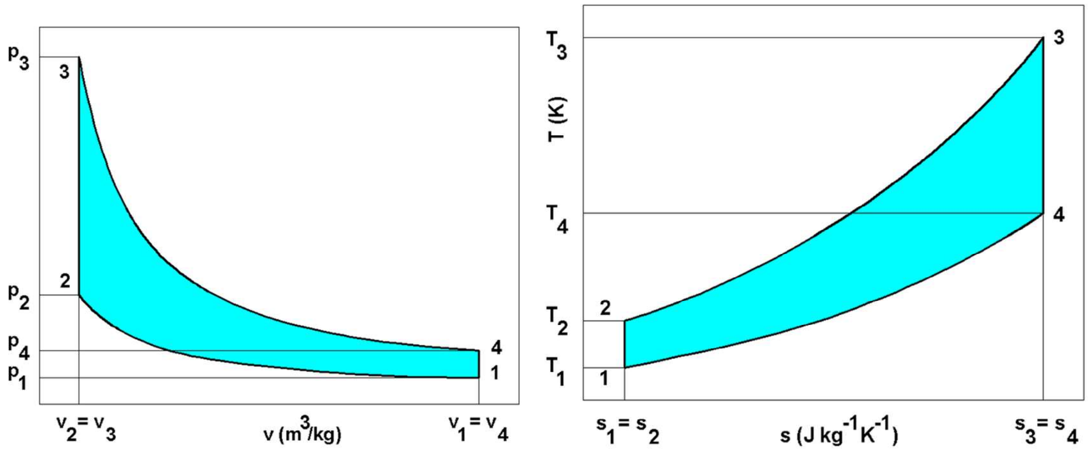
4.1. Otto cycles

The Otto-engine was the first four-stroke internal combustion engine. Made by Nikolaus August Otto in 1876. On the global scale, this internal combustion engine is the most popular and is the most fuel-efficient petrol engine.



Operation of the Otto engine
Figure 4.1

The operation of the engine can be divided into four stages. The first stroke is suction: the piston moves downward in the cylinder, while the air suction mixture is blown through the open suction valve or the more modern types only air flows into the cylinder space. At the second stroke, the plunger moves upwards, compresses the air mixture or only the air and all the valves are closed. At the start of the third stage, the compressed air fuel mixture is ignited by the candle (in the case of modern engines the fuel is injected into the combustion chamber before the spark). The pressure of the gas heated by the rapid combustion increases and then starts moving the plunger downwards; this is the rate of expansion or working stroke. In the fourth step, the piston moves upwards, the exhaust valve is open and the combustion products leave the cylinder. The cycle is restarted with a fresh gas mixture. The idealized process consists of two adiabatic and two isochoric processes. The p-V diagram of the cycle is shown in **Figure 4.2**. The first step is 0-1. There is no need for this section to discuss the cycle, so it is not always shown on the chart. The second stage corresponds to section 1-2, which is adiabatic compression. The third stage includes the isochoric state change in the 2-3 stage and the 3-4 adiabatic expansion section. The fourth stage is the isochoric phase (cooling) and stage 1-0 (exhaust) on a constant volume of 4-1, which are not always represented in the round. During the cycle, the heat takes place in sections 2-3, and the heat release is in section 4-1. During the discussion so far, we have not yet commented on the isochronous state change in the T-s diagram over a constant volume. This is a steeper power function. The pressure constant curves are also power functions, only a bit flatter in the isochoric lines. Later on, these will be discussed.



The Otto theoretical cycle in p-v and T-s diagrams
Figure 4.2

Examine the circle process in the T-diagram.
The heat inlet is carried out at the constant volume at section 2-3. Since there is no work on the first high-grade, it is possible to write that the heat injected increases the internal energy of the house. Assuming ideal gas, this can be written in the following format:

$$q_{be} = \Delta u_{32} = c_v \cdot (T_3 - T_2) \qquad 4.1$$

Heat absorption in section 4-1 also at constant volume.

$$q_{el} = \Delta u_{41} = c_v \cdot (T_4 - T_1) \qquad 4.2$$

Phases 1-2 and 3-4 are adiabatic changes in the state, there is no heat intake or heat outtake. The work carried out is similar to the Carnot circle. This cycle can be split into the sum of the elementary Carnot cycles, hence the above statement.

$$q_{be} - q_{el} = c_v \cdot (T_3 - T_2) - c_v \cdot (T_4 - T_1) \qquad 4.3$$

The efficiency of the cycle can also be calculated similarly to the Carnot cycle.

$$\eta_{Otto} = \frac{q_{be} - q_{el}}{q_{be}} = \frac{c_v \cdot (T_3 - T_2) - c_v \cdot (T_4 - T_1)}{c_v \cdot (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \qquad 4.4$$

There is an adiabatic change of state between 1-2 and 3-4, therefore

$$T_1 \cdot v_1^{\kappa-1} = T_2 \cdot v_2^{\kappa-1} \text{ and } T_4 \cdot v_4^{\kappa-1} = T_3 \cdot v_3^{\kappa-1}$$

Divide the two equations together

$$\frac{T_4}{T_1} \cdot \left(\frac{v_4}{v_1}\right)^{\kappa-1} = \frac{T_3}{T_2} \cdot \left(\frac{v_3}{v_2}\right)^{\kappa-1}$$

Use it because of the two isoforms due to the change of state $v_1 = v_2$ and $v_3 = v_4$.

$$\frac{T_4}{T_1} \cdot \left(\frac{v_4}{v_1}\right)^{\kappa-1} = \frac{T_3}{T_2} \cdot \left(\frac{v_4}{v_1}\right)^{\kappa-1}$$

Which gives rise to that

$$\frac{T_4}{T_1} = \frac{T_3}{T_2} \qquad 4.5$$

Transform *equation 4.4* to include temperature ratios and replace the 4.5 relationship.

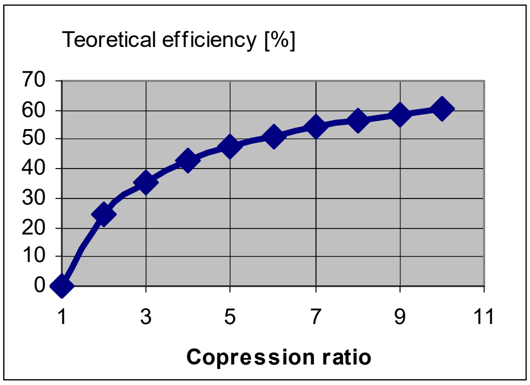
$$\eta_{Otto} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{\frac{T_4}{T_1} - 1}{\frac{T_3}{T_2} - 1} \cdot \frac{T_1}{T_2} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{v_2}{v_1}\right)^{\kappa-1}$$

For further conversion of efficiency, introduce the compression relationship:

$$\varepsilon = \frac{v_1}{v_2} \qquad 4.6$$

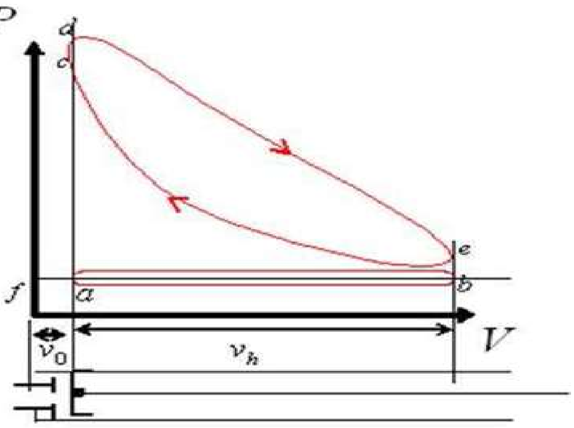
$$\eta_{Otto} = 1 - \left(\frac{v_2}{v_1}\right)^{\kappa-1} = 1 - \frac{1}{\left(\frac{v_1}{v_2}\right)^{\kappa-1}} = 1 - \frac{1}{\varepsilon^{\kappa-1}} \qquad 4.7$$

Efficiency increases by increasing the compression ratio. **Figure 4.3** shows the change in theoretical efficiency depending on the compression ratio.



Theoretical efficiency in relation to the compression ratio
Figure 4.3

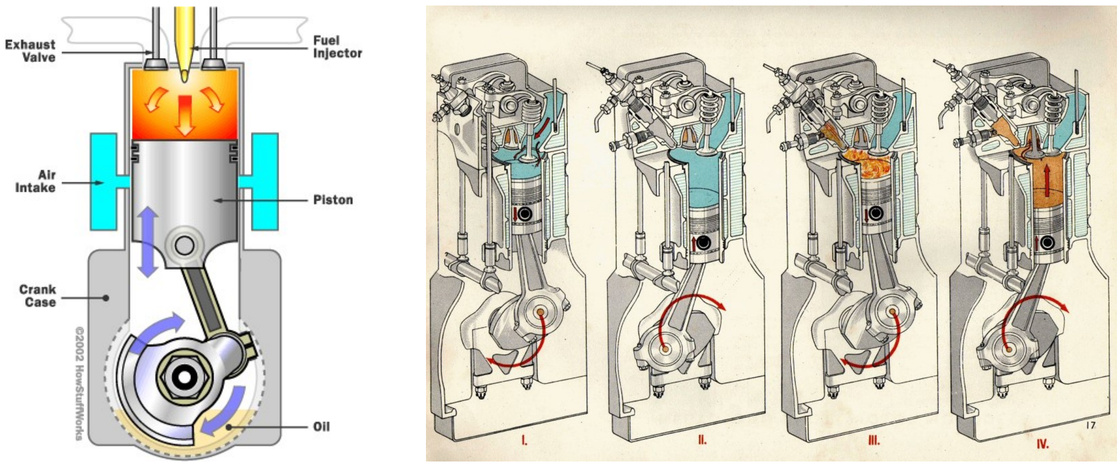
This is limited by the ignition of the fuel-air mixture, which should be avoided. For gasoline engines the compression ratio is 1:7 to 1:12. The efficiency of real engines is lower than the calculated value. In the case of petrol engines, the compression pressure is 12-17 bars, the combustion peak pressure is 40-60 bars, and the combustion peak temperature is 2000-2500 ° C. The actual efficiency of the engines is 24-35%.
For real gasoline engines, the p-V diagram is a complicated curve whose mathematical discussion is complicated. **Figure 4.4** shows the actual pressure conditions, also called the indicator diagram, as a function of the stroke.



The real Otto cycle in p-V diagram (indicator diagram)
Figure 4.4

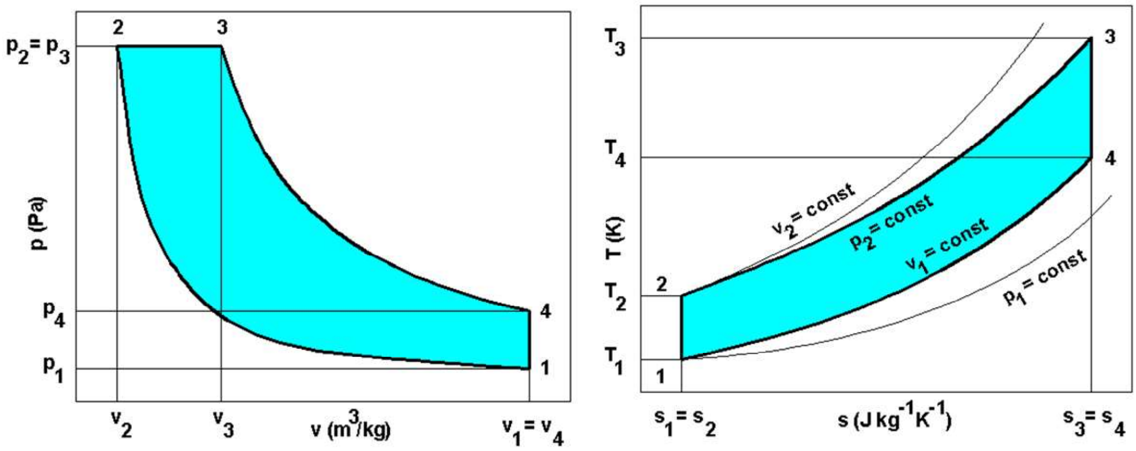
4.2. The Diesel cycle

The operation of the diesel engine (Rudolf Christian Karl Diesel, 1893) can be divided into four stages.



Diesel engine operation
Figure 4.5

The first stroke is suction: the piston moves downward in the cylinder, and air flows through the air valve through an open valve. At the second stroke, the plunger moves upwards, thickens and warms up the air, and each valve is closed. At the beginning of the third stage, diesel fuel is injected into the compressed hot air, which is ignited by the hot air and starts to push the plunger under constant pressure. Burning is slower than combustion of gas in the gasoline engine. At the end of the combustion, adiabatic expansion will pass the plunger to the lower dead center. In the fourth step, the piston moves upwards, the exhaust valve is open and the combustion products leave the cylinder. The cycle is restarted with fresh air. The idealized process consists of two adiabatic, one isobar and one isochoric process. The p-v diagram of the cycle is shown in **Figure 4.6.**



The Diesel Theoretical Circuit in p-v and T-s diagrams
Figure 4.6

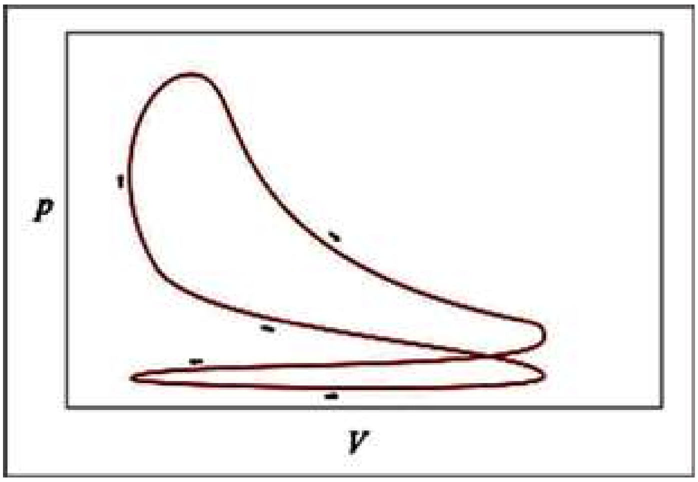
Efficiency can be derived based on principles similar to the Otto engine. Here, the situation is much more complicated as the round process consists of five sub-processes. And with the compression ratio $\varepsilon = \frac{v_1}{v_2}$, the

$$\varphi = \frac{v_3}{v_2} \qquad 4.8$$

at the end of the combustion and at the beginning of the brewing volume. Using these, the theoretical efficiency of the Diesel cycle:

$$\eta = 1 - \frac{1}{\kappa} \cdot \frac{1}{\varepsilon^{\kappa-1}} \cdot \frac{\varphi^{\kappa} - 1}{\varphi - 1} \qquad 4.9$$

Efficiency increases by increasing the compression ratio, decreasing the increase in the volume ratio ($\varphi < 1$). Compared to the Otto cycle, its efficiency is superior to the Diesel cycle when it comes to the same compression ratio. However, everybody knows the fact that the fuel consumption of a diesel engine is less (and hence its better efficiency) than that of Otto-powered cars. This is true because the compression ratio of the Otto motors is significantly lower than the diesel engines. The gasoline-air mixture would suffer from a self-infliction at a lower temperature (lower compression ratio). The other reason is that the petrol engine is controlled by throttling the air intake and the throttle causes energy loss. The third reason is that one litter of Diesel oil has more energy than a litter of gasoline. (Calculation value per kilogram: diesel $43\frac{MJ}{kg}$, petrol $43\frac{MJ}{kg}$, or per litter: diesel $36\frac{MJ}{\ell}$, gasoline $30\frac{MJ}{\ell}$). Of course, due to thermal, mechanical and other losses, the true overall effect of both engines is significantly lower than the theory.



The real Diesel cycle in the p-V diagram (indicator diagram)
Figure 4.7

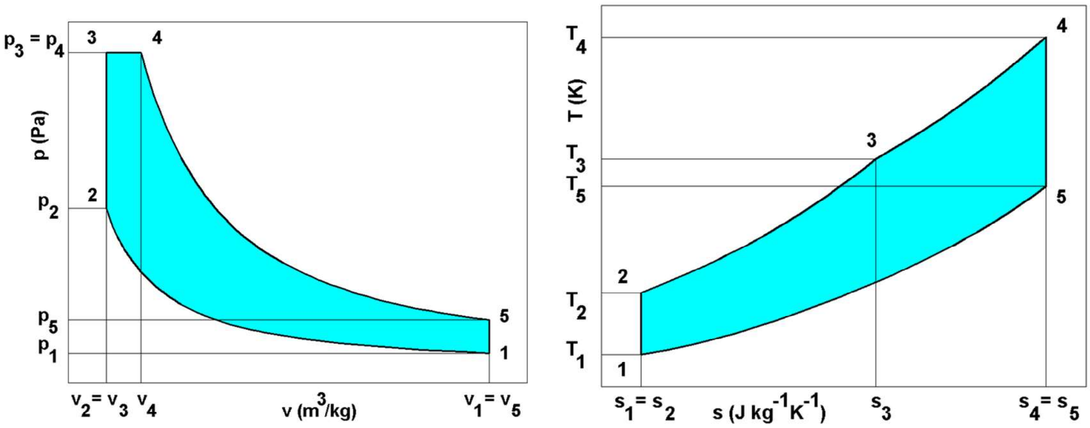
For true diesel engines, the p-V diagram resembles the curve obtained for gasoline engines but has no peaks, less angular. In case of diesel motors, the compression ratio is 1: 16-1: 22, the compression has a final pressure of 30-55 bars, the combustion peak pressure is 60-80 bars, and the combustion peak temperature is 2000-2500°C. The actual efficiency of the engines is 32-43%. One of the main reasons for the greater efficiency of diesel engines is the fact that diesel engines have much higher compression rates.

4.3. The Siegler-Sabathé Circuit

Both the Otto and the Diesel Theoretic Cycle provide some realization with some approximations. To improve the approximation, the authors offered a Seiliger-Sabathé cycle, or a mixed cycle, a dual cycle. During the process, the combustion takes place on a constant volume and at constant pressure. Thanks to this, this cycle best models the changes in the Otto motors and Diesel engines.

•

1 to 2 isentropic compressions.
- 2 to 3 constant (isochoric) state changes (first part of combustion)
- 3 - 4 constant pressure (isobar) change of state (second part of combustion)
- 4 to 5 isentropic expansion
- 5 to 1 constant volume (isochoric) heat dissipation



The Siegler-Sabathé theoretical cycle in p-v and T-s diagrams
Figure 4.8

Expression of theoretical efficiency without derivation is as follows:

$$\eta = 1 - \frac{1}{\varepsilon^{\kappa-1}} \cdot \frac{\varphi^{\kappa} \cdot \lambda - 1}{\lambda \cdot (1+\kappa) \cdot (\varphi - 1)} . \qquad 4.10$$

The meanings previously defined, expressed in the expression, mean:

$\varepsilon = \frac{v_1}{v_2}$ the compression ratio,

$\varphi = \frac{v_3}{v_2}$ the quotient of the volumetric volume at the end of the combustion and at the beginning

$\lambda = \frac{p_3}{p_2}$ pressure ratio, the quotient of the combustion pressure and initial pressure.

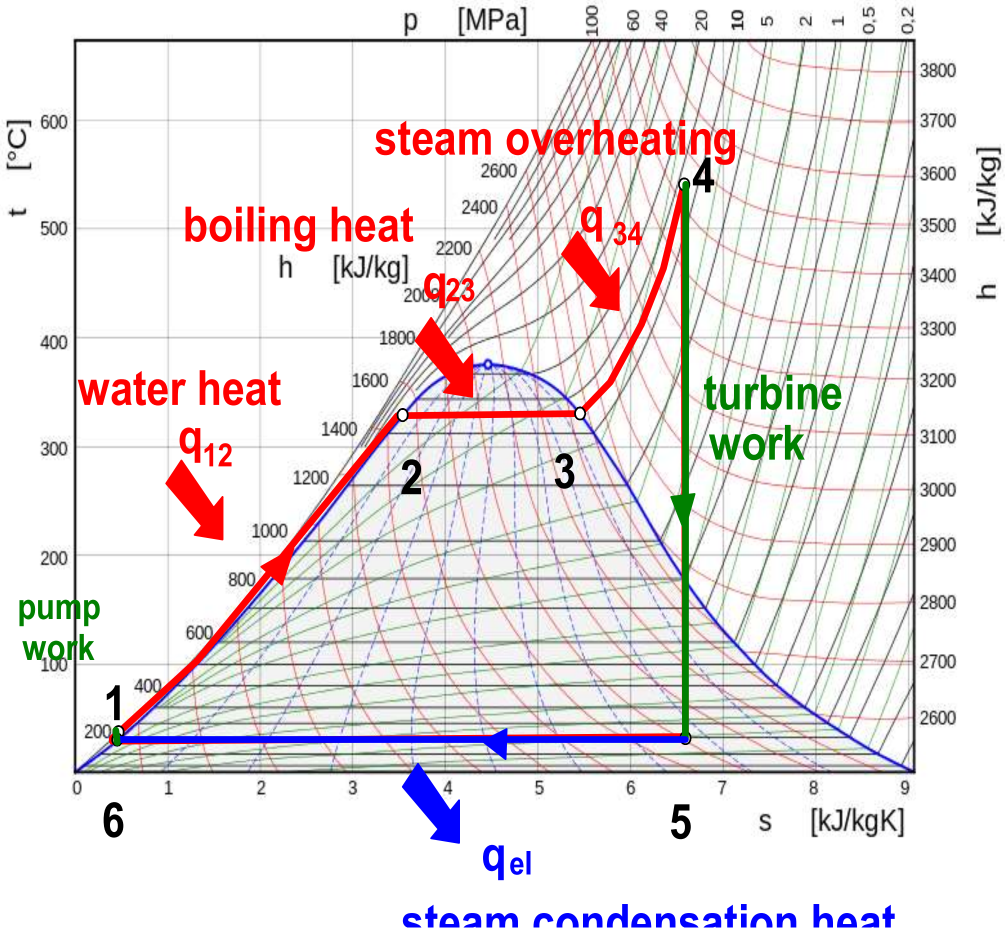
4.4. Steam cycle and steam diagrams

In the previous chapters, the continuum in the circular processes was a gas and even an ideal state of gas during each state change. Calculations can therefore easily be made because the medium is assumed to be more or less compatible with the ideal gas law. However, we had to face the unfavourable properties of the gases that change their temperature both during heat transfer and heat withdrawal.
To achieve better efficiency, we use mediums in our caloric machines that change their state of being, as the temperature does not change during the state of change of state, so theoretical possibility is a certain approach to the Carnot cycle. This means that the round process has to be moved to the area where the so-called border curves are found. In practice, water was first used for such purposes. Even when making a steam engine, they benefited from the phase

transition. In **chapter 1**, we already deal with the equilibrium diagram of the water. One of the most frequently used phases of phase change is the Claisius-Rankine water vapour cycle.

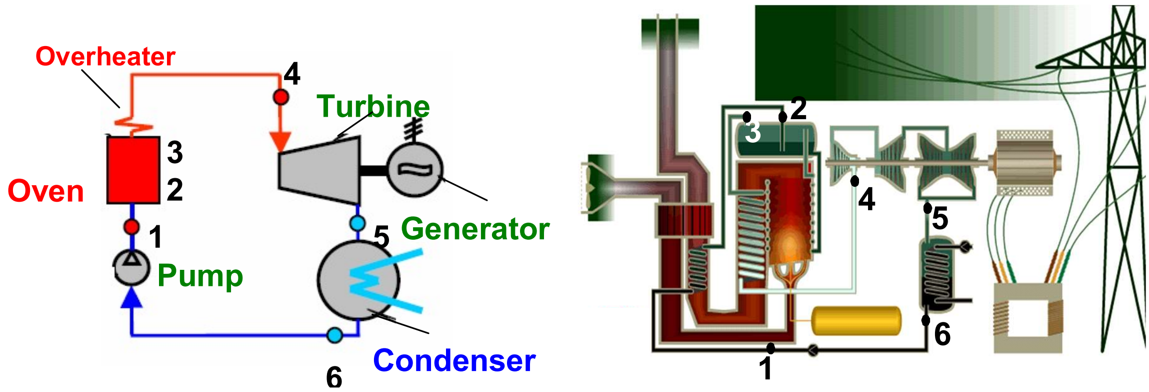
4.4.1. The Rankine-Clausius cycle

Before talking about the circle, let's say a few words about the T-diagram of water, as shown in **Figure 4.9**, with real-time data. The water does not act as an ideal gas even in the vapour state, so in the steam phase tables and charts provide the status indicators. Like the ideal gases, in the case of multi-phase systems, it is also desirable to give them the T-s diagram. In the T-s diagram of the two-phase systems, besides the usual thermal and caloric status indicators, the ratio of the phases composition to the specific vapour content (x) is also used. These are permanent composition lines. In the T-s diagram of the water-water vapour medium most practically practiced, the curves are similar in the vapour field in the steam field, moving from the boundary curve. As the ideal gas behaviour is approached by the properties of the medium, p and v are constant lines (permanent species) exponentially, and h, enthalpies constant lines become horizontal.
In the temperature entropy diagram, water and water vapour data were depicted per unit mass. On the vertical axis, the temperature in degrees Celsius can be seen on the horizontal axis by the entropy of unit weight. Under the blue curve (bell curve) there is water + steam mixed phase, water to the left of the curve and steam to the right. The right-hand scale of the diagram shows enthalpy, and the enthalpy lines inside the diagram are shown in red. They are approaching the horizontal in the overheated vapour field, with a strong upward bend in the wet area. At the top of the diagram you can see the pressure scale. In the box, these are black lines. They move horizontally in the wet box than the constant temperature lines, in the superheated steam field they raise strongly, similar to the T-diagram of the ideal gases. The steady-state volume lines are marked with green in the diagram, which are steeper than constant-pressure lines.
Dotted lines in the wet field indicate the vapour content in the wet field. Left-to-right on the left border curve is 0%, i.e. no steam is just water. Its first dashed line is 10% steam (% by mass), the second 20%, and so on. The right margin curve has 100% steam, the water is completely evaporated.
The top point of the bell curve is the critical point for water. Above the critical point there is no longer any distinction between steam and water, there is no phase change. To calculate the water vapour status indicators, a lot of computer programs have been made, from which you can accurately determine the status indicators.
After that, look at the round process. The Rankine cycle is a viable version of the Carnot cycle. The main difference is that here a pump is used to increase the pressure of liquid water. This requires about a hundred times less energy than compressing gas with a compressor than the Carnot cycle. (However, the efficiency of the Rankine cycle is not better, but worse in a Carnot cycle running within the same temperature range.) The efficiency of the Rankine cycle is usually limited by the physical characteristics of the working fluid. In order to avoid the critical value, the temperature limits are limited in water. Before entering the turbine, the steam temperature is generally 565°C (this is the temperature at which the steady flow of stainless steels does not yet occur) and the steam temperature in the condenser is about 30°C. In addition, the efficiency of the theoretically best Carnot cycle is about 63%, while the efficiency of a modern coal-fired power plant is about 42%. Because of the relatively low steam temperature, it is often used to build a combined cycle power plant at which higher temperatures are a gas turbine and at a lower temperature a Rankine cycle is used.



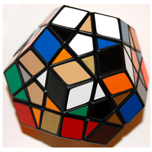
Rankine-Clausius Steam Turbine Circuit T-s diagram
Figure 4.9

The heat transfer process starts in the liquid field, from "1". Here the liquid is in an unsaturated state, i.e. its temperature is lower than the saturation temperature associated with its pressure. When the liquid is heated (at constant pressure) reaches the saturation temperature of its pressure, source "2" starts. The area under curvature "1-2" is proportional to the amount of heat required to heat the liquid.
The "2-3" curve represents the change in the state of the water, where water is poured out and steam will flow. The process takes place at constant pressure and at a constant temperature. The area beneath the curve is proportional to the hidden heat or foreign heat taken in the source, with latent heat.
The "3-4" curve, similar to the exponential curve of the gases, indicates the overheating of the steam generated. The area below the curve is the heat heater. The heat absorbed during the heat transfer is therefore the area under the three curves "1-4".
Starting from "4", the assumed expansion of high temperature (overheated) vapor is expected to be an adiabatic underneath the upper curve to a small point up to "5". Heat dissipation is a constant change of state and temperature during the condensation of the working medium (from steam back to water). The rectangular area "5-6" gives the abstract heat hidden during condensation. Finally, the work was carried out and it was conditioned, so the fluid in the liquid state again had to be pumped to the pressure of the heat transfer. Section 6-1. As the liquid is virtually uncompressed (constant pressure curves in the fluid field run in close proximity!), The amount of work required for this is practically negligible compared to the work done during the expansion.
The cycle shown in the figure is shown in the XIX. At the end of the 19th century, William John Macquorn Rankine was described by Scottish engineer and physicist for the first time and in memory of the Rankine Circuit (locally known as Rankine-Clausius Circuit). In fact, the original Rankine cycle did not include overheating.



Block iagram of the Rankine-Clausius Steam Turbine Circuit [9]
Figure 4.10

4.4.2 Clausius-Rankine circulates in T-s diagram



The data of a Rankine-Clausios cycle close to reality are in **Figure 4.9** of the water vapor t-s diagram.

Data: $p_1 = 165\,\text{bar}$; $t_1 = 35°C$; $t_2 = 330°C$; $t_4 = 545°C$; $t_5 = 30°C$; $p_5 = 0{,}035\,\text{bar}$

Questions:

a. Determine the status indicators in each section of the diagram!
b. Determine the amount of heat, heat consumed by the kilograms steam and the amount of work generated by the turbine and the pump!
c. Determine the efficiency of the circular process!

Solution: For each point in principle, the following row of data is shown in the table. (We have provided data from a computer program for accuracy)

a./

	1 point	2 point	3 point	4 point	5 point	6 point
p [bar]	130	130	130	130	130	0,05
t [C]	35	330	330	545	33	33
h [kJ/kg]	159	1527	2669	3456	2009	145
s [kJ/kg K]	0,5	3,55	5,44	6,59	6,59	0,5

Based on the above table we can calculate the amount of heat and work.
Use the I. therem of thermodynamic for Open Systems **2.10. equation** $H_{12} = Q_{12} + W_{T12}$, more precisely, its shape of this per unit of mass:

$$h_{12} = q_{12} + w_{T12} \qquad 4.11$$

b./ Based on these a

$$q_{12} = h_2 - h_1 = 1527 - 159 = 1368\frac{kJ}{kg}$$

$$q_{23} = h_3 - h_2 = 2669 - 1527 = 1142\frac{kJ}{kg}$$

$$q_{34} = h_4 - h_3 = 3465 - 2669 = 787 \frac{kJ}{kg}$$

In all three state changes, the technical work is zero because there is no pressure change, therefore the input heat equals the enthalpy change.
The heat input into the cycle is the sum of the three heat inputs:

$$q_{be} = q_{12} + q_{23} + q_{34} = 1368 + 1142 + 787 = 3297 \frac{kJ}{kg}$$

The work carried out by the turbine is an adiabatic expansion in section 4-5, while there is no heat transfer, so the enthalpy change is equal to the turbine's work.

$$w_{45} = h_4 - h_5 = 3465 - 2009 = 1456 \frac{kJ}{kg}$$

In section 5-6, the size of the extracted heat in the condenser

$$q_{el} = h_5 - h_6 = 2009 - 145 = 1854 \frac{kJ}{kg}$$

$$q_{34} = h_4 - h_3 = 3465 - 2669 = 787 \frac{kJ}{kg}$$

Pumped work on section 6-1 is also an adiabatic change, but it is compression.

$$w_{61} = h_1 - h_6 = 159 - 145 = 14 \frac{kJ}{kg}$$

c./ The efficiency of the round process is described in **Section 3.10**. equation:

$$\eta = \frac{Q_F - Q_A}{Q_F} = \frac{Q_{be} - Q_{el}}{Q_{be}} = \frac{3297 - 1854}{3297} = 0{,}4376 = 43{,}76\%$$

The work on the turbine is practically the same as the difference in heat. This difference is caused by the pumped work. But this can be neglected most of the time.

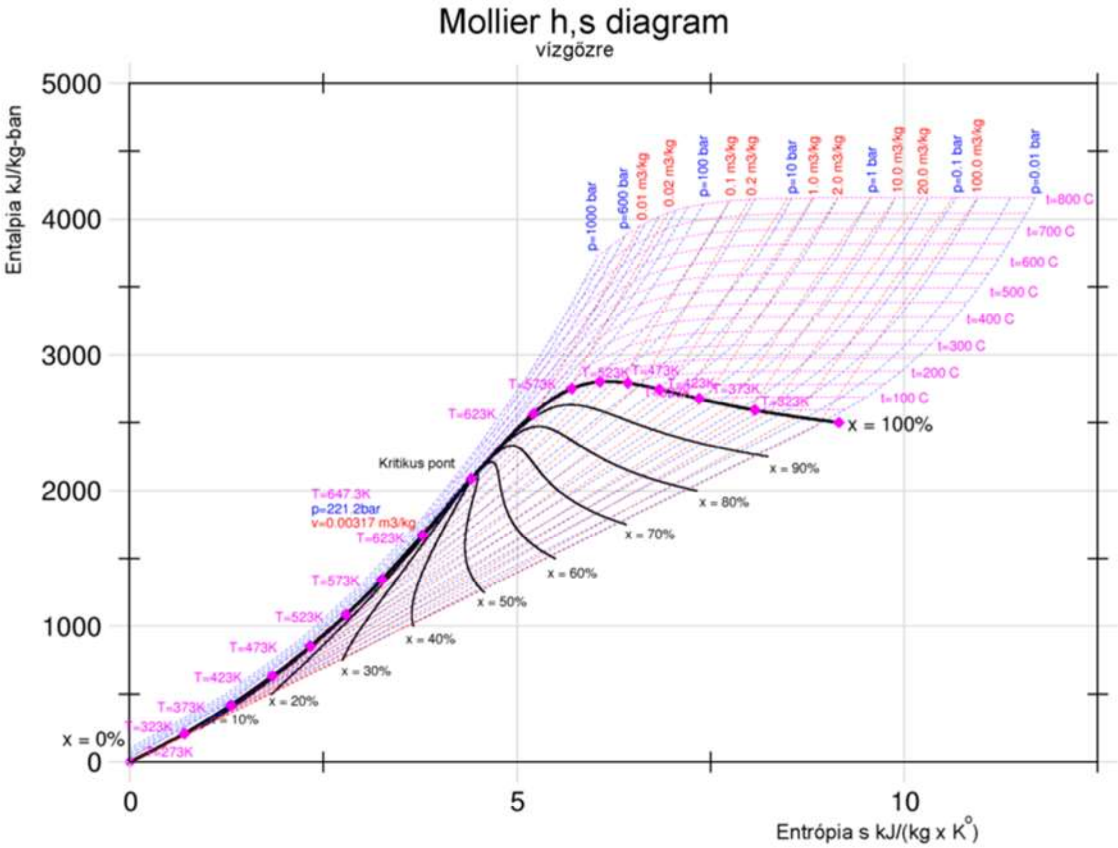
$$w_{45} - w_{61} = 1456 - 14 = 1442 \frac{kJ}{kg}$$

$$Q_F - Q_A = 3297 - 1854 = 1443 \frac{kJ}{kg}$$

(The two quantities are the same, the reason for the difference is rounding error!)
The efficiency of a real cycle is smaller in theory because of different losses.

4.4.3 Water vapor h-s diagram

Water vapor status indicators are often depicted not only in T but also in H diagrams. **Figure 4.11** shows such a diagram. The border curve slightly turns over the T-s diagram. The line system in it contains the same status indicators as the T-s diagram.
Here you can see the flow of constant pressure (blue) and constant volume (red) lines. Here are the "x", steam content lines (black).



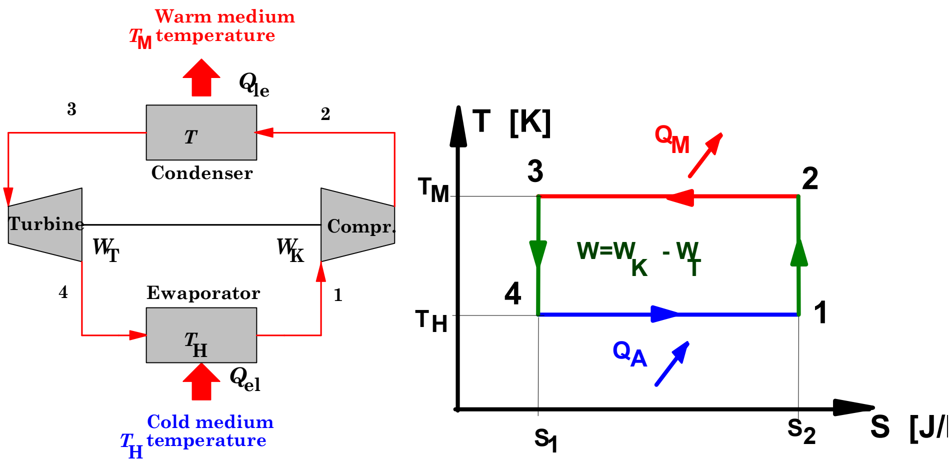
H-s diagram of water vapor
Figure 4.11

5. Refrigerators and heat pumps

In **Section 3.3** we have already discussed the reverse Carnot cycle. The power cycle processes discussed in **Chapter 4** have transformed part of the heat introduced into work. By turning the direction of these cycles, we can create heat by investing work. The most important field of application of this type of cycle is cooling, when we need to create and maintain a system with a lower temperature than the ambient temperature.
Since our goal is to create the greatest possible heat transfer (inhomogeneity) at the cost of the smallest possible work effort, we will not introduce the thermal efficiency, but the factor called the efficiency of the cooling circuit, to evaluate the energy efficiency of these cycles. The interpretation of this is in **Section 5.1**. Using the markings of Fig.

$$\varepsilon = \frac{Q_{el}}{W_{net}} = \frac{Q_{el}}{|W_K - W_T|} .$$

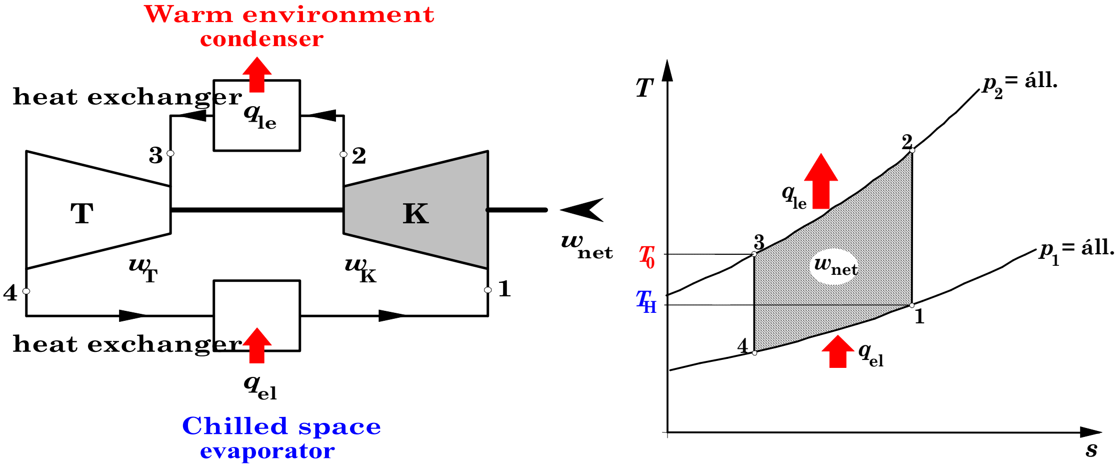
Figure 5.1 shows the name of each equipment. The process, whose T-s diagram is based on a series of state changes:



Theoretical structure of the reverse Carnot cycle
Fihure 5.1

1-2: adiabatic compression of working fluid,
2-3: isothermal heat loss, working fluid condensation,
3-4: adiabatic expansion of working fluid,
4-1: isothermal heat, evaporation of working fluid.

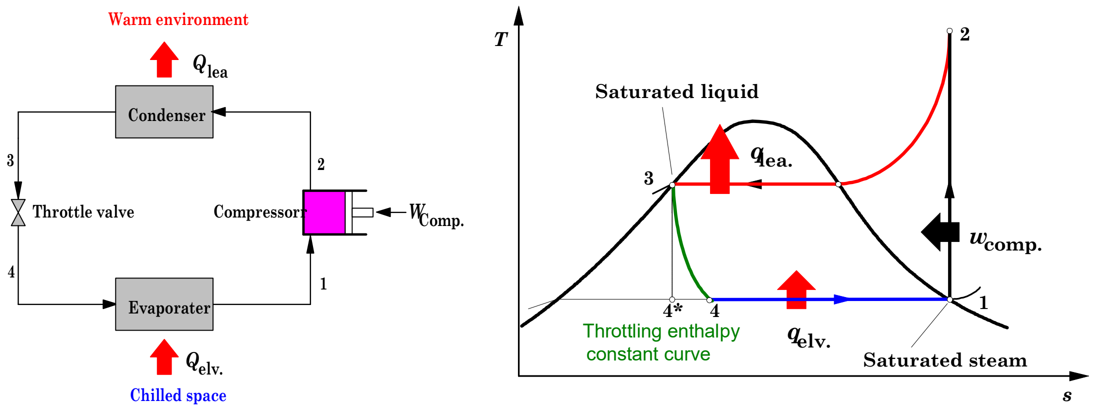
The reverse Carnot refrigerator can not really be put into practice. In practice, refrigerators according to the reverse cycle processes that are already familiar with steam or gas cycles are in operation. The conceptual design of the reverse Brayton cycle is shown in **Figure 5.2** and in Figure T-s is also shown in **Figure 5.2**. If heat loss and heat uptake take place without a temperature difference, the minimum temperature of the refrigerated space and the ambient temperature may be at most.



The basic structure of the reverse Brayton cycle and its T-diagram
Figure 5.2

In practice, the most commonly used refrigerant circuits are steam. The Steam Schematic and T-s diagram of the Steam Circuit Diagram is the **Figure 5.3** shows. Working fluid is undergoing the following state changes:

1-2: isothermal heat recovery, evaporation of working fluid,
2-3: adiabatic compression of working fluid,
3-4: isothermal heat loss, working fluid condensation,
4-1: adiabatic expansion of working fluid.



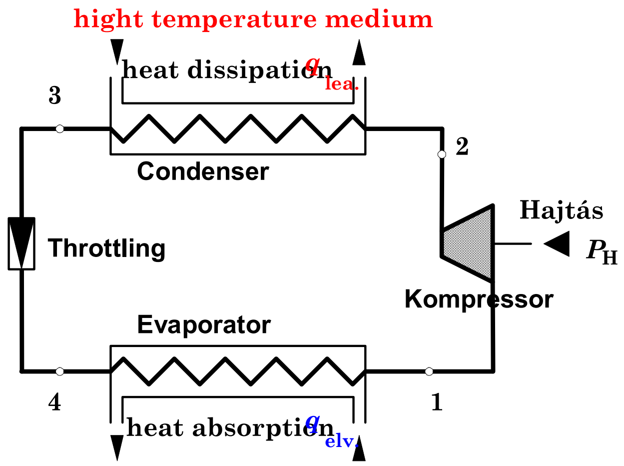
Outline and circuit of compressor refrigerator in T-s diagram
Figure 5.3

In the case of steam working fluid cooling circuits, the turbine due to the difficult feasibility of the expansion of the mixed biphasic medium expands the expansion by throthling expansion (4*), and post-depression state at point (4). Efficiency of the cooling circuit process expressed by the enthalpy values from the status diagram:

$$\varepsilon = \frac{h_1 - h_4}{h_2 - h_1}$$

5.1 Compressor refrigerator

The refrigerator and heat pump are heat engines which, during operation, perform processes with the refrigerant to deliver heat from a lower temperature to a higher temperature.
The use of a one-component two-phase working fluid means that in this case the heat output is almost isothermal and the heat input is isothermal. The principle connection of the device is shown in **Figure 5.4**.
The workflow consists of the following steps: at a sufficiently low temperature level in the evaporator, the working fluid evaporates and drains heat from the space to be cooled. Steam from the evaporator is compressed by the compressor at a pressure corresponding to the required high temperature level so that it is delivered to the condenser where the working fluid is liquefied. It gives the environment heat. Liquid coolant flows through the throttle to the evaporator. The cooling circuit is shown in diagram in **Figure 5.5**, where the points given in the switching sketch and the irreversibility of the cycle are checked. Compressor cooling circuits are usually illustrated in a log p-h diagram, because in this diagram the specific heat input and output as well as the mechanical work to be invested can be illustrated with sections, as shown in **Figure 5.6**.



Compressor refrigerator outline
Figure 5.4

The energy balance of the compressor refrigerator cycle is expressed by the following equations. The refrigerant is in the evaporator specifically

$$q_{elv.} = h_4 - h_1 \qquad 5.1$$

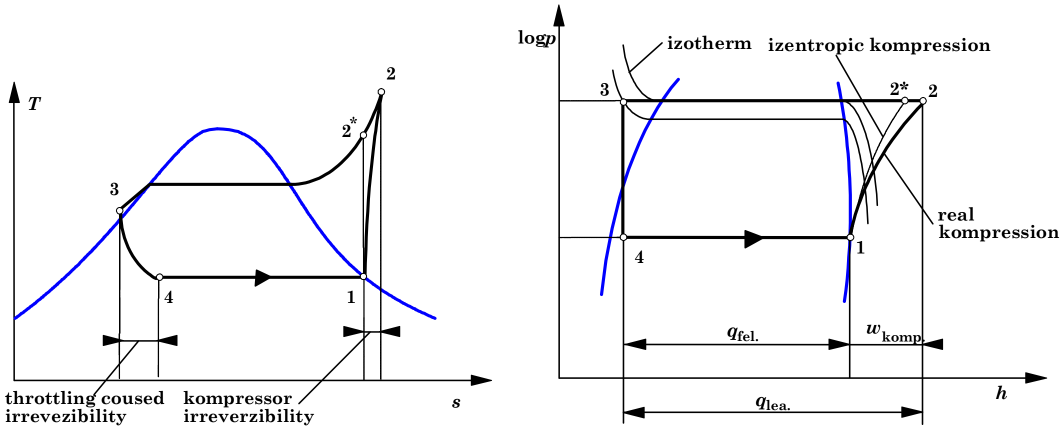
heat give down, in the condenser in particular

$$q_{lea.} = h_2 - h_3 \qquad 5.2$$

heat while the compression is specific

$$w_{komp.} = h_2 - h_1 \qquad 5.3$$

requires work.

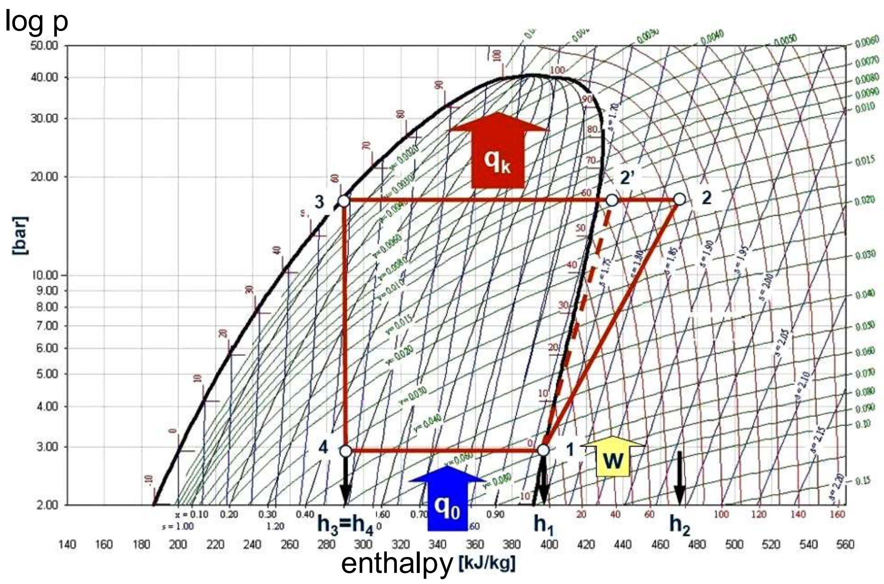


Compression refrigerator cycle T-s and log p-h diagram
Figure 5.5

In addition to compressor refrigerators, absorption refrigerators are also used. We do not deal with these details here. Find out more about these [1] literature.

5.2 Heat pumps

A heat pump is a device, a caloric machine, designed to extract heat from a lower temperature environment and to transport it to a higher temperature. The principle is completely similar to refrigerators. Its purpose is to manage heat energy by using cooling energy in heating (eg in hot water production) or utilizing environmental heat. A heat pump is, in principle, a refrigerator in which heat is not made use of on the cold side but heat generated on the warm side is used. Everything is based on the physical principle of heat pumps, which are also used in refrigerators. Steam compression equipment is most common, but there are also absorption heat pumps.



A log p-h diagram of a heat pump system
Figure 5.6

Figure 5.6 shows a heat pump circuit in a log p-h diagram. The temperatures can be seen on the left border curve marked with brown inscriptions. The lower temperature is 0°C, the upper temperature is 60°C. 2 'represents the line of perfectly adiabatic compression, and point 2 is the true compression line.

Heat pumps can also operate in reversed mode, so they can be used to cool the warmer space. Heat pumps can also be considered as power heaters in the reverse mode of operation as energy meters. [1]

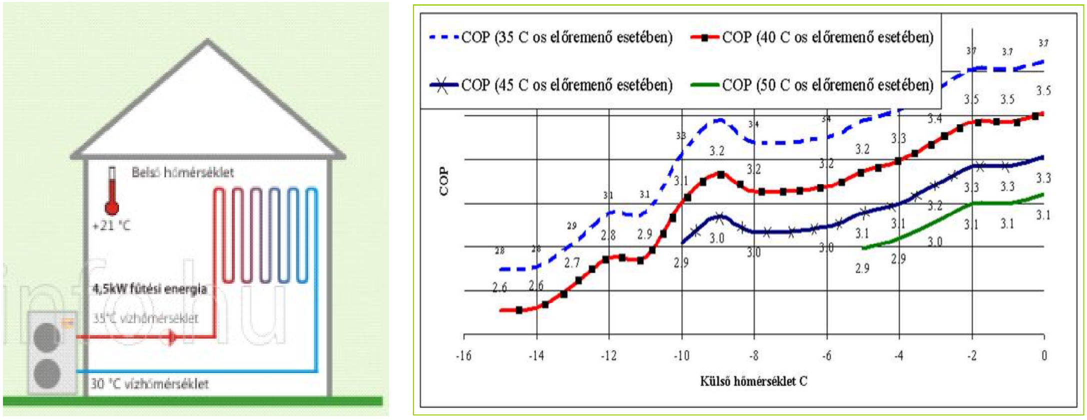
In the case of a soil collector system, hundreds of meters of special hard PVC jacketed tubes or polyethylene pipes are laid down 1 to 2 meters deep. Its disadvantage is that the large area (the heated floor area 1,5-3 times) has to be broken off when the pipes are laid, so it is most likely to be in case of newly constructed houses. (see **Figure 5.7**)

Groundwater: After draining the water from the groundwater well by a submersible pump, the water is either drained into another well or surface water (stream, lake, river) or drained through drainage pipes. Nearly constant temperature of the groundwater (7-12°C) and good thermal conductivity are the ideal sources of heat.

Air: The outside air is sucked in by the fan (s) and the heat pump cools down. Its advantage is easy installation, but its disadvantage is dependence on outdoor air, which does not have constant temperature. Therefore, the efficiency and heat output of the system are undesirable. The noise generated by the fan (s) may also be a problem. The COP of the air source heat pump (Coefficiency of Operation Proces) is the ratio of heat delivered by *equation 3.14* and compressed air compressor work. Or, according to **Figure 5.1**, the recovered heat relative to the invested work.

$$COP = \frac{Q_M}{W} \qquad 5.4$$

The heat pump is the more economical, the larger the number of COP is. This practically means how many kilowatt-hours of heating energy we produce by 1 kWh of electricity. Generally, a system is economical if this value is at least three or more. **Figure 5.8** shows how the COP depends on the temperature of the outside air in a heat pump system. On each curve, the parameter is the temperature of the heating water circulating in the heating system. The system is economical for low heating water temperatures, but larger radiators are required for good operation. Another thing which can be seen from the curvesis that the external temperature drop strongly reduces the efficiency of the air heat pump system.



Air collector heat pump heating

Figure 5.8

Waste heat: Waste water, spent thermal water, electrical equipment and industrial equipment to be cooled can be considered as heat sources.

Seasonal storage: During the summer season heat from cooling is transferred to the soil, stored in a large volumetric layer formed in this area, and in the winter, heat is taken from this layer in the heating plant. Heat source is the waste heat of cold stores, coolers and summer air conditioning. Summer heat is "gone to winter".

5.3 Refrigerants

The first implemented refrigerators were used for ammonia (NH_3), which is still common in large refrigeration systems today. Later, propane (C_3H_8), methyl chloride (CH_3Cl) sulfur dioxide (SO_2) and several other compounds were used. The Freon family is the commercial name of halogenated hydrocarbons that DuPont started to manufacture and is commonly used in the refrigeration industry due to its excellent properties. Examples include Freon-12 difluorodichloromethane (CF_2Cl_2), Freon-11 trifluorochloromethane (CF_3Cl) or Freon-2 Fluorodichloromethane ($CHFCl_2$). Freons (or CFC- s) have been widely used because of their excellent stability and safe use: they are non-flammable, less toxic than the refrigerants they have replaced. Later it turned out that their properties were very dangerous: if the freon escaped, in the upper atmosphere, their chlorine content destroyed the ozone layer, which protects the surface of the Earth from the strong ultraviolet radiation of the Sun. The chlorine atom promotes the degradation of ozone as a catalyst. The chlorine remains an active catalyst until it enters into another atom and does not form a stable molecule. CFC refrigerants are also common in refrigerators today, but their volume is decreasing. More and less polluting refrigerants are hydrochlorofluorocarbons (HCFC-s), such as R-22, which are used in most modern household refrigerators and HFC-s (eg R134a) that are spread in cars; they replaced the former CFC-s. The Montreal Convention also places HCFC-s on the list of substances to be gradually subtracted, which will be replaced by HFC (hydrofluorocarbon), for example, R-410A, which no longer contains chlorine. Today, sub- and supercritical carbon dioxide is also increasingly spreading as a refrigerant, labeled with R-744.

6. Humid air status indicators

6.1 The law of ideal gas mixtures

For the clear definition of the mixtures, as for each multi-element system, it is necessary to know the quantity of each constituent. There are two ways to give the number of creators, which differ in their bases: mass (kg) or amount of material (mol). We'll only get acquainted with the mass-based reference system. The behaviour of the ideal gas mixture is described in the Dalton and Amagat Laws. The most important features of the gas mixture (mass ratio, molar ratio) are discussed. We present the high-priority gas mixture, wet air behaviour, characteristics and state changes, and correlations and status diagrams needed to manage them.

$$m = m_1 + m_2 + m_3 + \ldots + m_i + \ldots + m_N \qquad 6.1$$

So the mass of the mixture: $m=\sum_{i=1}^{N} m_i$

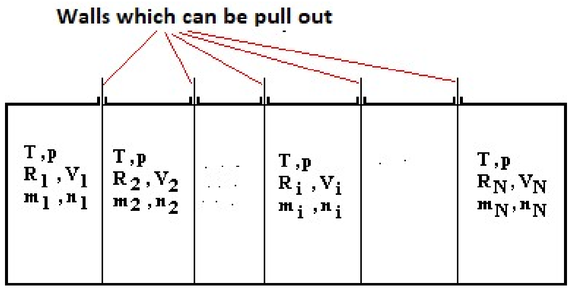
Consider the following system.
In the system, as seen, N gas mixtures are enclosed in a container with bulkheads. The same is their pressure and temperature. By pulling the bulkheads, some of the components of the gas mix start to mix and the mass are added.
The volume of the gas mixture is the sum of the volume of each component

$$V=\sum_{i=1}^{N} V_i \ . \qquad 6.2$$

The mass and other features are similarly calculated. The average material properties (gas constant, specific heat, etc.) can be calculated by weighting the mass ratio.

Partial pressure is nothing more than a particle pressure of a given component in the blend. Thus, the partial pressure of the i-component should be thought to be how much a component would produce a pressure in the total volume if the other components were removed. That is, the part of the total pressure is generated by that component.
Thus, the pressure of the mixture can be produced as a sum of the partial pressures of each component.

$$p = p_1 + p_2 + p_3 + \ldots + p_i + \ldots + p_N \qquad 6.3$$



The ideal gas mixture
Figure 6.1

6.2 Humid air

Multi-component air is often used in practice as a two-component mixture of water vapour and dry air, and in this case, it is called wet air. Knowing the amount of airborne H_2O, changing it during each state change is important in technical practice. One of the most important things in climate conditioning is to provide adequate air humidity at the temperature. Creating the right comfort (for humans, animals, and plants) means not only providing the right temperature, but also the right humidity. Another important application is industrial drying, which means removing water from different materials (e.g. paints, foods, etc.), which is an important process in many industries.

6.3 The h-x diagram of humid air

In practice, the h-x or (i-x) diagram is used to follow the changes in the wet air state. The absolute moisture content of the wet air x is the ratio of the moisture content (m_v) and the dry air (m_ℓ) mass in the given air mass.

$$x = \frac{m_v}{m_\ell} \qquad 6.4$$

The h_{1+x} enthalpy of 1 kg of dry air and x kg of water equals the amount of enthalpy of dry air (specific heat temperature) and the enthalpy of x kg water. The latter enthalpy consists of the sum of evaporation heat (r_0) and gas/vapour water enthalpy (temperature, temperature):

$$h_{1+x} = c_p \cdot t + x \cdot \left(r_0 + c_{p_g} \cdot t\right) \qquad 6.5$$

(Existing pressures and temperatures both components and the blend itself can be considered ideal gas.)
By encoding h_{1+x} in this way, t=0 °C is assigned to h1 + x = 0 kJ / kg. By replacing the value of the coefficient values of the air (c_p = 1 kJ / kg · K) and the water vapour (c_{pg} = 1.86 kJ/kg K) and the evaporating heat (r_0 = 2501 kJ / kg), we get the following expression:

$$h_{1+x} = t + x \cdot \left(2501 + 1{,}86 \cdot t\right) \qquad 6.6$$

The slope of the constant temperature curves in this case, $\left(\frac{\partial h_{1+x}}{\partial x}\right)_{t=const.} = 2501 + 1{,}86 \cdot t$ the proportions diverging in the temperature increase, as shown in **Figure 6.2**, is a h_{1+x}–x diagram.

Note that the change in slope caused by temperature change in relation to the slope of isotherms is very small according to the order of magnitude r0 and cp_g. Thus, the slope of *0°C* is 2501 and the same value is at 2687 for t=100 °C. This means that the range useful for us is between two small straight slopes, which is an unusable small area on a given scale diagram. Solving this problem is the coordinate transformation suggested by Mollier, that is to rotate the x axis with the angle α. (The α angle is a function of the scales applied in the diagram.) In the resulting truncated coordinate system, the enthalpy constant lines will be parallel to the rotated x_α axis. From the temperature of constant lines, the isotherm of 0°C becomes horizontal and the temperature increases slightly diverging lines, as shown in **Figure 6.1**. In the diagrams used, we see the usual rectangular coordinate system whose scale is derived from the rotated x_α axis.
The diagram just gives the isotherms, and we do not know anything about the moisture. This is a further consideration. To prescribe the relation between the total pressure of the wet air, the moisture content and the vapour pressure in the air, the ideal gas state equations for the components are described and their ratios:

$$\frac{V_k p_\ell}{V_k p_g} = \frac{m_\ell \frac{\Re}{M_\ell} T}{m_g \frac{\Re}{M_g} T} \qquad 6.7$$

where: $\Re$ the universal gas constant (8,314 J/mol·K)
M_ℓ the mol mass of air
M_g the mol mass of water wapor

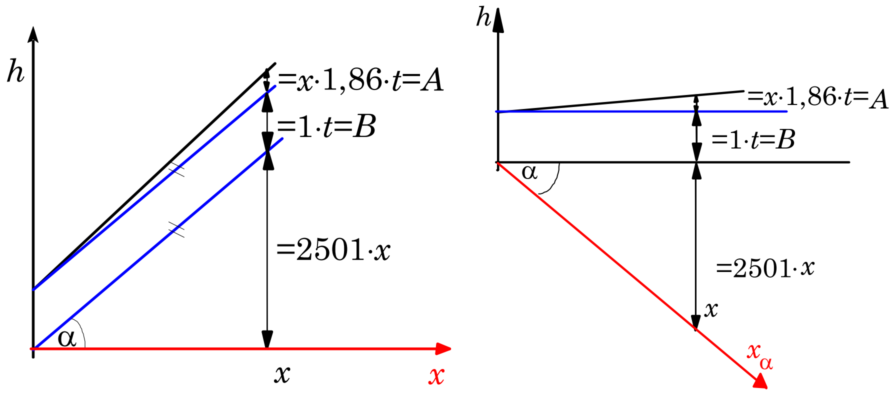
$$\frac{p_g}{p_\ell} = \frac{m_g}{m_\ell} \cdot \frac{M_\ell}{M_g} = x \cdot \frac{29}{18} = 1{,}61 \cdot x\,, \qquad 6.8$$

so

$$p_{\ell+x} = p_\ell + p_g\,,$$

so we get that

$$p_g = p_{\ell+x} \cdot \frac{x}{0{,}622 + x} \qquad 6.9$$



The transformation of the enthalpy axis is obtained by Mollier diagrams
Figure 6.2

The increase in moisture content thus results in a vapour pressure increase of *equation 6.9*. At a given temperature, vapour higher than the saturation pressure of the temperature cannot occur, so the moisture content in the air may increase at a given temperature until the vapour sub pressure reaches the saturation pressure for the given temperature. That is, every temperature has a moisture content (x_s) dependent on p_{1+x} when the vapour sub pressure reaches the saturation pressure corresponding to the given temperature, so that the value of x_s is determined by the following expression:

$$x_s = 0{,}622 \frac{p_s}{p_{1+x} - p_s}. \qquad 6.10$$

Any further moisture added to the mixture can only be present in liquid state. (Smooth liquid phase, evenly distributed in the mixture, is the fog.)
Enthalpy in this case is the next

$$h_{\ell+x} = c_{p_\ell} \cdot t + x_s \cdot \left(r_0 + c_{p_g} \cdot t\right) + \left(x - x_s\right) \cdot c_{p_v} \cdot t \qquad 6.11$$

where c_{pv} is the water heat of the water.

The steepness of the temperature constant slopes $c_{p_v} = 4{,}17 \frac{kJ}{kg \cdot K}$ with the substitution:

$$\left(\frac{\partial h_{\ell+x}}{\partial x}\right)_{t=áll.} = 4{,}17 \cdot t\,. \qquad 6.12$$

The temperature is a set of constant lines, summarizing the previous ones, ie:
For zero moisture content, enthalpy is equal to the product of the isobaric type of dry air temperature and the total pressure (at 1 bar $c_{p\ell} = 1 \frac{kJ}{kg \cdot K}$).

By increasing the moisture content, the isotherm is straight at a small angle with the horizontal, reaching the saturation moisture content, the slope of the isotherm is suddenly changed and almost enthalpy is parallel with constant lines.
Enter the saturation degree

$$\psi = \frac{x}{x_s} \qquad 6.13$$

Moisture content can also be characterized by relative humidity in meteorological practice.

$$\varphi = \frac{p_g}{p_s} \qquad 6.14$$

What is the percentage of the partial pressure of the steam in the air in the saturation pressure for the given air temperature. Relationship between relative humidity and saturation degree is as follows.

$$\varphi = \frac{p_{\ell+x} \cdot \dfrac{x}{0{,}622 + x}}{p_{\ell+x} \cdot \dfrac{x_s}{0{,}622 + x_s}} = \frac{x \cdot (0{,}622 + x_s)}{x_s (0{,}622 + x)} = \psi \cdot \frac{(0{,}622 + x_s)}{(0{,}622 + x)},$$

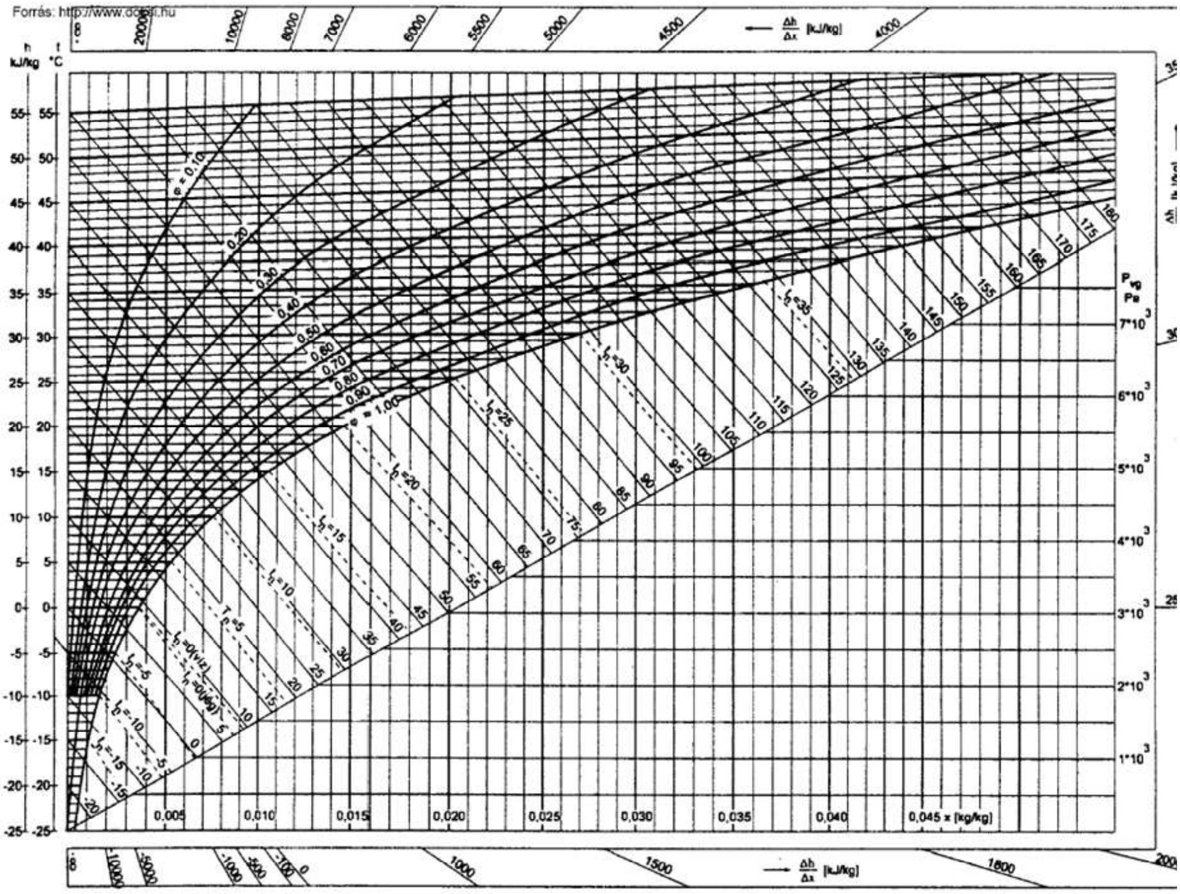
or the pressure utilizing the connection

$$\psi = \frac{0{,}622 \cdot \dfrac{p_g}{p_{\ell+x} - p_g}}{0{,}622 \cdot \dfrac{p_s}{p_{\ell+x} - p_s}} = \varphi \cdot \frac{p_{\ell+x} - p_s}{p_{\ell+x} - p_g}$$

in the form. While the temperature of the mixture is much lower than the saturation temperature for the total pressure, x and x_s are << 1 and thus $p_g << p_{\ell+x}$ and $p_s << p_{\ell+x}$ in practice $\varphi \approx \psi$.

In the isotherms, the curves describing the various saturation states (φ=0,1; 0,2;...) can be depicted in the range between the saturated state (φ=1)and the completely dry (φ=0) air.

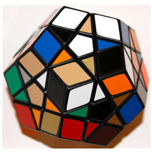
The curve connecting the isotherm breakpoints for the saturation values is called the saturation curve. The position of the saturation curve is affected by the humidity compression value. For higher total pressures, according to *equation 6.10*, the moisture content of the saturation state is smaller, so the saturation curve is shifted to the left. The range below the saturation curve is called the dimensional uniformity of the uniformly distributed liquid phase.



Mollier h-x air humidity diagram

Figure 6.3

6.4 Air condition test during heating



During a climate problem, the external air condition is t_e/φ_e =+10ºC/50%. The air should be heated to t_i=25 ºC.

Questions:

a. Draw the air state change in h-x diagram and read the absolute humidity and heat content values.

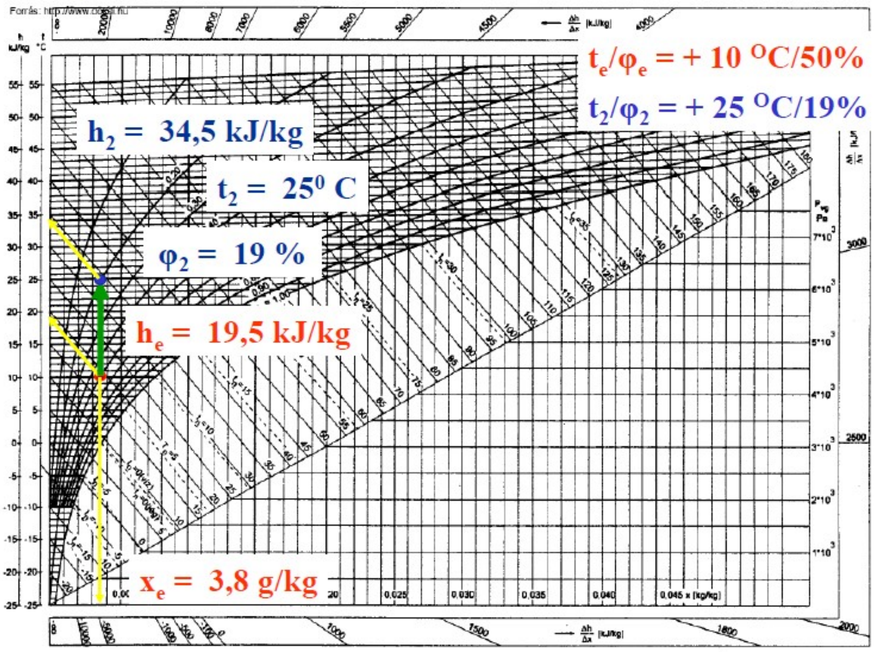
b. Then the air is moistened in an air washer at a relative humidity of φ = 100%. What values can be read at time t, φ, x and h?

c. / After moistening, add the air in the preheater to t_4 = 22 ºC. What values do we get in the h-x diagram?

Solution: In principle, the diagram shows the air state for each point.

a. / In the preheating the air is heated. Its absolute moisture (x) does not change, so a change of state goes up along a vertical line. The temperatures, enthalpies and relative humidity in points 1 and 2 were entered in the figure. The heating increased the enthalpy of the mixture and the relative humidity decreased.

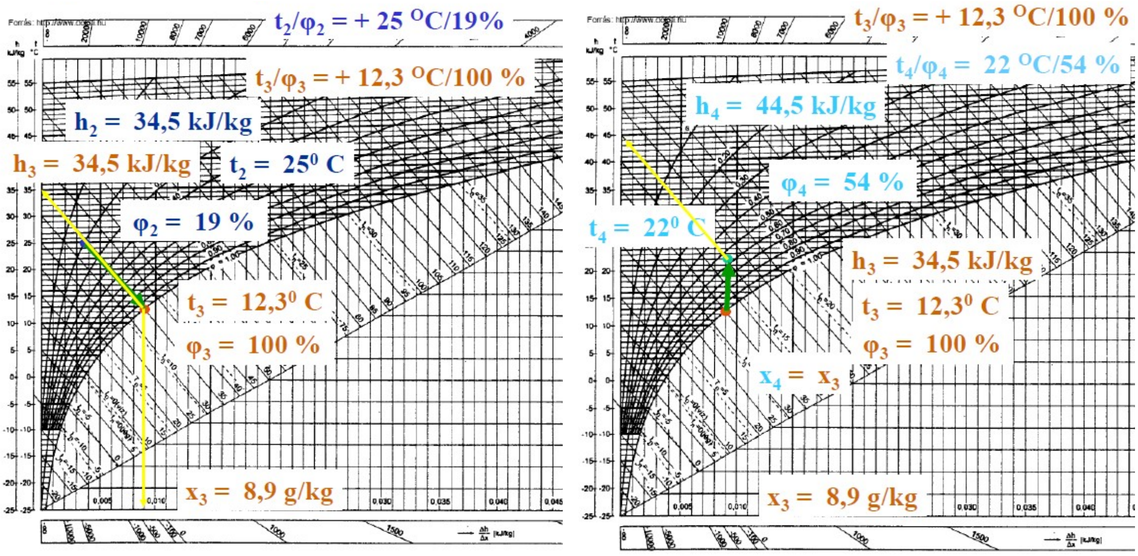
Solution: a./ preheating 25^0 C



Preheating diagram
Figure 6.4

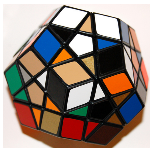
b./ During the humidification, the direction of change of state depends basically on the temperature of the water. The wetting shown in the figure was carried out on approximately constant enthalpy. At this time, the sprayed water temperature is the same as the air temperature. Cooling is caused by the evaporation of water. Latent heat reduces the temperature of the mixture.
The temperatures, enthalpies and relative humidity in points 2 and 3 were entered in the figure. By humidification, the enthalpy of the mixture remained unchanged and temperature and 100% relative humidity remained. The absolute humidity of the air also increased, which is the purpose of humidification.
c./ After the heater has a similar change of state than the preheating, the air is heated. Its absolute moisture (x) does not change, so a change of state goes up along a vertical line. The temperatures, enthalpies and relative humidity in points 3 and 4 were entered in **Figure 6.5**. Heating has increased the enthalpy of the mixture and decreased its relative humidity.
Moisture and post-heating can be carried out in one step, in which case steam is humidified by air. This is also a standard procedure for air conditioning.
In the case of low-pressure steam from an electric steam generator, the line of state change is slightly different from t =constant line. For simplicity, the edits are done in this way.

Solution: b./ humidification **Solution: c./ after-heating**



Humidification and after-heating in the diagram
Figure 6.5

6.5 Air condition test during cooling



It is a common air conditioning task in summer to cool the air. During cooling, the humidity is often precipitated because the air is cooled to the dew point. During an air conditioning task, the ambient air of the air is t_e/φ_e=+32 ºC/40%. The air should be cooled to t_i =14 ºC to saturation.

Questions:

a./ Draw the air state change in h-x diagram and read the absolute humidity and heat content values.

b./ If the refrigerating unit supplies $\dot{m}_{air} = 48\,000\frac{kg}{h}$ air every hour, how much water will air out of the air every hour?

c./ What is the cooling capacity of the unit?

d./ What is the latent and perceptible cooling power?

Solution: In principle, the diagram can be read for each point.

a./ During cooling, the same humidity as the same for the first time decreases the temperature and the relative humidity increases. During cooling, reaching the saturation state, which is $t_2^* = 17°C$ approx. the water starts to extract from the air. Cooling the air, more and more water will precipitate and cool on the saturation curve to the required $t = 14°C$ temperature. As a result of the decay and cooling, its enthalpy decreases.

b./ Water kills $x_e - x_2 = 11{,}8 - 9{,}9 = 1{,}9\left[\frac{g}{kg}\right]$ one kilogram of air. The change to 1 kg shown in the diagram is to be performed on the total amount of air used during cooling. So we can calculate the amount of water that is precipitation hourly.

$$\dot{m}_w = \dot{m}_{air} \cdot (x_e - x_2) = 48000\left[\frac{kg}{h}\right] \cdot (11{,}8 - 9{,}9)\left[\frac{g}{kg}\right] = 91\,200\left[\frac{g}{h}\right] = 91{,}2\left[\frac{kg}{h}\right]$$

A large amount of water is generated during cooling.

c./ The cooler must ensure a reduction in the enthalpy of air, which can be expected as follows:

$$\dot{Q}_{cool} = \dot{m}_{air} \cdot (h_e - h_2) = 48000\left[\frac{kg}{h}\right] \cdot (62{,}3 - 39)\left[\frac{kJ}{kg}\right] = 1118400\left[\frac{kJ}{h}\right]$$

$$\dot{Q}_{cool} = 1118400\left[\frac{kJ}{h}\right] \cdot \frac{1}{3600}\left[\frac{h}{s}\right] = 310{,}6\left[\frac{kJ}{s}\right] = 310{,}6[kW]$$

d./ The refrigeration capacity of a refrigerator can usually be divided into two parts: one part reduces the air temperature, this is the perceptible cooling power, and the other part is turned to divert the hidden heat of the precipitating water, this hidden cooling power.

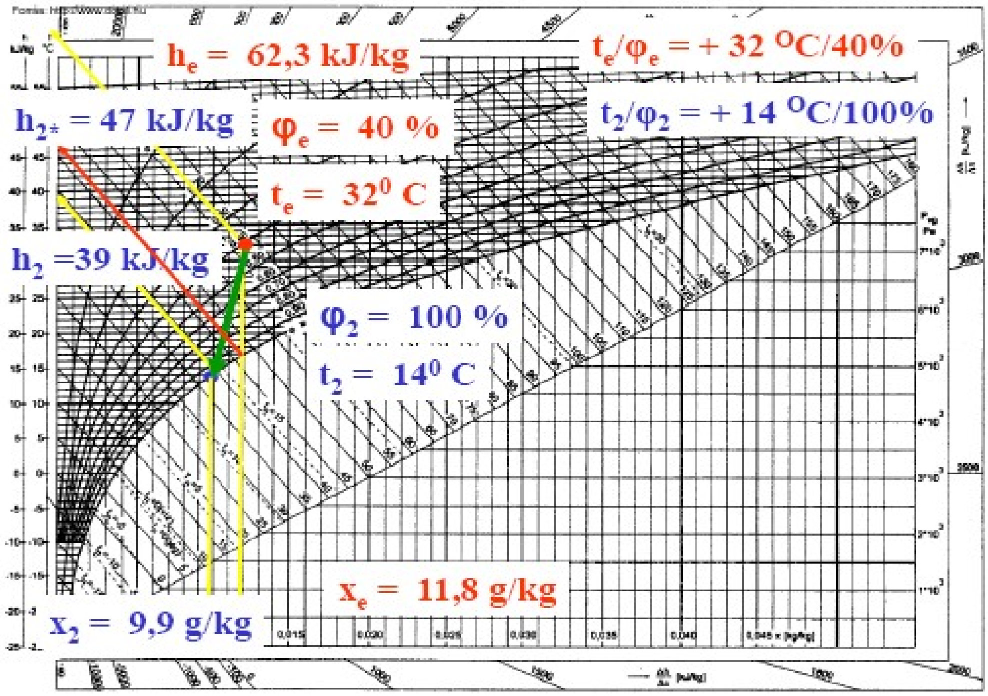
$$\dot{Q}_{cool} = \dot{Q}_{perceptibleő} + \dot{Q}_{latentt}$$

$$\dot{Q}_{latent} = \dot{m}_{air} \cdot (h_{2*} - h_2) = 48000\left[\frac{kg}{h}\right] \cdot (47 - 39)\left[\frac{kJ}{kg}\right] = 384\,000\left[\frac{kJ}{h}\right]$$

$$\dot{Q}_{latent} = 384000\left[\frac{kJ}{h}\right] = 384000\left[\frac{kJ}{h}\right] \cdot \frac{1}{3600}\left[\frac{h}{s}\right] = 106{,}6\left[\frac{kJ}{s}\right] = 106{,}6[kW]$$

$$\dot{Q}_{perceptible} = \dot{Q}_{cool} - \dot{Q}_{latentt} = 310{,}6[kW] - 106{,}6[kW] = 204[kW].$$

Solution: a./ cooling



Cooling process in the diagram
Figure 6.6

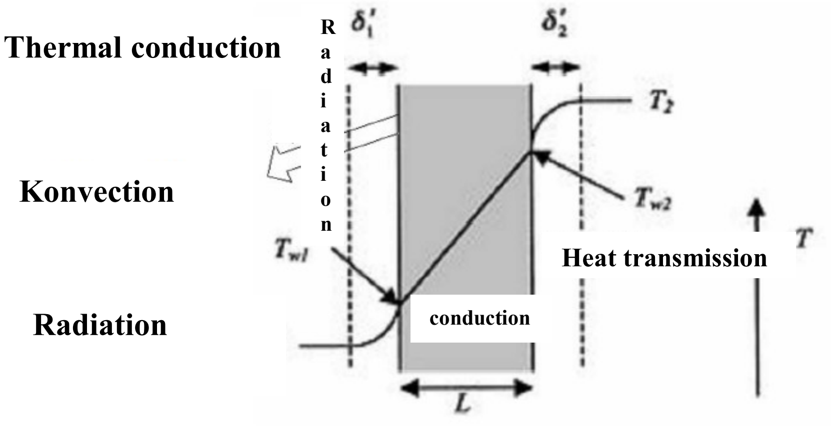
7. Heat Spreading

The spatial spread of thermal energy due to temperature differences is usually the result of highly complex processes. For the quantitative description of the heat spread, the following three elemental processes are distinguished.

"**Conduction**" - the way in which energy is propagated when "flowing" from one part of a medium at a higher temperature to another part of the medium causes the media to shift the particles to be not significant or unordered. (For example, the other end of a heated rod at one end will heat up, the energy from the warmer end of the rod will get heat to the other end.)

Heat transfer (convection) - the form of energy propagation when it is realized by the orderly displacement (flow) of the particles forming the medium. Energy flows along with the material.

Thermal radiation is the process of electromagnetic waves in the spatial distribution of energy, which is a mechanism without necessity of a medium.
The three forms of energy flow often occur simultaneously. We use the form of negotiation that is dominant in terms of energy flow.

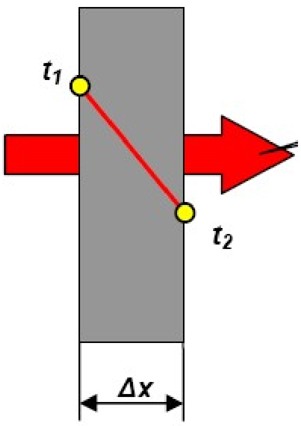


Different forms of heat spread
Figure 7.1

7.1 Thermal conductivity

However, the specific mechanism of thermal conductivity for the different media differs considerably from each other. In the gases, the energy is spread through collisions (and diffusion) due to the dislocation of atoms and molecules. In metals, heat is propagated by two parallel, almost independent mechanisms, on the one hand by the vibration of the atoms forming the crystal grid and by the diffusion of free electrons. In the case of non-metallic materials and liquids, the spread of energy is achieved through flexible elemental waves.

The French physicist and mathematician, Jean Baptiste, Joseph Fourier, set up an experiential relationship between the XIX. at the beginning of the century. According to their observations, the amount of heat flow (Q) through the driving unit is directly proportional to the difference in temperature (t_1-t_2), the surface area (A) and inversely proportional to the distance of the surfaces (Δx) and is proportional to the so-called (λ) with thermal conductivity. The minus sign indicates that the heat flow flows to the decreasing temperature.



Thermal conductivity in a solid body
Figure 7.2

$$\dot{Q} = -\lambda \cdot A \cdot \frac{(t_1 - t_2)}{\Delta x} \; [W] \qquad 8.1$$

$$q = -\lambda \cdot \frac{dt}{dx} \qquad 8.2$$

$$\underline{q} = -\lambda \cdot \text{grad}\, t \qquad 8.3$$

The Fourier thermal conductance equation contains a material attribute, the thermal conductivity factor.

7.1.1 The thermal conductivity factor (λ)

Expressing the thermal conductivity factor, we get the unit of measure from the Fourier equation. It means that the amount of heat flow in the material starts up to one meter in the Kelvin temperature difference.

$$\lambda = -\frac{\dot{q}}{\text{grad}\, t} \left[\frac{W}{m \cdot K}\right]$$

The heat conduction factor is mostly non-constant, but varies according to temperature. This change is negligible for smaller temperature differences. Due to higher temperature differences due to the fact that the thermal conductivity is mostly linear in the temperature, the mean value corresponding to the average temperature can be calculated. There is an analogy between the thermal conductivity and electrical conductivity. Electrically conductive materials (metals) have good thermal conductivity. In contrast, the non-metallic materials have a significantly lower thermal conductivity, which have thermal insulation properties. The basic explanation for this is that the metals have a lot of free electrons that flow through the electric voltage difference, and that they also play a role in the heat flow.
The following table shows the thermal conductivity of some substances:

7.1 Table Thermal conductivity of materials

Material	Conductivity factor λ [W/mK]	Material	Conductivity factor λ [W/mK]	Material	Conductivity factor λ [W/mK]
Metals		**Other materials**		**Gases**	
Aluminum	210	Aluminum	210	Air	0,026
Brass	85	Brass	85	Carbon dioxide	0,017
Copper	386	Copper	386	Nitrogen	0,025
Gold	393	Gold	393	Oxygen	0,027
Iron	73	Iron	73		
Lead	35	Lead	35		
Platinum	70	Platinum	70		
Silver	408	Silver	408		
Steel	48	Fiber	0,04		
Liquids		Granite	2,1		
Acetone	0,20	Ice	2,2		
Petrol	0,16	Linen	0,088		
Ethyl alcohol	0,17	Paper	0,13		
Mercury	8,7	Soft rubber	0,14		
Engine Oil	0,15	Dry sand	0,39		
Vaseline	0,18	Silk	0,04		
Water	0,58	Solid snow	0,21		
		Dry soil	0,14		
		Wood	0,13		

The table shows that metals condust heat well. Among metals, copper and aluminum are good heat conductors, so they are preferred for the production of heat exchangers. The heat conductivity of the liquids is worse by the order of magnitude and the gases are very good thermal insulation. (Provided there is no flow in them, because they convey the heat by convection.) The hair and feathering of the animals have good thermal insulation properties due to their closed air.

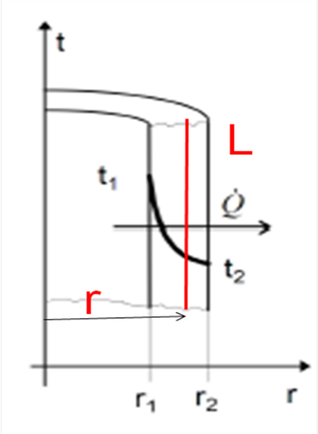
7.1.2 Stacioner heat conduction in solid bodies

The following are the main practical cases of the differential equation of heat conduction. The temperature distribution in flat walls, pipes and spheres. Thermal conductivity through flat walls, in a stationary case, results in a simple linear temperature distribution. This is discussed later in the heat release. Thick-walled pipes have a more complicated temperature distribution.

Thermal conductivity in a stationary case in a thick pipe wall.

A warm fluid flows inside the pipe. As a result, the flow of heat to the cooler outer space begins through the pipe wall. Assuming stationary flow, the same heat stream of each tube "r" of the tube flows through. The magnitude of this $\dot{Q}$.

$$\dot{Q} = -\lambda \cdot A \cdot \frac{dt}{dr} = -\lambda \cdot 2\pi \cdot r \cdot L \cdot \frac{dt}{dr}$$



Thermal conductivity in the pipe wall
Figure 7.3

Express the temperature change of the equation along the radius. Openly in the lower inner part of the body, the temperature varies more rapidly than the outer outer shell. Separate the differential equation.

$$\dot{Q} \cdot \frac{dr}{r} = -\lambda \cdot 2\pi \cdot L \cdot dt,$$

then integrate between the inner and a running beam:

$$\dot{Q} \cdot \int_{r_1}^{r} \frac{dr}{r} = -\lambda \cdot 2\pi \cdot L \cdot \int_{t_1}^{t} dt.$$

$$\dot{Q} \cdot \ln \frac{r}{r_1} = -\lambda \cdot 2\pi \cdot L \cdot (t - t_1)$$

Express the temperature function:

$$t = t_1 - \frac{\dot{Q}}{\lambda \cdot 2\pi \cdot L} \cdot \ln \frac{r}{r_1}$$

The resulting temperature is the largest on the inner surface and then logarithmically decreases outwardly. The figure is shown in **Figure 7.3**.

The heat flow can be expressed by temperature difference and material and geometry characteristics.

$$\dot{Q} \cdot \ln \frac{r_2}{r_1} = -\lambda \cdot 2\pi \cdot L \cdot (t_2 - t_1)$$

$$\dot{Q} = 2\pi \cdot \lambda \cdot L \cdot \frac{t_1 - t_2}{\ln \frac{r_2}{r_1}} \; [W] \qquad 8.4$$

Thermal conductivity in a stationary case, in a spherical wall

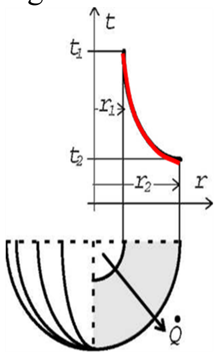
There is a warm medium inside a spherical container. This causes the flow of heat to the colder outer space through the wall of the sphere. In the stationary case, each of the spheres of the sphere "R" radiates the same heat flow. The magnitude of this $\dot{Q}$.

$$\dot{Q} = -\lambda \cdot A \cdot \frac{dt}{dr} = -\lambda \cdot 4\pi \cdot r^2 \cdot \frac{dt}{dr}$$

Express the temperature change of the equation along the radius. Openly, the inner, smaller spheres change temperature more rapidly than the outer larger radius. Separate the differential equation.

$$\dot{Q} \cdot \frac{dr}{r^2} = -\lambda \cdot 4\pi \cdot dt,$$

then integrate between the inner and a running radius.



Thermal conductivity in the spherical wall
Figure 7.4

$$\dot{Q}\cdot\int_{r_1}^{r}\frac{dr}{r^2}=-\lambda\cdot 4\pi\cdot\int_{t_1}^{t}dt$$

$$\dot{Q}\cdot\left[-\frac{1}{r}+\frac{1}{r_1}\right]_{r_1}^{r}=-\lambda\cdot 4\pi\cdot\left(t-t_1\right)$$

Express the temperature function:

$$t=t_1-\frac{\dot{Q}}{\lambda\cdot 4\pi}\cdot\left(\frac{1}{r_1}-\frac{1}{r}\right)$$

The resulting temperature is the largest on the inner surface and then hyperbolically decreases outwardly. The characteristic is shown in **Figure 7.4**.

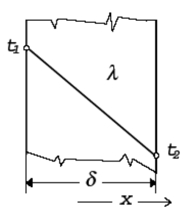
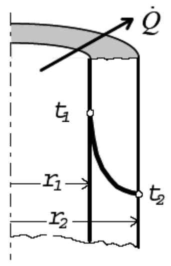
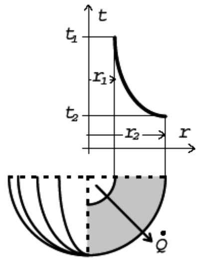
The heat flow can be expressed by temperature difference and material and geometry characteristics.

$$\dot{Q}=\frac{4\pi\cdot\lambda\cdot\left(t_1-t_2\right)}{\frac{1}{r_1}-\frac{1}{r_2}}=\frac{4\pi\cdot\lambda\cdot r_1\cdot r_2\cdot\left(t_1-t_2\right)}{r_2-r_1} \qquad 8.5$$

We can summarize the heat flows generated through different geometric walls.

7.1.3 The thermal resistance

For metals, thermal conductivity and electrical conductivity are analogous.

Geometry			
Heatflux and temperature functions	$\dot{Q}=\lambda\cdot A\cdot\frac{(t_1-t_2)}{\delta}$ $t(x)=t_1-\frac{\dot{Q}}{\lambda\cdot A}\cdot x$ $t(x)=t_1-\frac{t_1-t_2}{\delta}\cdot x$	$\dot{Q}=\frac{2\pi\lambda L}{\ln(r_2/r_1)}(t_1-t_2)$ $t(r)=t_1-\frac{\dot{Q}}{2L\pi\lambda}\cdot\ln\left(\frac{r}{r_1}\right)$ $t(r)=t_1-\frac{t_1-t_2}{\ln(r_2/r_1)}\cdot\ln\left(\frac{r}{r_1}\right)$	$\dot{Q}=\frac{4\pi\lambda(t_1-t_2)}{1/r_1-1/r_2}$ $t(r)=t_1-\frac{\dot{Q}}{4\pi\lambda}\left(\frac{1}{r_1}-\frac{1}{r}\right)$ $t(r)=t_1-\frac{(t_1-t_2)r_1r_2}{r_2-r_1}\left(\frac{1}{r_1}-\frac{1}{r}\right)$
Equivalent area	$\dot{Q}=\frac{\lambda}{\delta}\cdot A_e\cdot(t_1-t_2)$	$\delta=r_2-r_1$ $A_e=\frac{2\pi L(r_2-r_1)}{\ln(r_2/r_1)}$ ha $r_2/r_1\le 2$ $A_e\approx(r_1+r_2)\pi L$	$\delta=r_2-r_1$ $A_e=4\pi r_1 r_2$
One layer	$R_\lambda=\frac{\delta}{A\lambda}$	$R_\lambda=\frac{\ln(r_2/r_1)}{2L\pi\lambda}$	$R_\lambda=\frac{r_2-r_1}{4\pi\lambda r_1 r_2}$
More layers result heat resistence	$R_\lambda=\frac{1}{A}\sum_1^n\frac{\delta_i}{\lambda_i}$	$R_\lambda=\frac{1}{2\pi L}\sum_1^n\frac{\ln\left(\frac{r_{i+1}}{r_i}\right)}{\lambda_i}$	$R_\lambda=\frac{1}{4\pi}\sum_1^n\frac{r_{i+1}-r_i}{r_{i+1}r_i\lambda_i}$

Thermal conductivity for different geometies [5]
Figure 7.5

Good heat conducting metals are also good electric conductors. By using analogy, thermal resistance can be defined, analogous to electrical resistance.
The equilibrium of the heat conduction plane plane is adjusted to keep the difference between the temperatures on the right side, the result (use $-\Delta x = \delta$).

$$\dot{Q}=\lambda\cdot A\cdot\frac{(t_1-t_2)}{\delta}\;[W]$$

Transform the equation into a shape such that the temperature difference is on one side and the heat flow on the other side.

$$t_1-t_2=\frac{\delta}{\lambda\cdot A}\cdot\dot{Q} \qquad 8.6$$

The term is $R_h=\frac{\delta}{\lambda\cdot A}$ **thermal-** with the introduction of thermal resistance, the analogies of Fourier and the Ohm Act are obvious:

$$U = R \cdot I$$

The calculation connection of thermal resistance of bodies with a simple geometry with constant thermal conductivity are also included in **Figure 7.5** for single- and multilayer structures. For layered structures, the values in the table are valid only if each layer is ideally connected to one another, i.e. the contact between them does not represent resistance to the heat stream.

7.1.4 Instacioner heat flow

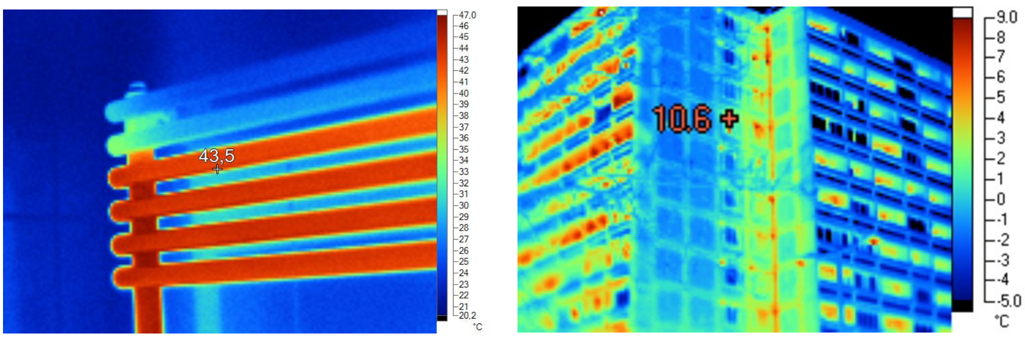
Heating or cooling materials is a very common phenomenon. Just think about cooking, but if you look at the field of engineering, hot working of materials, steels are also a process. In these processes, the materials are heated and cooled, and this operation is often carried out very quickly. In this case, we must also count on the spread of heat over time. This describes the differential equation of heat conduction in general form.

$$\frac{\partial}{\partial t}(\rho c T) = \frac{\partial}{\partial x_i}\left(\lambda\left(\frac{\partial T}{\partial x_i}\right)\right) + q_v$$

where ρ (density), c, (specific heat) λ (thermal conductivity factor) and q_v (volume heat source) from the site (possibly also from time) (x, y, z coordinates) and temperature parameters dependent on T temperature. Differential equation for thermal conductivity with constant λ thermal conductivity:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_p}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{q_v}{\rho c_p} \qquad 8.7$$

Define the term "a" temperature coefficient or temperature coefficient factor. $a = \frac{\lambda}{\rho c_p}$



heat loss of radiator fault **housing**

Thermal cameras

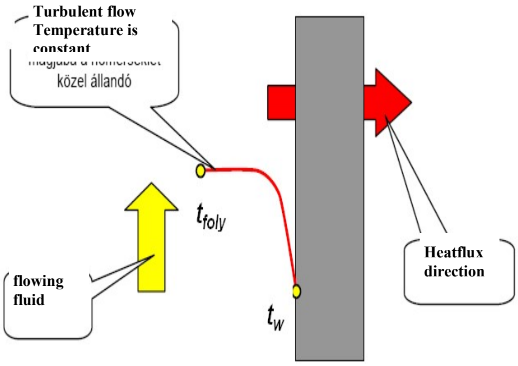
Figure 7.6

[http://www.pairform.hu/index.php?thermog]

The solution of the above differential equation is very complicated. In some cases there is an analytical solution as well. It is also important that the precision of the initial and marginal conditions is very important for the clarity of the solution. Nowadays, there are many softwares available to solve the equation very well. The solution can be visualized in the form of a heat image, as well, which can be directly compared with thermal imaging measurements.

7.2 Heat transmission

In most cases, thermal conductivity in solid bodies is caused by contact with liquid (gas) at temperatures other than their surface temperature. Heat transfer between the solid surface and the fluid through the cross is the heat transfer. Generally, it is created as a combined effect of a heat conduction and a convection.



Heat transfer in wall flow
Figure 7.7
http://5mp.eu/web.php?a=redonv&o=Mcbz5RmPEil

The basic equation of heat transfer prescribed by Newton:

$$\dot{Q} = \alpha \cdot A \cdot \left(t_{foly} - t_w\right) [W] \quad 8.8$$

where:

Q the heat flow on the surface of the solid body, [W]
A the liquid contact surface, [m^2]
$\mathbf{t_w}$ the surface temperature of the body, [°C], or [K.]
$\mathbf{t_{foly}}$ the liquid temperature, [°C], or [K]
α the heat transfer factor $\left[\frac{W}{m^2 \cdot K}\right]$

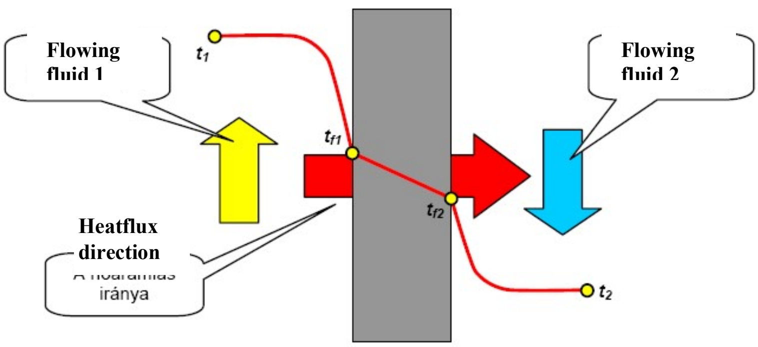
In the above description of the heat flow between the surface of the body and the liquid it was assumed that the total surface temperature is the same (isothermal) and the liquid can be characterized by a single temperature. By introducing the heat transfer factor in this way, the two most important parameters of a complex process, the temperature difference and the surface are highlighted, all other physical effects (nature of flow, velocity, etc.) are included in the heat transfer factor.
The result of these effects is expressed by the numerical value. Calculation of the heat transfer coefficient can be very complicated, and we will discuss later.

7.2 Table Thermal transfer factors used in architecture

The name of the structure and its position relative to the spatial or the heat flow	α external [W/m^{2}K]	α internal [W/m^{2}K
Exterior wall and doors	24	8
Interior wall and doors	8	8
Flat roof and skylight	24	10
Interior slab (uphill), attic floor	12	10
Interior slab (downhill), cellar ceiling	8	6
Ceiling above the arcade	20	6

7.3 Heat Transfer

In engineering practice, it is a common task that heat in the system is to be transmitted or heat is drawn through the solid wall of the system and the heat comes from the environment or is passed. This process, that is, from a heat transfer (between the liquid in the system and the wall of the system) takes place through a heat conduction (through the wall of the system) and an additional heat transfer (between the wall of the system and the environment). This is briefly termed heat transfer.
Such heat transfer takes place between the interior space of the buildings and the environment. Today, this heat transfer is being made to reduce as much as possible by architects. The cooling and heating of buildings has a very high energy input. This is a very effective way of reducing energy consumption. We also take our examples from this topic. In mechanics, the opposite is often the task, so the more efficient, faster heat transfer is the goal. Consider, for example, the cooling problem of engines.



Heat transmission on a flat wall
Figure 7.8
[http://5mp.eu/web.php?a=redony&o=Mcbz5RmPEj]

Let's first examine heat transfer through a flat wall. For buildings, this is the most common case.
For the three temperature differences shown in Figure 7.8, the corresponding relationships can be described. On the left side of the wall, a warmer medium passes its heat through heat to the wall. The heat is passed through the wall through heat, and the media on the right again heat the heat. We assume that the phenomenon is stationary. This means that the temperatures and

heat flows do not change over time. The heat flow from the warmer medium on the left fully reaches the cooler right side. That is, the heat flow in both media and in the wall is the same. First, write down the heat flow from the left medium by the Newton cooling law ($\dot{Q}$).

$$\dot{Q} = A \cdot \alpha_1 \cdot (t_1 - t_{f1}) \, [W] \tag{8.9}$$

Then the heat flow through the wall through the heat wave with the Fourier Law,

$$\dot{Q} = A \cdot \lambda \cdot \frac{t_{f1} - t_{f2}}{\delta} \, [W] \tag{8.10}$$

and finally again with the Newton cooling law:

$$\dot{Q} = A \cdot \alpha_2 \cdot (t_{f2} - t_2) \, [W]. \tag{8.11}$$

Express the temperature differences in all three contexts:

$$t_1 - t_{f1} = \frac{\dot{Q}}{A \cdot \alpha_1} \qquad t_{f1} - t_{f2} = \frac{\dot{Q}}{A \cdot \lambda/\delta} \qquad t_{f2} - t_2 = \frac{\dot{Q}}{A \cdot \alpha_2},$$

then add the three equations. On the left is the average temperature of the hot and cold media $t_1 - t_2$.

$$t_1 - t_2 = \dot{Q} \cdot \frac{1}{A} \cdot \left(\frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2} \right) \tag{8.12}$$

Write the equation in a shape similar to the Newton cooling law

$$\dot{Q} = A \cdot \frac{1}{\frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2}} \cdot (t_1 - t_2) \tag{8.13}$$

$$\dot{Q} = A \cdot k \cdot (t_1 - t_2) \tag{8.14}$$

Let us introduce the factor "k", which is termed a heat transfer factor (heat release coefficient). By using the heat transfer factors and the heat conduction of the wall and its thickness, k is obtained by:

$$k = \frac{1}{\frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2}} \tag{8.15}$$

The structure of the relationship results from the fact that in the case of a given wall thickness and heat conduction factor, the heat transfer factor is in any case less than any of the heat transfer factors on both sides of the medium. The natural consequence of this is that in case of a given wall the increase or decrease of the heat transfer factor is most efficiently achieved by changing the worst heat transfer factor.

It is also common to write the formula with thermal resistances. These individual equations are:

$$t_1 - t_{f1} = R_{\alpha 1} \cdot \dot{Q} \qquad t_{f1} - t_{f2} = R_{\lambda} \cdot \dot{Q} \qquad t_{f2} - t_2 = R_{\alpha 2} \cdot \dot{Q}$$

where:

$$R_{\alpha 1} = \frac{1}{A \cdot \alpha_1} \qquad R_{\lambda} = \frac{1}{A \cdot \lambda/\delta} \qquad R_{\alpha 2} = \frac{1}{A \cdot \alpha_2}$$

Adding the top three equations, we get:

8.16

$$t_1 - t_2 = \left(R_{\alpha 1} + R_{\lambda} + R_{\alpha 2}\right) \cdot \dot{Q}$$

It can be seen from the formula that thermal resistances are added together. The resulting sum of the parts. It corresponds to a networked system. So, for any resistance, the resulting resistance will be greater. Compared with the heat transfer factor, it can be seen that the thermal resistance also includes the surface size. Increasing surface reduces thermal resistance, and increasing heat transfer factors also reduce heat resistance. So if you want to increase heat resistance at buildings, you either reduce the surface or reduce the heat transfer factor. Architects in the latter strive mainly with increasing thermal insulation.

$$\frac{1}{k \cdot A} = R_{\alpha 1} + R_{\lambda} + R_{\alpha 2} \qquad k = \frac{1}{\frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2}}$$

For multi-layered walls, the heat transmitted through heat conduction must pass through each layer. In the calculation, the relationship between **Figure 7.5** should be used in this case.

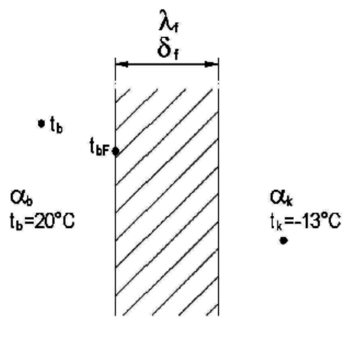
7.4 Heat transmission over multilayer wall

Through an architectural example, let's examine the details of the calculation and also the energy benefits of thermal insulation of the wall structure.

We start out from the isolation of a 38 brick wall of a traditional brick building. Wall 38 is a 38 cm thick wall structure built with conventional brick and mortar with the following parameters. We know the total heat transfer factor of the wall, k. In architecture, they are marked with U. The first question is the thermal conductivity of the wall. Then we heat the wall from the outside with a different thickness of heat insulating material and examine the effect on the wall temperature and heat loss.

Data:

$$t_b = 20°C \; ; \; t_k = -13°C \; ; \; \alpha_k = 20\left[\frac{W}{m^2 \cdot K}\right]; \; \alpha_b = 8\left[\frac{W}{m^2 \cdot K}\right]; \; k(U) = 1{,}5\left[\frac{W}{m^2 \cdot K}\right]$$



Wall structure and temperature data
Figure 7.9

Questions:

a. /What is the thermal conductivity of the wall if its thickness is 38 cm?
b. What is the temperature of the inner surface of the wall?

c. What insulation is required for thick thermal conductivity so that the heat transfer factor of the wall structure is halved?
d. / In this case, what is the internal temperature of the wall?
e. / What thick thermal insulation should the new specifications meet the wall construction? (Decree 7/2008 (V.24.) TNM),
f. / What is the internal temperature in this case?

Solution:
a./ The thermal conduction factor of the wall 38 cm is obtained from the expression of the heat release factor *equation 8.1*

$$k = \frac{1}{\frac{1}{\alpha_b} + \frac{\delta}{\lambda} + \frac{1}{\alpha_k}}$$

Explain the wall data and replace the heat transfer factors:

$$\frac{\delta}{\lambda} = \frac{1}{k} - \frac{1}{\alpha_b} - \frac{1}{\alpha_k} = \frac{1}{1,5} - \frac{1}{8} - \frac{1}{24} = 0,5 \quad \left[\frac{m^2 \cdot K}{W}\right]$$

Then the desired thermal conductivity is:

$$\lambda = \frac{\delta}{0,5} = \frac{0,38[m]}{0,5\left[\frac{m^2 \cdot K}{W}\right]} = 0,72\left[\frac{W}{m \cdot K}\right]$$

This wall structure is an average (brick and mortar) thermal conductivity factor.

b./ The temperature of the inner wall

In the next question, the inner temperature of the wall is determined. To do this, we first calculate the density of the heat flow through the wall.

$$\dot{Q} = k \cdot A \cdot (t_b - t_k)\,[W], \qquad 8.17$$

expression of the heat stream per unit of surface

$$\dot{q} = k \cdot (t_b - t_k)\ [W/m^2]$$

replacing the data, we get it

$$\dot{q} = k \cdot (t_b - t_k) = 1,5 \cdot (20 - (-13)) = 49,5\ [W/m^2].$$

The heat flux density is the same for the individual parts, so $\dot{q} = \alpha_b \cdot (t_b - t_{bf})\ [W/m^2]$ the interior wall temperature required is:

$$t_{bf} = t_b - \frac{\dot{q}}{\alpha_b} = 20 - \frac{49,5}{8} = 13,8^{\circ}C \qquad 8.18$$

In winter, such a cold wall causes a malaise in the apartment, causing a feeling of coldth through the radiation.

c./ The thickness of the required thermal insulation is to reduce the "k"

We have to write the heat transfer coefficient of the thermal insulation wall (two layers), which should be reduced to the original half.

$$k' = \frac{k}{2} = \frac{1{,}5}{2} = 0{,}75 = \frac{1}{\frac{1}{\alpha_b} + \frac{\delta}{\lambda} + \frac{\delta_{sz}}{\lambda_{sz}} + \frac{1}{\alpha_k}}$$

Express the characteristics of the insulating material and enter the data into the formula:

$$\frac{\delta_{sz}}{\lambda_{sz}} = \frac{1}{k'} - \frac{1}{\alpha_b} - \frac{\delta}{\lambda} - \frac{1}{\alpha_k} = \frac{1}{0{,}75} - \frac{1}{8} - \frac{0{,}38}{0{,}72} - \frac{1}{24} = 0{,}6389$$

Knowing the thermal conductivity of the heat-insulating material, determine its thickness:

$$\delta_{sz} = 0{,}6399 \cdot \lambda_{sz} = 0{,}6389 \left[\frac{m^2 \cdot K}{W}\right] \cdot 0{,}04 \left[\frac{W}{m^2 \cdot K}\right] \cong 0{,}026\,[m] = 2{,}6\,[cm]$$

So with a heat insulation material of 2.6 cm, the heat transfer factor **can be reduced by half!** Which means that the heat loss is also reduced to half! This follows *equation 8.17.*

$$\dot{q}' = \frac{\dot{q}}{2} = \frac{49{,}5}{2} = 24{,}75 \left[\frac{W}{m^2}\right]$$

d./ How much does the internal temperature of the wall change with the effect of thermal insulation? The course of the solution as in **b/** question. The *equation 8.18* applied mutatis mutandis:

$$t'_{bf} = t_b - \frac{\dot{q}'}{\alpha_b} = 20 - \frac{24{,}75}{8} = 16{,}9^\circ C.$$

Note that the temperature difference from room temperature has dropped by half from 6.2ºC to 3.1ºC.

e./ Decrease the heat transfer coefficient of the wall to the extent required by Decree 7/2008 (V.24.) TNM.To answer this question, look at the cited provision. The enclosed table contains the required values.

7.3 Table The values of the layered heat transfer factor

Requirements for Layer Thermoconductor Factors [Decree 7/2006 [V.24.]

Building blocking structure	
Exterior wall	0,45
Flat roof	0,25
attic Ceiling	0,30
Heated roofs	0,25
Lower closing ceiling above the arcade	0,25
Lower closing ceiling above unheated cellar	0,50
Facade glazed doors [with wood or PVC frame structure]	1,60
Facade glazed doors [with metal framework]	Z,00
Facade glazed windows with a nominal surface of less than 0,5 m	2,50
Facade glass	1,50
Roof top illuminated	2,50
Roof flat window	1,70
Facade glazed door	3,00
Door between facade or heated and unheated spaces	1,30
Wall between heated and unheated spaces	0,50
Wall between adjacent heated buildings	1,50
Ground contact wall between 0 and -1 m	0,45
Floor lying on the floor in a 1.5 m wide lane along the perimeter (can be replaced by a heat insulation of the same resistance on the plinth)	0,50

According to the table, the heat transfer of the outer wall $k(u) \le 0{,}45 \frac{W}{m^2 \cdot C}$

The solution can be performed as described in **c**/ above. That is, calculating the thickness of the thermal insulation layer by the required heat transfer factor.

$$k'' = 0{,}45 = \frac{1}{\frac{1}{\alpha_b} + \frac{\delta}{\lambda} + \frac{\delta_{sz2}}{\lambda_{sz}} + \frac{1}{\alpha_k}}$$

$$\frac{\delta_{sz2}}{\lambda_{sz}} = \frac{1}{k''} - \frac{1}{\alpha_b} - \frac{\delta}{\lambda} - \frac{1}{\alpha_k} = \frac{1}{0{,}45} - \frac{1}{8} - \frac{0{,}38}{0{,}72} - \frac{1}{24} = 1{,}528$$

$$\delta_{sz2} = 1{,}528 \cdot 0{,}04 = 1{,}528 \left[\frac{m^2 \cdot K}{W}\right] \cdot 0{,}04 \left[\frac{W}{m^2 \cdot K}\right] \cong 0{,}061\ [m] = 6{,}1\ [cm]$$

Note: Heat insulation of this thickness is not available, but the nearest size is 10 cm.

f./ How much does the inner wall temperature change with the new, thicker thermal insulation?

$$\dot{q}'' = k'' \cdot (t_b - t_k) = 0{,}45 \cdot (20 - (-13)) = 14{,}85\ [W/m^2]$$

$$t''_{bf} = t_b - \frac{\dot{q}''}{\alpha_b} = 20 - \frac{14,85}{8} = 18,14°C$$

With the new thermal insulation, the inner temperature of the wall $t_{bf} = 13,8^{\circ}C$ is approx. 4 degrees warm and approx. We achieved a 70% decreesing of heat loss !!!!

Note: When renovating a building's thermal insulation, it is not enough to insulate the walls, but also to replace the doors and windows with better thermal insulation! Most of the heat loss of the apartment is through the doors and windows.

8. A Calculation of heat transfer, heat transfer factor

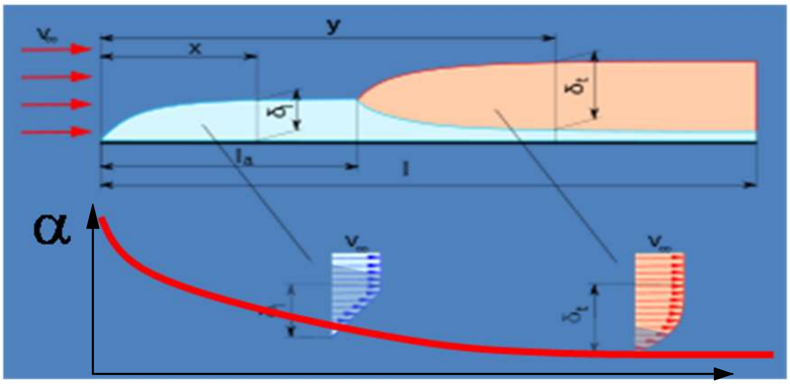
In mechanical engineering, a lot of heat transfer tasks have to be done. Different heat exchanger calculations can produce a wide range of heat transfer conditions. Heat transfer is a way of spreading heat when the flowing continuum gives off or takes heat on a solid surface or a solid surface, interface. For this case, other laws apply to heat conduction. In addition to the thermal conductivity of its material properties, the continuity flow (laminar or turbulent) is of decisive importance. When the heat transfer occurs, phase change (source, condensation) occurs.
The case of laminar flow is closer to heat conduction, while the case of turbulent flow associated with intense mixing. The heat transfer factor is therefore not material. In this chapter we present some calculations without the need for completeness.
Its definition is predominantly based on model experiments, with the so-called "similarity criteria" criterion equations. In the modeling of heat transfer processes, similarity with the Re-number needs to be fulfilled. These non-unit (dimensionless) similarity numbers expressing similarity terms are generally the following:

$Nu = \frac{\alpha \cdot L}{\lambda}$ Nusselt number "L" is the characteristic length

$Pr = \frac{\mu \cdot c}{\lambda} = \frac{\nu}{a}$ Prandtl number where $\nu = \frac{\mu}{\rho}$ „ν" kinematic viscosity,
„μ" dynamic viscosity,
"$a = \frac{\lambda}{\rho \cdot c}$ "the thermal conductivity factor
(see *equation 8.7)*

$Re = \frac{v \cdot \rho \cdot L}{\mu} = \frac{v \cdot L}{\nu}$ Reynolds number

The purpose of the calculations is to define the function Nu = f (Re, Pr), from which the heat transfer factor "α" can be calculated.
For example, let's look at the Nu number calculation on the plane plate. Flat plate parallel to flow. At the beginning of the page the flow is laminar. A boundary layer is formed between the main flow and the sheet. In this layer, the v_∞ away from the sheet decreases to zero. This layer thickens continuously as we progress along the page. Then, at some distance, the boundary layer becomes thicker and the inner layer of the wall continues to remain laminar and the outer layer becomes turbulent. The appearance of the turbulent boundary layer is approx. $Re < 10^5$ above. Calculated $Re = \frac{V \cdot L}{\nu}$ here, L is the distance from the beginning of the plate. (Caution: At pipe flow the L is usually the diameter of the pipe!)



Calculation of the heat transfer coefficient on the floor
Figure 8.1

In this case, the Nusselt number shall be counted as follows:

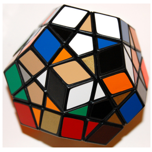
$Re < 10^5$ (laminar flow)

$$Nu_{lam} = 0,664 \cdot \sqrt{Re} \cdot \sqrt[3]{Pr}$$

$Re > 10^5$ (turbulent flow)

$$Nu_{turb} = \frac{0,037 \cdot Re^{0,8} \cdot Pr}{1 + 2,443 \cdot Re^{-0,1} \cdot \left(Pr^{0,66} - 1\right)}.$$

Figure 8.1 shows the change of heat transfer factor "α". It is worth noting that the heat transfer coefficient is very good (high) at the beginning of the rib. If a small rib is used instead of a large rib, on which each boundary begins again, then the same surface can produce a much larger heat transfer. This is the principle of the Forgó small ribbed heat exchanger. See later Heller-Forgó cooling system.



Calculate the heat transfer coefficient of a flat plate heat rib and calculate how much the delivered heat flow along one 1m wide sheet! Air flowing along the flat plate (cooler rib).

Data:

$$v = 2\frac{m}{s};\ \rho = 1,2\frac{kg}{m^3};\quad \mu = 17 \cdot 10^{-6}\,Pa \cdot s;\quad \lambda = 0.026\frac{W}{m \cdot K};\quad L = 10\,cm\quad;\quad c_p = 1008\frac{J}{kg \cdot K};$$

$$\alpha_k = 20\left[\frac{W}{m^2 \cdot K}\right];\ t_\ell = 20°C;\ t_{borda} = 50°C$$

Solution: First we need to calculate the Reynolds number to determine the flow type.

$$Re = \frac{v \cdot L}{\nu} = \frac{v \cdot L \cdot \rho}{\mu} = \frac{2 \cdot 0,1 \cdot 1,2}{17 \cdot 10^{-6}} = 14117$$

So the boundary layer is laminar because $Re < 10^5$.

Next, we need to determine the Prandtl number:

$$Pr = \frac{\nu}{a} = \frac{\mu \cdot c}{\lambda} = \frac{17 \cdot 10^{-6} \cdot 1008}{0,026} = 0,66$$

Then we can apply the related context:

$$Nu_{lam} = 0,664 \cdot \sqrt{Re} \cdot \sqrt[3]{Pr} = 0,664 \cdot \sqrt{14117} \cdot \sqrt[3]{0,66} = 68,18$$

The heat transfer factor can then be calculated from the definition of the Nusselt number:

$$\alpha = Nu \cdot \frac{\lambda}{L} = 68,18 \cdot \frac{0,026}{0,1} = 17,72\ \frac{W}{m^2 \cdot K}.$$

Let's remember that: the heat transfer factor inside the building ($8\frac{W}{m^2 \cdot K}$) and outside the building ($24\frac{W}{m^2 \cdot K}$) was given in this order of magnitude for calculating the heat loss of the building.

And finally, with the Newton expression, we can calculate the heat flow being transmitted:

$$\dot{Q} = A \cdot \alpha \cdot (t_{rib} - t_{\ell}) = 0{,}1 \cdot 1 \cdot 17{,}72 \cdot (50 - 20) = 53{,}16 [W]$$

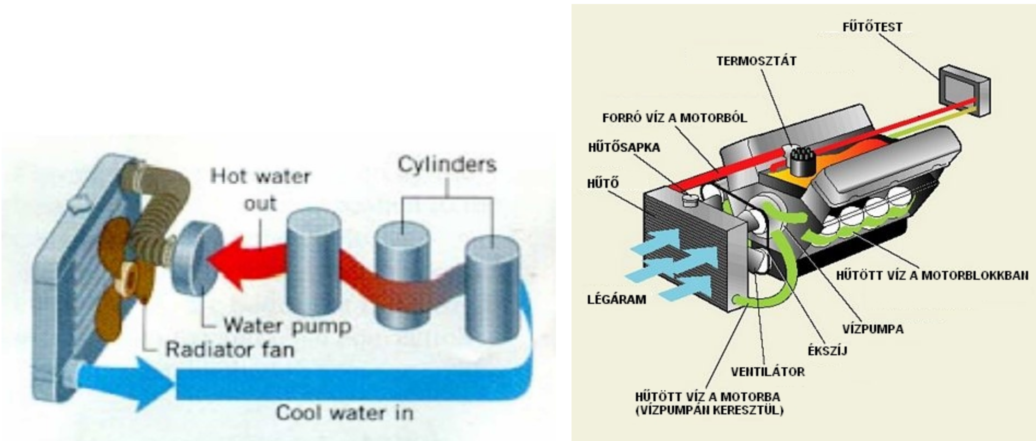
$$\dot{q} = \frac{\dot{Q}}{A} = \frac{53{,}16}{0{,}1 \cdot 1} = 531{,}6 [W]$$

Compared to the building's heat output, apparently there is almost two orders of magnitude greater than the delivered heat flow. The heat sink is designed for good heat loss.

8.1 Heat transfer in forced flow

For forced circulation in circular pipes, the magnitude of the heat transfer factor depends most on the nature of the flow, because the nature of the flow mainly affects the physical parameters of the boundary layer at the pipe wall, and determines the degree of heat transfer.
In the nature of the flow when critical speed is reached - Reynolds results - switching suddenly occurs, when the **laminar (layered)** flow becomes **turbulence**.
In the case of fluids flowing in pipes, the transition value is Re≈2320. The velocity in the tube changes in the cross-section as a function of the radius. The number in the Re-number is the average velocity of the medium, which is the ratio of the flowrate (flowrate Q) and the cross-section (A) of the flow per unit of time v = Q/A. In duct flow, "D" is the inner diameter of the tube and the kinematic viscosity in the denominator. Instead of the diameter D, the typical size may be the pipe length or the outside diameter of the tube.

$$Re = \frac{v \cdot D}{\nu}$$

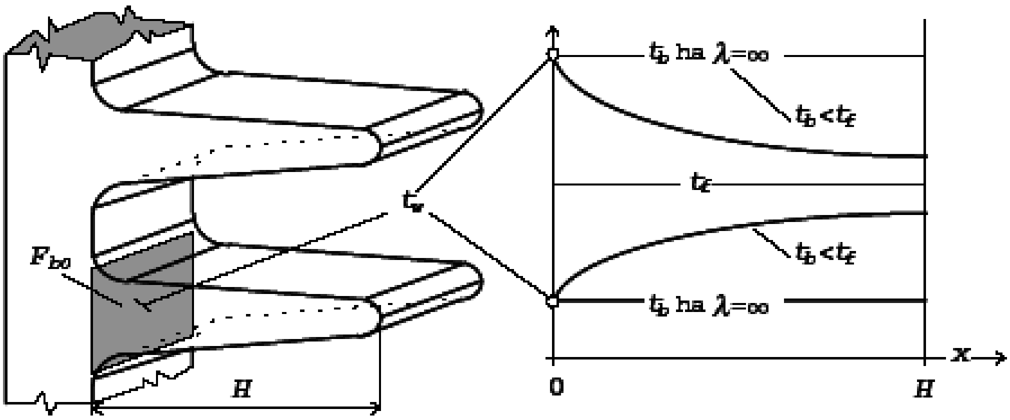


Forced flow of refrigeration system
Figure 8.2

Cooling rib
To improve the transfer of heat, the surface ribbing is often used in mechanical structures. **Figure 8.2** also shows such a solution, such as a refrigerator. Surfaces over which the

resulting heat transfer coefficient is too small are provided with ribbing to increase the size of the surface and thus the size of the heat stream. However, this increased surface can not be taken into account in its total size, as the temperature decreases along the ribs from the root of the rib. This decrease in temperature is dependent on the formation of the rib and the so-called ri-efficieny are taken into account.

$$\eta_{brib} = \frac{t_{rib\,average} - t_2}{t_{root} - t_2}$$



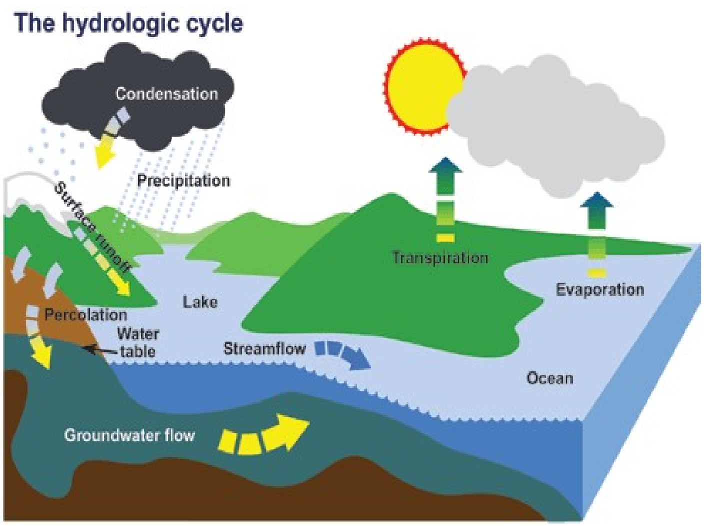
Cooling rib efficiency
Figure 8.3

8.2 Heat transfer in free flow

During heat transfer, it is often the case that heat transfer occurs during heat transfer because of the density difference. We are talking about free flow. The Gr number is the ratio of the buoyancy and friction force (Franz Grashof). Consequently, the Gr number is characteristic of the flow as the Re number, but for flows where the flow is not forced, but is generated by the difference in density caused by the temperature difference.

$$Gr = \frac{\beta \cdot g \cdot \Delta t \cdot \ell^3}{\nu^2}$$

In the context, "β" is the cubic thermal expansion factor of the medium, "ℓ" the linear size characteristic of the flow (for example, in the case of a vertical pipe, the length of the pipe, but in the case of a horizontal pipe diameter). The denominator has kinematic viscosity. The difference in temperature 'Δt' is the difference between the temperature characteristic of the flow medium and the typical temperature of the wall. The different material characteristics must be taken at the temperature corresponding to the arithmetic mean of the two temperatures mentioned above.



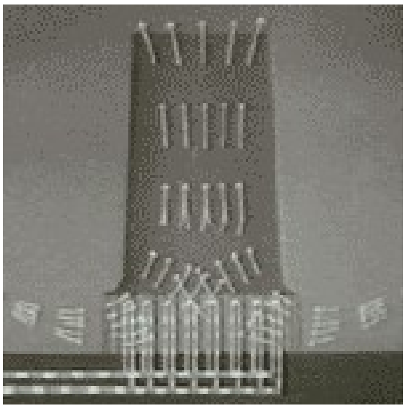
Free flow of air

https://jeremyjschmidt.com/2013/03/21/the-hydrologic-cycle-where-are-the-people/

Figure 8.4

Heller-Forgó's cooling system, which is a brilliant invention of two Hungarian engineers, is very much used in our thermal studies. Here is a description of the Wikipedia article below:

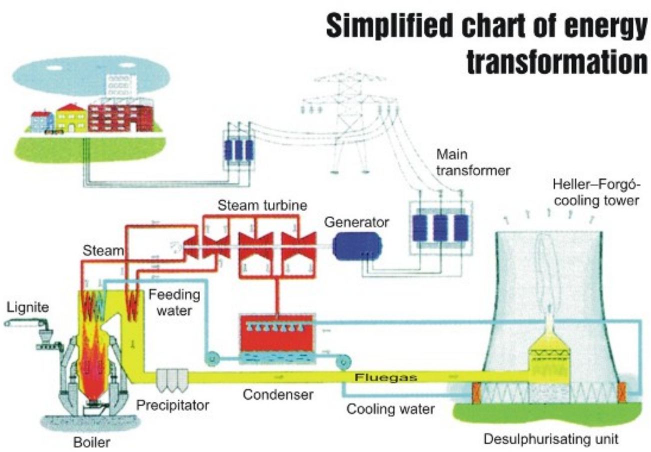
"The Heller-Forgó refrigerator is the invention of two Kossuth-winning Hungarian engineers. This is an indirect air-cooled system in which condenser water coolant is cooled down in a closed system, free of water and air. The two main components of the system are the mixing condenser and the preheated heat exchanger for cooling the cooling water.

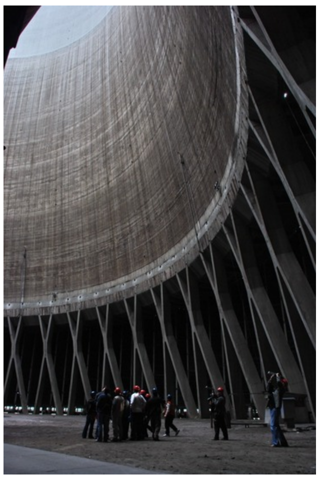


Free flow in cooling towers (wet cooling towers)

http://www.matud.iif.hu/2011/08/16.htm

Figure 8.5





The Heller-Forgó cooling system, the inside of the cooling tower

http://www.mert.hu/en/power-station

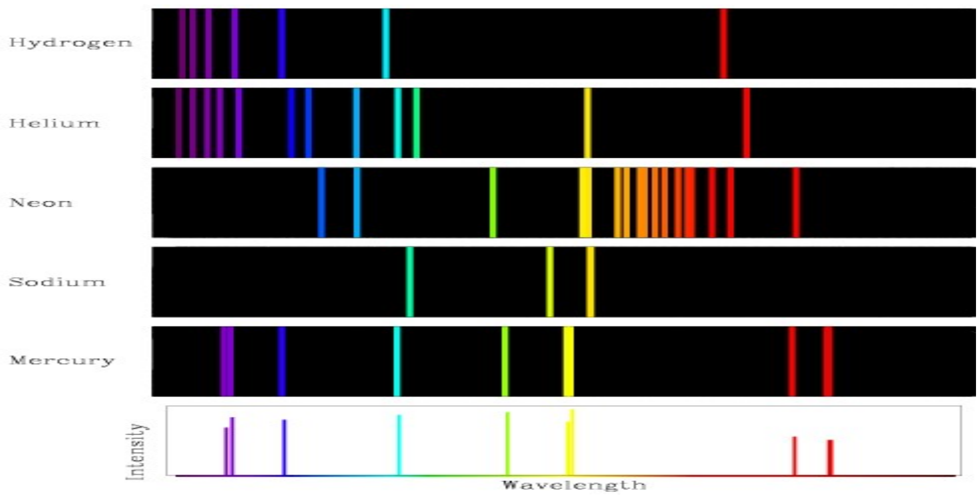
Figure 8.6

Its significance was recognized by Ignác Gorjanc and László Károly of the Jászberény Metal Press and Millery Factory - the legal predecessor of the LEHEL Refrigeration Plant - and with the creation of the production world-famous world news became truly world-famous. The refrigerator is the product - besides the refrigerator - to which the Lehel Refrigeration Plant can be proud of. Two Hungarian engineers, László Heller and László Forgó created plans for air-condensing equipment to solve the problem of the high water demand of thermal power plants so far. The Heller-Forgó refrigerator - so spread its popular name - is a major invention, which has been observed in many parts of the world.
The principle and the mixing capacitor are applied to László Heller, and the heat exchanger for cooling the heat exchanger is edited by László Forgó. The essence of their invention was that condensed steam from steam turbine from the power plant, vacuum steam, by cold water injecting and liquefying. The still warm water goes to the preheated heat exchanger, cools down and becomes recyclable in a closed loop without evaporation.
It is a one-time press release, which is why the essence of the invention is well-known: "The tremendous water demand of thermal power plants stems from the fact that in coal-fired thermal power plants, 50% should be removed by water cooling. Therefore, the daily water demand of a medium thermal power plant corresponds to the total daily water consumption of Budapest. "
During this period of 1957-1958 the news of this very significant Hungarian invention spread in wider circles. In the first case, he presented the prototype of Heller-Forgó's refrigerator here at the 1957 Budapest International Fair. In May 1958, the invention was awarded the Grand Prix of the World Expo in Brussels and soon set off on the path of the world champion.

First, the largest electric company in the UK, English Electric Company, concluded a very large patent contract and in 1958 China ordered the Heller-Forgó equipmqnt. The Lehel equipmqnt.was well-conceived when it saw excellent fuel economy in refrigeration production. In the summer of 1959, the production of refrigeration was started and at the end of the year the first refrigeration units were installed in the Dunaújváros cooling tower. Then, in September 1959, the order in England started to run. Then, at the time in West Germany (Germany), Ibbenbüren built a cooling tower in 1967 following the work of Jászberény. The Pearl Visonta plant was built with Heller-Forgó cooling. Iran was built in 1985, where a Hungarian assembly team worked on the installation of cooling towers. The refrigeration plant supplies a large amount of refrigeration components for domestic and foreign equipment. Exportations were initially carried out by KOMPLEX and then by TRANSELEKTRO Hungarian Electricity Foreign Trade Company. In 1991, Electrolux purchased the refrigeration plant, the refrigeration production did not fit into the Electrolux group's profile and did not claim it. In 1992, EGI purchased the machines and tools, the right to manufacture and leased the factory hall. In 1992, GEA purchased EGI, then established GEA EGI Energy Management Co. and built a new factory hall next to the Refrigeration Plant. "

9. Heat radiation

When energy is communicated with a substance, its internal energy is increased, the state of motion of its molecules and atoms changes, for example the rotation of atoms in the molecules, the vibration of the atoms, the path of the electrons within the atoms, the molecules and the molecules, atoms will be higher. These levels can only be of definite value, according to the fact that the particles can only pick up a certain amount of energy. If the energy transfer ceases or is below an equilibrium level, the bodies seek to return to a lower energy level. In doing so, photons are emitted, electromagnetic waves are emitted.
Various bodies radiate differently. The spectrum of the electromagnetic wave emitted is considered to be continuous if the spectral intensity of radiation is a continuous function of wavelength, wavelength, and wide wavelength range from zero. Such radiation is shown by bulbous bodies and liquids. The atoms of the bulbous gases show a line-like spectrum where the spectral intensity density of the radiation differs from zero only in a small wavelength range; in this, a spectrum is visible. **Figure 9.1** shows some line spectra.

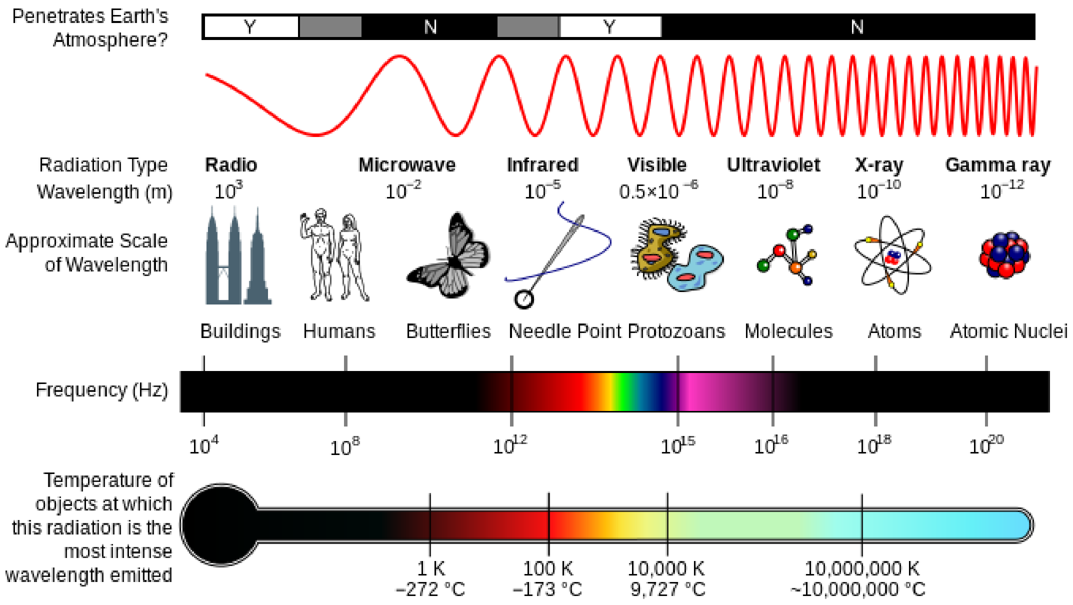


Linear spectrum of gases

http://www.mozaweb.hu/Lecke-mozaWeb-A_feny-Szinkepek-99590

Figure 9.1

We continue to deal with continuous spectra.
Each body emits electromagnetic radiation. At low temperatures (up to room temperature), the energy thus emitted is virtually negligible while in the range of high temperatures it becomes significant. Due to the complexity of the mathematical apparatus required for a precise quantitative description of the energy in the form of electromagnetic waves in the form of electromagnetic waves and the transformation to other forms of energy, we use a simplification descriptor model to determine the correlations necessary for calculating the heat flow calculation of the technical practice. Transmission material or medium without heat spread (electromagnetic radiation). Electromagnetic radiation: the transformation of the electrical and magnetic field strength changes over time. Electrical field strength, magnetic field strength, and propagation direction form a right-angled, right-angled vector system. The wavelength distribution of electromagnetic radiation can be seen in **Figure 9.2**.



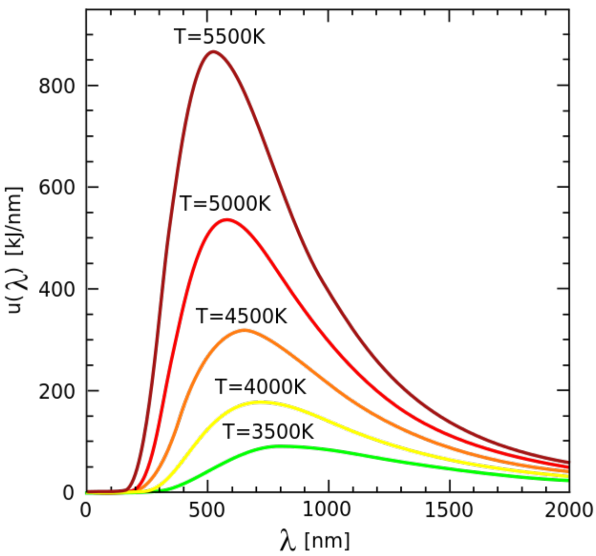
Allocation of electromagnetic radiation

https://en.wikipedia.org/wiki/Electromagnetic_spectrum#/media/File:EM_Spectrum_Properties_edit.svg

Figure 9.2

Linear spectrum of gases

From the low-frequency radio waves the electromagnetic rays are shown through the range of visible light to gamma rays. The wavelength of the visible light ranges from 750-400 nm from



Planck's law

http://hu.wikipedia.org/wiki/Elektrom%C3%A1gneses_sug%C3%A1rz%C3%A1s

Figure 9.3

the red to the violet range (10^{-9} the nano multiplier).

Electromagnetic waves are partially transmitted by a body (diathermicity factor d≤1), reflects (reflection factor $r \leq 1$), absorbs (absorption, absorption coefficient of a≤1).

$$a + r + d = 1$$

Based on the temperature-dependent radiation curves of the black body, it first came empirically to the appropriate correlation. Later, the atoms are treated with harmonic oscillators vibrating at a frequency "ω" that can only "hω" (or a finite amount of) energy at a time.
It has also provided a derivation of the relation of radiation and can be considered as the first well-derived quantum mechanic relationship. According to the Planck Act, the black body is diffuse radiant, and the energy emitted depends to a great extent on the absolute temperature of the body. The radiation emitted to the whole halftone intensity as a function of temperature and wavelength is shown in **Figure 9.3**.
The figure shows that with increasing temperature, the intensity of the radiation increases and the wavelength for the maximum intensity is shifted to the left, to the decreasing values.

The basic relationship between the radiation of radiation by the Austrian physicist Joseph Stefan and Ludwig Eduard Boltzmann in the XIX. At the end of the century, and today, Stefan-Boltzmann's law, which gives the radiant density of the unit

$$\dot{q} = \varepsilon \cdot \sigma_0 \cdot T^4 \left[\frac{W}{m^2}\right]$$

where $\sigma_0 = 5.67 \cdot 10^{-8} \left[\frac{W}{m^2 \cdot K^4}\right]$ the Stefan-Bolzmann factor.
All radiations of each body are compared to the absolute black body, expressed by the relative blackness degree.
Those bodies whose emission factor is independent of λ, is called a color body. If the emission factor is independent of the wavelength, it is a gray body in terms of heat radiation. Gray bodies are therefore diffuse radiators that transmit a constant proportion of the energy of the black body on every wavelength, such as the energy emitted by the gray bodies on the basis of the Stefan-Boltzmann law:

$$\dot{q} = \varepsilon \cdot \sigma_0 \cdot T^4 \left[\frac{W}{m^2}\right].$$

Hereinafter, we only deal with gray bodies. The relationship between emission and absorption capability is described in the KIRCHHOFF Law, which states that the absorption and absorption capacities of the bodies in the given direction and wavelength radiation are the same. From this lawfulness it follows that the black body, which means that the black $\varepsilon = a = 1$ body is not only the body with maximum absorption but also the maximum energy emitter. Since the latter is related to the radiation of other bodies, $\varepsilon < 1$ the emission factor exists.
For convenience only, the Stefan-Boltzmann law is usually written in such a way that the order of magnitude of the Boltzmann constant is written to the absolute temperature divider, so it can be counted with smaller numbers, more comfortable

$$\dot{q} = \varepsilon \cdot 5{,}67 \cdot \left(\frac{T}{100}\right)^4 \left[\frac{W}{m^2}\right]$$

The heat exchange between radiating bodies is, according to the Stefan-Boltzmann law, proportional to the difference between the four powers of the two temperatures

$$\dot{q} = \varepsilon_{red} \cdot 5{,}67 \cdot \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] \cdot \varphi_{12} \left[\frac{W}{m^2}\right]$$

In the context, 'ε_r' is defined as the so-called 'blackness' of the two surfaces. A reduced degree of blackness, and 'φ_{12}' can be defined as the size and relative position of the two surfaces (the normal positioning of the normal ones) irradiation factor.
In the context of the heat flow, the reduction of the reduced blackness and the irradiation factor usually requires very complicated geometric calculations. Therefore, here, without the deductions, the correlation of the reduced degree of blackness is given only in two cases where the irradiation is perfect or nearly perfect ($\varphi_{1.2} = 1$),

- the two surfaces are parallel, close to each other and approx. of the same size,
- a body ('1') with one surface is located inside a closed surface ('2') formed by the other body.

Thermal imaging has developed a lot in recent decades. Today, temperature measurement based on radiation measurement has become almost commonplace.
The details, principle and diagnostic use of the thermal imaging measurement are described in more detail in later subjects. Here are just a few details of the possibilities. **Figure 9.4** illustrates the warming of an electric motor and the heating of a circuit element with a thermal camera.

The following chapters will deal with different chapters of flow technology. To date, our discussion on heat will be linked to many chapters in the chapters.

10. Hydrostatic

Most of the following chapters are made based on notes [16] and [17]. Where I did not specifically mark it. I used the pictures and texts of these notes.
In the first chapter of the book we discussed the medium, continuum expressions. The subject the mechanics of fluids. This includes resting and moving liquids. In the previous chapters, if not emphasized, we examined the so-called quasi-static state changes. These are relatively slow processes. Mass forces do not play a decisive role in the process. The flow technology also has a chapter where motion does not have a role, and it only deals with dormant fluids, this is hydrostatic.

10.1 The concept of pressure

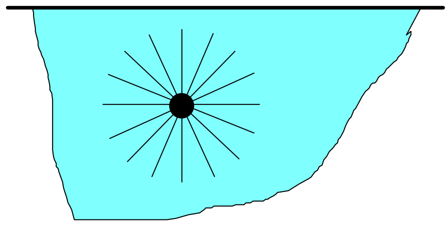
The main characteristic of fluids is pressure. Pressure is the force perpendicular to the surface per unit surface or otherwise the perpendicular pressures force and the surface ratio:

$$p = \frac{F}{A} \qquad 10.1$$

Two important principles have been put forward by ***Blaise Pascal*** (1623-1662) French mathematician and philosopher in relation to pressure (the SI unit of the pressure is named after him):

- At one point, pressure is the same in all directions, as illustrated in **Figure 10.1**.
- On the solid wall defining the liquid, the force from the pressure acts perpendicularly.

These statements are often called Pascal laws.



The effect of pressure at one point
Figure 10.1

Tensile stresses in rest liquids only occur very rarely, and never for Newtonian liquids. In solitary fluids, only tensile stresses occur. Pressure is a scalar quantity, which is generally a function of space and time $p = p(r,t)$, that is, the scalar space, that is $p = p(x, y, z, t)$, the four-variable function, which is called scalar vector space. (The independent variable scalar, the dependent variables are the vector, the later vector-vector spaces, where the independent variables and the dependent variables are also vectors. For example, the velocity vector can be characterized by a vector space.) Similarly, the scattering can be used to describe the temperature $T = T(x, y, z, t)$ distribution, or the density distribution in the airspace.

Scalar spaces are characterized by level surfaces (contour lines) that connect the points of the plane (or plane) in which the physical variable is the same. (E.g. isotherms are constant temperature points in space.)
To characterize the location change of the scalar spaces, we use a vector quantity, whose "x, y and z" components are proportional to the size of the described physical quantity in x, y and z direction:

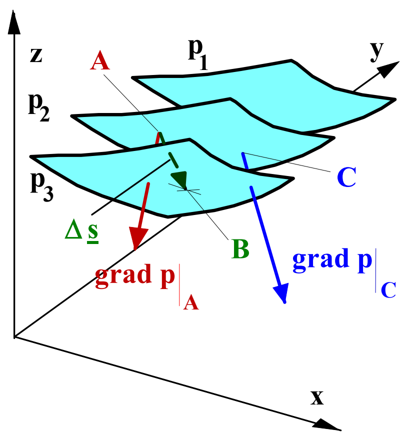
10.2

$$\text{grad}p = \underline{\nabla} \cdot p = \frac{\partial p}{\partial x}\underline{i} + \frac{\partial p}{\partial y}\underline{j} + \frac{\partial p}{\partial z}\underline{k} = \frac{\partial p}{\partial \underline{r}}, \text{ ahol } \underline{\nabla} = \frac{\partial}{\partial x}\underline{i} + \frac{\partial}{\partial y}\underline{j} + \frac{\partial}{\partial z}\underline{k}$$

Properties of the gradient vector:

- points to the fastest growing scalar site,
- its length is proportional to the size of the change and
- perpendicular to the level surface (level line).

Figure 10.2 shows a spatial pressure distribution, its three levels of surface. The pressure increases downwards $p_3 > p_2 > p_1$. Select two close "A" and "B" points on two close levels. The vector $\Delta \underline{s}$ binds them with the base point at point "A". The "p" skeleton "Δp" change between "B" and "A" in linear approximation is given $\Delta p = p_3 - p_2 \cong \text{grad}p|_A \cdot \Delta \underline{s} = \frac{\partial p}{\partial x}\Delta x + \frac{\partial p}{\partial y}\Delta y + \frac{\partial p}{\partial z}\Delta z$ by the scalar formula.



Gradient vector
Figure 10.2

If the quotient is formed $\frac{\Delta p}{\Delta s}$ and the "Δs" size holds for zero, then we get the direction derivative for that direction, equal to

$$\left.\frac{dp}{ds}\right|_s = \text{grad}p|_A \cdot \frac{\Delta \underline{s}}{|\Delta \underline{s}|} = \text{grad}p|_1 \cdot \underline{e}$$

expression. (See the scalar product of two vectors in **Figure 12.6**). "$\underline{e}$" is a unit vector pointing to the selected direction. (Referring to the concept of the derivative in relation to the substantive or full derivative of the velocity vector, we return to **Chapter 12**.) If the unit vector "$\underline{e}$" is parallel to the gradient vector, then the direction derivative is the largest change in the given point.

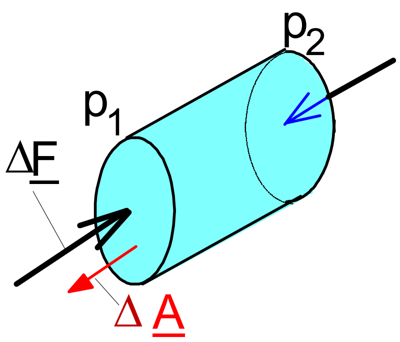
The absolute value of the gradient vector starting from "C" is greater than the start point "A" because in the vicinity of the "C" point the pressure changes at a shorter distance than the "A" point. Here, too, the pressure changes $\Delta p = p_3 - p_2$, but in a shorter way, because the distance between the two points "B" and "C" is smaller than before.

10.2 Calculating the compressive force from the pressure

The pressure from compression in the most common case a

$$\underline{F} = -\iint_A p \cdot \underline{dA} \qquad 10.3$$

can be given. The surface "A" is vector normal from the surface to the outside, and the compression force can only press the surface, so it is directed perpendicularly to the surface, so the negative sign must be in front of the integral signal. In some cases, the pressure change due to fluid weight is negligible compared to the pressure inside the fluid. In a thought, delimit a cylindrical part inside the fluid. The position of the cylinder is arbitrary. Examine the resultant force of the cylinder axis.



Pressure in weightless fluid
Figure 10.3

The forces acting on the lower and upper sides are opposite, but they are of equal size to balance

$$\Delta A \cdot p_1 = \Delta A \cdot p_2 \ .$$

It is easy to see that the condition has to be fulfilled. After the position and height of the cylinder were chosen as desired, the pressure in the continuous weighted liquid space was the same everywhere.
This is utilized by hydraulic presses and hoists. The essence of their structure is a small and a large piston that extends into a common fluid space. The pressure of the fluid is the same at all points, so the force acting on the small plunger increases the structure in proportion to the pistons. On this principle, with a small structure, a 100-fold or even 1000-fold increase in power is easily available.
The even distribution of pressure is also used by the pneumatic tires. When pressed, the pressure is virtually no increase, the supporting force increases proportionally with the bearing surface. Tires with a lower tire require a larger surface, so it gets bigger.

10.3 Basic Equation of Hydrostatic

If the weight of the liquid cannot be neglected at the prevailing pressure then the distribution of pressure in the liquid will not be constant. This is usually the case for still water. Test the former fluid cylinder in the gravity field as shown in Figure 10.4 below. The density of the fluid is "" the difficulty acceleration "g" pointing in the direction of the downward "z" axis. By describing the vertical forces acting on the cylinder, we get the following equation:

$$p \cdot \Delta A + \Delta z \cdot \Delta A \cdot \rho \cdot g - (p + \Delta p) \cdot \Delta A = 0$$

In addition to the compressive forces, we had to take into account the weight of the liquid in the cylinder, which is the second member.
Simplified and arranged we get it

$$\frac{\Delta p}{\Delta z} = \rho \cdot g \ .$$

If $\Delta z \to 0$, then a

$$\frac{\Delta p}{\Delta z} = \frac{dp}{dz} = \rho \cdot g$$

$$\lim \Delta z \to 0$$

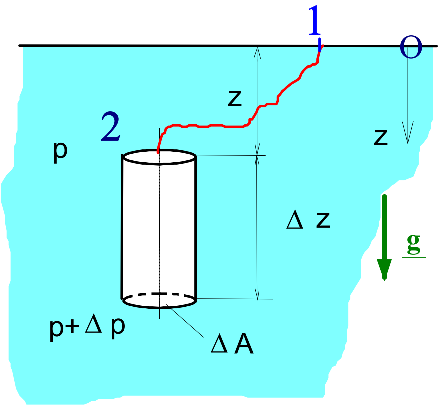
expression is obtained. It is easy to see that in the case of a general position coordinate system or a generic field vector, the above expression can be rewritten

$$\operatorname{grad} p = \rho \cdot \underline{g} \qquad 10.4$$

vector equation, which is called **the basic equation of hydrostatic**.

The basic equation of hydrostatic 10.4 states that a
- the greatest change in pressure is in the direction of the field strength vector,
- the size of the change is proportional to the force of the field vector and the density.

10.3.1 Hydrostatic equation in gravity field



Balance in gravity force
Figure 10.4

It is known from physics that if a force field is conservative, then there is a potential "U" that has the following relation to the field vector (negative sign agreement result):

$$\underline{g} = -\text{grad}U \qquad 10.5$$

As we know in the conservative field on a closed curve, it integrates the field vector with a zero result. Working on a unit of mass, any closed curve (ℓ), is a zero, mathematical expression

$$\oint_{\ell} \underline{g} \cdot d\underline{s} = 0$$

An equivalent mathematical condition for the existence of potential is that

$$\text{rot}\underline{g} \equiv \underline{0}$$

The mathematical and physical meaning of rotation is discussed in **section 4.4** in connection with velocity. Hydrostatic equilibrium can only be imagined in conservative forces. (Among other things, the Earth's atmosphere is constantly in motion because there is a Coriollis field that rotates the Earth, which is whirling).

Replace Equation *10.5 in Equation 10.4*

$$\text{grad}p = -\rho \cdot \text{grad}U \qquad 10.6$$

expression is obtained.
It can be seen that the constant potential surfaces coincide with the constant pressure surfaces. (Evidence is excluded, but it can be found in [2]).
If the density $\rho = \text{const.}$ **i.e. density is constant**, then distributed by density and typing behind the gradient signal, then rearranging the

$$\text{grad}\frac{p}{\rho} + \text{grad}U = 0$$

expression is obtained. Let us get a common gradient sign behind the scalar values so we get the

$$\text{grad}\left(\frac{p}{\rho} + U\right) = 0$$

expression.

The change in a scalar quantity can be zero if the volume itself is constant in space,

$$\frac{p}{\rho} + U = \text{constant}$$

There is also a relationship between any two points in the liquid space, in Figure 10.4 can also be written between point 1 on the surface and 2 points on the top plate of the cylinder, replacing the

$$\frac{p_1}{\rho} + U_1 = \frac{p_2}{\rho} + U_2 \qquad 10.7$$

Often, this latter equation is also called the basic equation of hydrostatic. Of course, it can only be applied to a constant density medium between two points in the fluid space that can be connected to a continuous line. In other words a

$$\frac{p}{\rho} + U = \text{constant}$$

which means that if the potential increases in a liquid space, the pressure drops and if the potential decreases, the pressure must increase.

As we know in the difficulty force field, the potential is to work on a unit of mass versus field strength. So if we move upwards, then the potential increases, if we go downwards, it will decrease. In Figure 10.4, in the case of a downward coordinate system, the potential is a

$$U = -gz + U_0$$

expression. Select $U_0 = 0$, which can be done without further action, because we will only be able to calculate potential differences in the equation. Replaced in *equation 10.7* and using the coordinate "z" in zero 1, we get the following:

$$\frac{p_1}{\rho} + 0 = \frac{p_2}{\rho} - g \cdot z \,. \qquad 10.8$$

from which the well-known term is that the pressure moves linearly from the surface of the liquid to the surface:

$$p_2 = p_1 + \rho \cdot g \cdot z \,.$$

10.3.2 Solving hydrostatic problem in general

Hydrostatic tasks can be solved by relatively well-defined steps:

1. First we choose a suitable coordinate system in which the potential function can be written.
2. We select suitable points (at least two). One point where we know the data, the other where we look for (for example) the pressure.
3. Writing the potential function is the next step.
Use Equation
4. $\frac{p_1}{\rho} + U_1 = \frac{p_2}{\rho} + U_2$ if the density is constant or $\text{grad}p = -\rho \cdot \text{grad}U$ the equation is used when the density changes. In the latter case, other additional equations or equations are required to change the density.

10.4 Pressure change in the atmosphere

In the atmosphere, moving from above the surface of the Earth, the pressure, temperature, and density of air varies. The change in air temperature is shown in **Figure 10.6**. In the troposphere, the temperature distribution is nearly linear. The troposphere or cloud zone is the main arena of weather changes. This is approx. 11 km thick layer of Earth's atmosphere. The near-surface temperature varies considerably in both space and time. According to an international agreement, the average surface temperature and the temperature decrease have been standardized by the INA (International Standard Atmosphere), so that the temperature change in the air can be approached $T = T_0 - a \cdot z$ by the term.

Earth and its atmosphere
Figure 10.5

"z is an upward pointing coordinate from the ground. The value of "a" gives the degree of temperature drop by meter, the standard value is a=0.00648ºC/m, i.e. approx. 6.5 ºC decrease in temperature per thousand meters.

Questions:

a./ Determine the pressure change in the troposphere.

b./ Verify that there is a polytrophic change in pressure and density in the atmosphere characterized by the temperature distribution. The polytrophic state change a

$$\frac{p}{\rho^n} = \text{const.}$$

can be characterized, where "n" is the polytrophic exponent, usually between **1** and the adiabatic exponent "κ". How does "n" and "a" depend on from each other?

c) How does the height depend on the pressure if the temperature does not change (isentropic atmosphere) i.e. "a" = 0? What is the "n" in this case?

d / 10 km in height; $n = 1$ $n = \kappa = 1.4$ and $n = 1.234$ (standard atmosphere) the ratio $\frac{p}{p_0}$ at the ground temperature of 15 ºC?

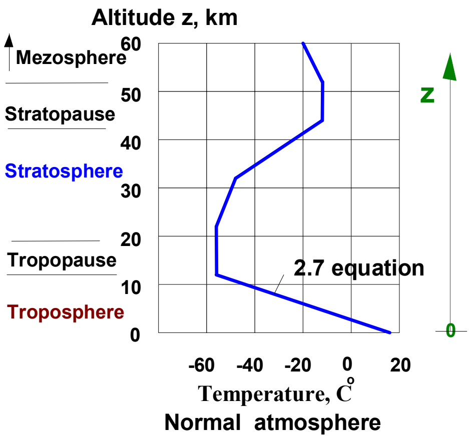


Figure 10.6

Air can be regarded as an ideal gas, its gas constant $R = 287 \frac{J}{kg \cdot K}$.

Solution:

To answer the above questions, it is appropriate to determine the dependence of the pressure on the height.

a./ In the atmosphere, air is assumed to be static equilibrium (no wind blowing). The density of the air varies upwards so that hydrostatic basic law can be used only in differential form, which is the following (*equation 10.4*):

$$\text{grad}p = \frac{\partial p}{\partial x} \cdot \underline{i} + \frac{\partial p}{\partial y} \cdot \underline{j} + \frac{\partial p}{\partial z} \cdot \underline{k} = \rho \cdot \underline{g}$$

Using the above equation for the given case, there is only a change in the upward direction "z", so the expression of the gradient is simplified only to the derivative "z", and the "only" can be written as "-g" pointing "z" axis is contrary to its direction, thus

$$\frac{dp}{dz} = -\rho \cdot g \quad 10.9$$

To $p = p(z)$ calculate the function, the dependence on the density of the pressure must also be given, so that the air is considered to be an ideal gas, i.e.

$$\frac{p}{\rho} = R \cdot T$$

can be described as an equation from which we express the density, 10.10

$$\rho = \frac{p}{R \cdot T}.$$

Finally, we use the standardized linear function of the temperature:

$$T = T_0 - a \cdot z \quad 10.11$$

The *equations 10.10* and *10.11* are replaced by *10.9*, the pressure variation a

$$\frac{dp}{dz} = -\frac{p}{R \cdot (T_0 - a \cdot z)} \cdot g$$

differential equation. By dividing the variables, we get a simple differential equation:

$$\frac{dp}{p} = -\frac{g \cdot dz}{R \cdot (T_0 - a \cdot z)}$$

Boundary condition: $z = 0$ $p = p_0$

Integrated from $z = 0$ to any "z", we get the

$$\ln \frac{p}{p_0} = \frac{g}{R \cdot a} \ln\left(\frac{T_0 - a \cdot z}{T_0}\right) \quad 10.12$$

function, transformed to the pressure change, the next exponential expression obtained from:

$$p = p_0 \cdot \left(\frac{T_0 - a \cdot z}{T_0}\right)^{\frac{g}{R \cdot a}}. \quad 10.13$$

The graph of the function for the various parameters shown in **Figure 10.7** exponentially decreases with the pressure upward.

b./ The relationship between the polytrophic state change and the derived expression is defined in the next subtask. The polytrophic state change is described in *equation 1.20*, which is:

$$\frac{p_0}{T_0^{\frac{n}{n-1}}} = \frac{p}{T^{\frac{n}{n-1}}},$$

which is further shaped to get the connection between pressure and temperature?

$$\frac{p}{p_0}=\left(\frac{T}{T_0}\right)^{\frac{n}{n-1}}.$$

The equation 10.13 can also be applied to a similar shape when used to obtain the resulting pressure distribution:

$$\frac{p}{p_0}=\left(\frac{T}{T_0}\right)^{\frac{g}{R\cdot a}}.$$

The identity of the latter two equations proves that there is a change in the polytrophic state between pressure and density, and here between pressure and temperature. From the comparison we get that

$$\frac{n}{n-1}=\frac{g}{R\cdot a},$$

from which

$$a=\frac{n-1}{n}\cdot\frac{g}{R} \quad \text{or} \quad n=\frac{1}{1-\frac{R\cdot a}{g}}. \qquad 10.14$$

For standard atmospheric conditions a = 0.0065 ºC/m, the corresponding polytrophic change is n = 1.234.

c. / If the temperature does not change, "a = 0", then it can be seen from *10.14* terms that n = 1 is the characteristic of the isothermal state change.

We cannot describe the change of pressure with the *expression 10.13* because at a=0 the function cannot be interpreted. In this case, the limit of the expression should be taken, using the Bernoulli-L'Hospital rule. This solution is a useful practice for the reader.

The other option is to determine the pressure function from equations *10.9* and *10.10*. Temperature is constant, so $T = T_0$. In *equation 10.9* substitute the ρ of *10.10*, then

$$\frac{dp}{dz}=-\frac{p}{R\cdot T_0}\cdot g$$

differential equation we have got.

Rearranged $\frac{dp}{p}=-\frac{g}{R\cdot T_0}$ then, using z = 0, $p = p_0$ and integrating from z = 0 to an arbitrary "z" height, changing pressure to

$$p=p_0\cdot e^{-\frac{g}{R\cdot T_0}\cdot z} \qquad 10.15$$

exponential function have been given.

This function was also illustrated in **Figure 10.7** (a = 0).

d./ Calculate the pressure ratio values for T_0 = 288 K using the terms *10.13* and *10.15.*

d./1 n=1, a=0, so the isothermal atmosphere, then the relation *10.15* is calculated, so

$$\frac{p}{p_0} = e^{-\frac{g}{R \cdot T_0} \cdot z} = e^{-\frac{9.81}{287 \cdot 288} \cdot 10^4} = 0.305$$

d./2 n = 1.4, the value of "a" is calculated from the expression $a = \frac{n-1}{n} \cdot \frac{g}{R}$

$$a = \frac{n-1}{n} \cdot \frac{g}{R} = \frac{1.4-1}{1.4} \cdot \frac{9.81}{287} = 0.00973 \frac{C^\circ}{m},$$

then with the *relation of 10.13* the desired pressure ratio

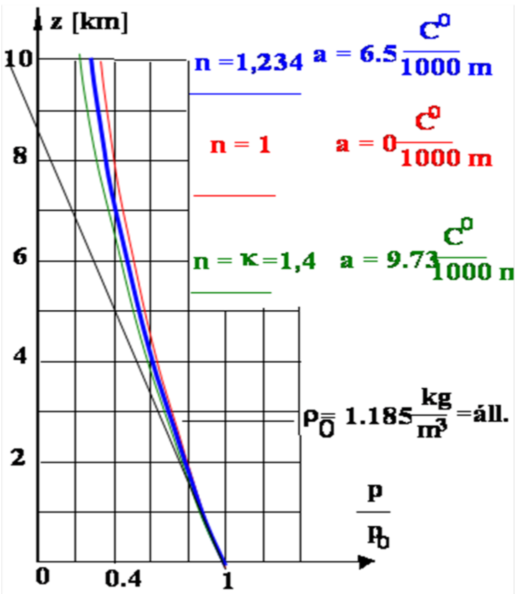
$$\frac{p}{p_0} = \left(\frac{T_0 - a \cdot z}{T_0}\right)^{\frac{g}{R \cdot a}} = \left(\frac{288 - 0.00973 \cdot 10^4}{288}\right)^{\frac{9.81}{287 \cdot 0.00973}} = 0.234$$

d./3 n = 1.234, $a = \frac{n-1}{n} \cdot \frac{g}{R}$ calculate the value of "a"

$$a = \frac{n-1}{n} \cdot \frac{g}{R} = \frac{1.234-1}{1.234} \cdot \frac{9.81}{287} = 0.00648 \frac{C^\circ}{m},$$

then with *the relation of 10.13* the desired pressure ratio

$$\frac{p}{p_0} = \left(\frac{T_0 - a \cdot z}{T_0}\right)^{\frac{g}{R \cdot a}} = \left(\frac{288 - 0.00648 \cdot 10^4}{288}\right)^{\frac{9.81}{287 \cdot 0.00648}} = 0.260$$



Changes in air pressure in the troposphere
Figure 10.7

d./4 If the density is considered as constant

$$p = p_0 - \rho_0 \cdot g \cdot z$$

the pressure can be calculated at a given height, just like the change in pressure in the water, except here with the opposite coordinate direction. With this assumption, even the top of Mt. Everest (8848 m) is running out of the air, although we know that even without breathing more people have been scoured, though at great difficulty, because at this altitude with a breath of air at sea level, 1/3 is received only by the human body.
Compare the graphs of results from different models.

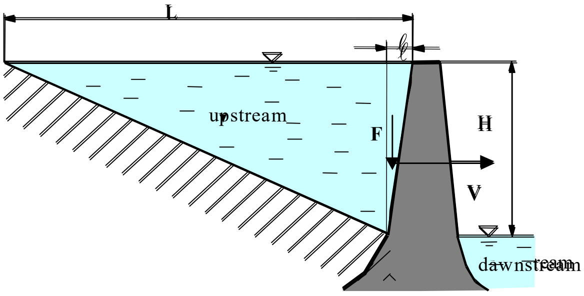
<u>**Figure 10.7**</u> depicts pressure variations with different temperature gradients. In case of constant density. At a height of 8.6 km, the pressure becomes zero, so this model is by no means a good approximation to reality. The other models differ from the INA model by no more than 20%.

Comment:

In reality, the temperature of the atmosphere moves around the layers differently, but the average value is set. The average state has been standardized. The pressure change chart is also used for altitude measurements in aviation. The height measurement is returned to pressure measurement.

10.5 Calculation of force from hydrostatic pressure, Hoover dam

The Hoover dam was built on the Colorado River in 1936. Between the lower and the upper water level "H" is the level difference. The dam is "Z" wide. Approach the volume of water behind the dam with a rectangular "H*Z" and an "L" trowel. In the artificial reservoir behind the dam V = 35 km3 water is located in an irregular shape at 184 km long. The maximum width is 13 km. The size "L" therefore differs greatly from the actual length, but as we will



The force applied to the Hoover dam

http://heaven81.mindenkilapja.hu/html/19288804/render/usa-varosok-indianok-zene

Figure 10.8

see, it is possible to estimate the stored energy. The dam is a convex shell structure in the upward direction. The contour side contour is neither vertical but also exploits the stabilizing effect of the weight of water above it. (Geometry is simplified and data is approximate, so the results are only for scale estimation.)

Data: $H \cong 200\text{m}$; $L \approx 1380\text{km}$; $Z \cong 380\text{m}$; $Z \cong 380\text{m}$

Questions:

a./ What is the horizontal pressure force on the dam and how high is the attack point?

b./ What is the vertical compressive force acting on the dam?

c./ About the amount of space energy stored in the water mass behind the dam? Consider the surface level of the sub-level as a reference level.

Solution:

a./ The overpressure acting on the barrier increases linearly to the depth "H", the resultant distribution system is downwardly linearly increasing and the source of the resulting force is awake at a depth of 2/3H and its magnitude:

$$\underline{F = \rho \cdot g \frac{H}{2} \cdot H \cdot Z = 10^3 \cdot 9.81 \cdot \frac{200}{2} \cdot 200 \cdot 380 = 74.5 \cdot 10^9 \, \text{N}}$$

b./ The vertical force "F" on the barrier is also easy to calculate because it is equal to the weight of the water above the barrier. The water above the barrier is approximately triangular, with one side of the triangle "l", the other "H" and the height of the column is "Z", so

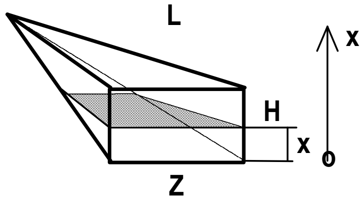
$$F = \frac{2}{3} \cdot H \cdot \ell \cdot 1 \cdot \rho \cdot g = \frac{2}{3} \cdot 50 \cdot 12.5 \cdot 10^3 \cdot 9.81 = 4.08 \cdot 10^6 \frac{N}{m}$$

Comment:

This dam is formed from a level of approx. 7.5 º downward forces, making the barrier more stable because it transfers a smaller torque to the base as if the horizontal "V" force is acting.

c./ The figure shows the same layer of water "x", which should be "dx".

The area of such a trapezoidal layer is:



$$\frac{Z \cdot L}{2} - \left(\frac{H - x}{H}\right)^2 \cdot \frac{Z \cdot L}{2}$$

By multiplying the weight of the layer, with the altitude measured from the underside and integrated, we get all the potential energy:

$$E = \rho \cdot g \cdot \int_0^H \left[\frac{Z \cdot L}{2} - \left(\frac{H - x}{H}\right)^2 \cdot \frac{Z \cdot L}{2}\right] \cdot x \cdot dx = \rho \cdot \frac{Z \cdot L \cdot H}{3} \cdot \frac{5}{8} \cdot g \cdot H$$

In the final formula, the volume of water stored behind the dam was introduced, V

$$E = \rho \cdot V \cdot \frac{5}{8} \cdot g \cdot H = 10^3 \cdot 35 \cdot 10^9 \cdot \frac{5}{8} \cdot 9.81 \cdot 200 = 4.29 \cdot 10^{16}\,\mathrm{Ws} = 11.9\,\mathrm{TWh}$$

The multiplier of 5/8 is obviously the result of the recorded pyramid shape, in reality this multiplier may vary, but the result is a magnitude estimator.

10.6 Pressure gauges and pressure gauges

Pressure measurement in the flow case is just as vital as the measurement of voltage and current in the electrification. In most cases, we do not measure the absolute pressure (vacuum value) but the differential pressure.

The following two main principles are used to measure the differential pressure:

a./ from the height of the liquid column holding the pressure under the hydrostatic law,

b./ Determine the size of the pressure by measuring the deformation of a solid body that changes its shape flexibly. Let's first look at what it is.

10.6.1 Absolute and overpressure

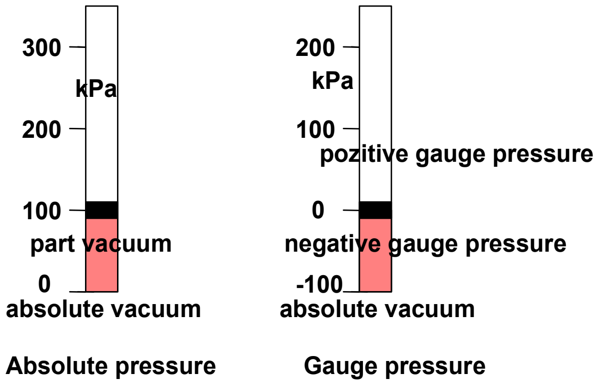


Figure 10.9

When calculating or measure pressure values, we need to know what the reference value of the pressure in the calculation or in the measurement was. In most cases, the reference pressure is the atmospheric pressure and the measured or calculated pressure is "overpressure" or „gauge pressure”. The pressure measured in absolute vacuum is called 'absolute pressure'. In each case it is important to know about the pressure value that is absolute or gauge pressure. There is a very simple connection between the two pressures:

$$p_{abs} = p_{gauge.} + p_{atm} \qquad 10.16$$

1. Absolute vacuum is the lowest possible pressure, so the absolute pressure is always positive.

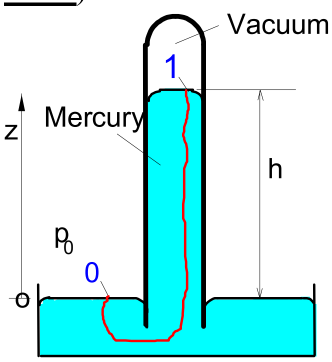
2. The gauge pressure can be negative even if the atmosphere is under pressure, and this is called a vacuum.

3. Atmospheric pressure varies depending on weather and weather conditions, not a constant value.

4. The change of atmospheric pressure varies between the 95 kPa (abs) and 105 kPa (abs) of the Earth's surface or see level. The normal atmospheric pressure is 101.3 kPa (abs).

10.6.2 Mercury barometer

By weight, the atmosphere exerts pressure on the bodies inside it. In the previous embodiment, the change in pressure was determined. The simplest tool for measuring atmospheric pressure is the mercury barometer. Air pressure was first measured by Italian physicist ***Evangelist Torricelli*** (1608-47) in 1643. Approximately 1 m long, one end of the sealed glass tube is filled with mercury to the ground and then put the end of the tube downwards into a container containing mercury. When the captured end is released, mercury is only partially off. Mercury in the tube is approx. 760 mm higher than the surface of the mercury in the outer vessel (see **Figure 10.10**).



Mercury barometer
Figure 10.10

Apply *equation 10.7* for the given task. The figure "0" in the figure is on the surface of the mercury in the vessel and the "1" points in the closed tube. Take the coordinate "z" upwards positive and the origin of the lower mercury surface. At point 1, the saturated vapour of mercury prevails (0.16 Pa, 20 °C), which can be virtually absolute vacuum. So according to the equation $\frac{p_0}{\rho_{Hg}} + 0 = \frac{0}{\rho_{Hg}} + g \cdot h$.

Rearranging $p_0 = \rho_{Hg} \cdot g \cdot h$

from which the height "h" can be expressed, $h = \frac{p_0}{\rho_{Hg} \cdot g}$.

At sea level the normal atmospheric pressure $p_0 = 101\,350\ \text{Pa}$, $\rho_{Hg} = 13600 \frac{kg}{m^3}$ and

$g = 9.81 \frac{m}{s^2}$,

and, thus, the mercury fibre in the barometer is h=0.761m or 761 mm. (An aqueous manometer would show 10.35 m. They use mercury because it is the heaviest fluid available.) As a pressure unit," torr" is also used for Torricelli's memory, although this is not the basic unit of the SI system.

$$1\,\text{torr} = 1\,\text{Hgmm} = 9.81 \cdot 13.6 = 133.4\text{Pa}$$

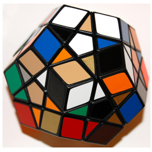
Blood pressure is still given in "torr", eg 120/80 torr of someone's blood pressure.

10.6.3 „U" tube as a manometer

The easiest fluid column pressure gauge is the U-tube. Its operation is based on the hydrostatic equilibrium principle. The difference in pressure between the two tanks shown in **Figure 10.11** shall be determined, p_1-p_2. The medium (water) in the tanks is "ρ". This completely fills the space above the measuring fluid in the U-tube and the measuring line. The measuring medium "ρ_m" does not mix with the density "ρ", so it is separated from the contact surface by a definite surface. If the same pressure on both surfaces of the measuring fluid and the surface tension on the two surfaces are the same, the two surfaces are on the horizontal surface. If different pressure is applied to the two connections of the manometer, the measuring fluid in the "U" tube is exposed. The "U" tube must be the basic law of hydrostatic

$$\frac{p}{\rho} + U = \text{const.}$$

The equation must be written between the appropriate points chosen. The equation should not be applied through the boundaries of liquids, since the density changes dramatically and is therefore not constant. Auxiliary points must be added to the interface and assumed the identity of the pressures.



In the two tanks shown in **Figure 10.11**, the water and the pressure in the water above the water are different at different altitudes. From the bottom of the tanks, pressure is fed to a mercury U-tube manometer. The pressure lines and the "U" pipe above the mercury are saturated with full water.

Data: $\rho_{víz} = 10^3 \frac{kg}{m^3}$; $\rho_{Hg} = 13.6 \cdot 10^3 \frac{kg}{m^3}$; $p_0 = 10^5 \, Pa$

Questions:

a./ What is the difference between the pressure in the two tanks?

Solution:

The solution used for the constant densities $\frac{p}{\rho} + U = const.$ should be used. Considering that the relationship is valid only in one medium. At the boundary boundary, only the pressure of the same is to be assumed. The potential function U can simply be written in a coordinate system whose axis is vertically upward and its origin is at the height of the bottom of the "U" tube at that point

$$U = g \cdot z + const.$$

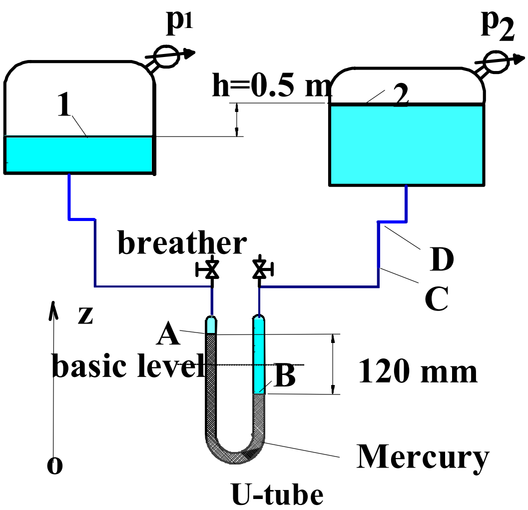


Figure 10.11

a./ Apply the Static Base Equation to the following points:

1-A points in water $\frac{p_1}{\rho_{víz}} + g \cdot z_1 = \frac{p_A}{\rho_{víz}} + g \cdot z_A$

A-B points between mercury

$$\frac{p_A}{\rho_m} + g \cdot z_A = \frac{p_B}{\rho_m} + g \cdot z_B$$

B-2 between dots in water

$$\frac{p_B}{\rho_w} + g \cdot z_B = \frac{p_2}{\rho_w} + g \cdot z_2$$

Coordinates "z" indicate the altitudes measured from the zero level.

Express the differential pressures from the equations above, at this time

$$p_1 - p_A = g \cdot \rho_w \cdot (z_A - z_1)$$
$$p_A - p_B = -g \cdot \rho_m \cdot (z_A - z_B)$$
$$p_B - p_2 = g \cdot \rho_w \cdot (z_2 - z_B)$$

then we add the three equations, so we get the desired differential pressure:

$$p_1 - p_2 = g \cdot \rho_w \cdot (z_A - z_B) - g \cdot \rho_m \cdot (z_A - z_B) + g \cdot \rho_w \cdot (z_2 - z_1).$$

Replacing the example data, the result is as follows:

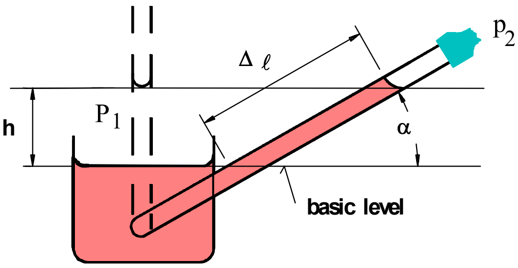
$$p_1 - p_2 = 9.81 \cdot 10^3 \cdot 0.12 - 9.81 \cdot 13.6 \cdot 10^3 \cdot 0.12 + 9.81 \cdot 10^3 \cdot 0.5 = -9.9276 kPa$$

It is preferable to give a positive differential pressure, which is

$$\underline{p_2 - p_1 = 9.927 kPa}.$$

10.6.4 Micromanometer

The micromanometers are based on the principle of the "U" pipe by increasing the reading length, e.g. with the help of incined or curved micrometers, to improve the accuracy of the pressure measurement. **Figure 10.12** shows the sketch of the slanted micromanter.

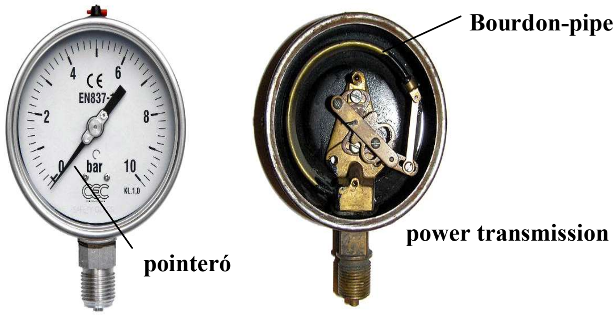


Inclined tubular micromanometer
Figure 10.12

In the case of a given differential pressure $p_1 - p_2$, the angle reading can be increased by changing the angle "α" and the accuracy of the pressure measurement can be increased.
The accuracy of reading can be increased by optical means. This principle is implemented by the Betz micromanometer. (For details, see the "Laboratory Measurement Flow Meters [7]").

10.6.5. Spring pressure gauges

Perhaps the most common pressure gauge is the Bourdon pressure gauge (see Figure 10.13). It was named by French mechanic and inventor ***Eugene Bourdon (1808-1884)***.



Bourdon pipe manometer

Figure 10.13

One end of the circular or spiral bent tube is welded and connected to a pointer. The other end is connected to the pressure measuring point. The pressure inside the tube seeks to straighten the tube. The free end of the tube is moved to a pointer to the pointer that it moves. The scale below the indicator is calibrated correctly. The widespread spread of the instrument is explained by its simple structure and ease of handling.

10.6.6 Pressure transmitters

The proliferation of electrical output devices is becoming increasingly widespread in industrial, laboratory use. This is due to the rapid spread of computer data processing control.

Pressure gauges with an electrical output signal can work differently. One of their kind is the liquid level micromanometers described in the previous chapter, which are converted into electrical signals, and can then be used with appropriate transformation.
A further set of electronic pressure gauges is the one where the pressure causes a resilient element to deform and the electrical voltage or current supplied by the generated deformation sensing is used as output signal. The most commonly deformable element is a membrane used to detect low pressures. The sensitivity and precision of the pressure gauge depends on the material and the geometric dimensions of the membrane. The membrane material greatly influences the accuracy of the measurement, the null error of the pressure gauge and the linearity of its characteristic.
There are also piezoelectric principle, magnetic pressure measuring devices, which are not described here.
After discussing the pressure gauge instruments, let us return to the basic equation of the hydrostatic apparatus.

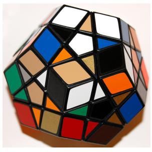
10.7 Fluid in an accelerating system

If a liquid container is standing or moves in a straight line at steady speed, the water contained therein remains calm to the tank. The free surface of the liquid remains horizontal, perpendicular to the field vector. However, if the tank is evenly accelerated, the surface of the liquid is dislodged from the horizontal and then relieves relative to the tank, but the surface changes depending on the direction of the acceleration compared to the original condition. Let us consider, for example, the horizontally accelerating, overturned tank, as shown in **Figure 10.14** during acceleration.
Examining the accelerating system, the phenomena, in addition to the difficulty, also have an inertia force. The effect of the inertia force is as if a mass in the direction of acceleration pulls the mass points in the system in the opposite direction. The field vector indicated by "$\underline{g}_a$" is completely similar to the Earth's gravity force vector. The result of the two field vectors is the role of the force field in the basic equation of hydrostatic. Thus, the liquid surface will be perpendicular to the resulting vectorial vortex, and the greatest change in pressure will also be in the direction of the resulting field.
To solve the hydrostatic task, it is necessary to prescribe the potential of the resulting force field. The potential function can be set as the sum of potentials of each force field. The coordinate system should be picked up with a horizontal "x" and a vertical "z" axis and placed its origin in one of the selected points. In this case, this should be "A".

It is easy to see that the resulting potential function is as follows:

$$U = g \cdot z + a \cdot x + U_0 \qquad 10.17$$

(The value of the potential is selected for zero and zero, and $U_0 = 0$.)
The first member of the function is the potency of the difficulty force, the second is the potential of the accelerating field. The sign of the potentials can be determined by increasing the direction of acceleration. He is growing toward what he can hardly ever go through when he gets into the system, in this case up and right.
If the acceleration and the difficulty field are not perpendicular to one another, they must be broken down into components parallel to and perpendicular to the other (see Áramlástan Példatár [18])



A tanker with water and horizontally accelerating with = 3 m/s2 acceleration. The car is 3 m long and the depth of water in the car is 1.5 m when the car is at rest.

Questions:

a./ What is the inclination angle "α" of the water surface relative to the horizontal?

b./ What is the maximum pressure on the bottom?

c./ What is the minimum pressure on the bottom?

Solution:

a./ First determine the inclination of the surface from the vector chart:

$$\tan\alpha = \frac{g_a}{g_g} = \frac{3}{9.81} = 0.306$$

from where $\alpha = 17^\circ$.

As a result of the acceleration, the rise of the water "h_1" level can also be seen from the figure:

$$h_1 = \tan\alpha \cdot \ell/2$$

$$h_1 = 0.306 \cdot 3/2 = 0.459\text{m}.$$

b./ For the calculation of the differential pressure, we know the shape of the surface as the coordinates of the "0" points "A" and "C" in the recorded system.

The potential function based on the above, according to e*quation 10.17*:

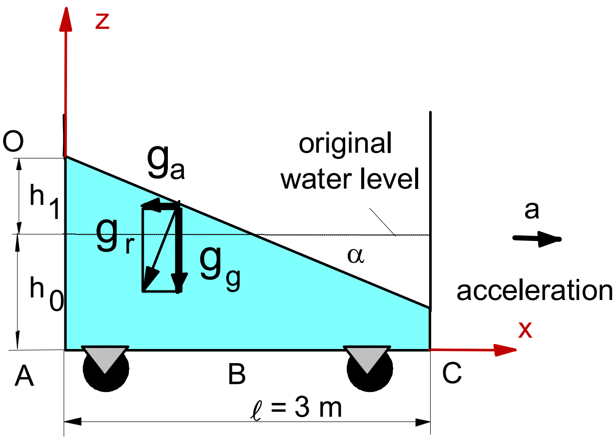
$$U = g \cdot z + a \cdot x$$

Next, you can apply static basic law, first between "0" and "A"

$$\frac{p_0}{\rho} + g \cdot (h_0 + h_1) + a \cdot 0 = \frac{p_A}{\rho} + g \cdot 0 + a \cdot 0$$

From this, the differential pressure:

$$\underline{p_A - p_0 = \rho \cdot g(h_0 + h_1) = 10^3 \cdot 9.81 \cdot (1.5 + 0.459) = 19.21\text{kPa}}$$



Horizontally accelerating car
Figure 10.14

c./ The minimum pressure wakes up at point "C" on the bottom. Similarly to the above, between the "0" and "C" points, the static basic law applies to the

$$\frac{p_0}{\rho} + g \cdot (h_0 + h_1) = \frac{p_C}{\rho} + a \cdot \ell$$

, of which the value of the differential pressure is expressed and replaced:

$$p_C - p_0 = \rho \cdot g(h_0 + h_1) - \rho \cdot a \cdot \ell =$$

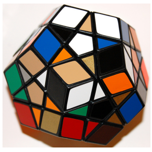
$$= 10^3 \cdot 9.81(1.5 + 0.459) - 10^3 \cdot 3 \cdot 3 =$$

$$\underline{= 10.21\,\text{kPa}}$$

The same result is obtained when the

hydrostatic pressure of a water column above point "C" is calculated at point "C".

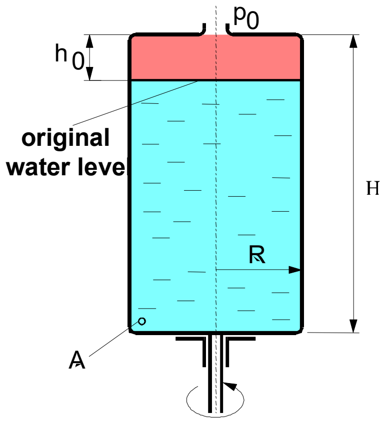
10.8 Liquis in rotating system



Liquid in a rotating tank around the vertical axis, looking from the rotating coordinate system, is also standing. In the accelerating system there is also an inertia force beyond the difficulty force. The effect of the inertia force is as if a mass in the direction of acceleration pulls the mass points in the system in the opposite direction. In the rotating system, the centripetal acceleration keeps the mass points in the system centered towards the center point. We need to capture a field vector opposite to the centripetal acceleration pointing outwards from the center point. The field strength vector marked with "$\underline{g}_a$" can be treated altogether similar to Earth's gravity field vector, its magnitude equals well-known centripetal acceleration.

$$\left|\underline{g}_r\right| = r \cdot \omega^2$$

However, in the rotating space, the force vector depends on the radius, so the potential function is obtained from the radial integral.



Liquid in rotating pot
Figure 10.15

The sign also applies here that the potential needs to increase in the direction of "harder" to go if one gets into the system, in this case it is more difficult to go to the center. The decreasing radius has an increasing potential.
The surface here is perpendicular to the resulting field vector.

$$U = -\int r \cdot \omega^2 \cdot dr = -\frac{r^2 \cdot \omega^2}{2} + \text{const.} \qquad 10.18$$

Figure 10.15 shows the pre-rotation of a water-filled centrifuge. In the distance between the water surface and the tank lid (h_0) air fills the vessel. The vessel is rotated around the vertical axis by the angular "ω" speed.

Data: $H = 300\text{mm}$; $h_0 = 20\text{mm}$; $R = 100\text{mm}$

Questions:

a. / What is the angular velocity of the water at the top of the vessel?

b./ How does the surface sink from "ω" when the top of the vessel has reached the water?

c./ Have an angular velocity $\omega = 20\frac{1}{s}$. What is the difference in pressure $p_A - p_0$?

d./ How much is the lift force acting on the container's top surface ? ($\omega = 20\frac{1}{s}$)

e./ How much is the liquid pressure force acting on the bottom of the vessel? ($\omega = 20\frac{1}{s}$)

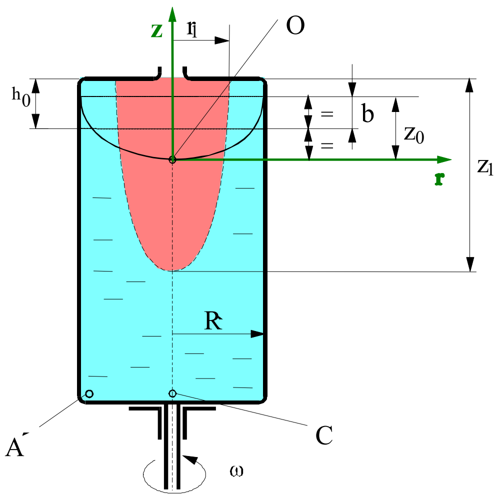


Figure 10.16

Solution:

When solving this problem, as in the case of the previous accelerating car, the selection of the right coordinate system and the writing of the potential must begin.

Assuming that the water surface is lowered in the middle as a result of rotation (eg, as in coffee mixed with glass), the lowest point on the axis is chosen as the origin. Upward pointing "z" and a horizontal "r" axis are in the roller coordinate system.

a./ <u>Figure 10.16</u> shows a state where the water has not reached the top of the vessel and another that it has reached. Let's start with the first case.

The height difference between the origin on the axis and the top of the surface is "z_0". At every point of the surface, the pressure, that is the basic static principle, is that the potential is constant at every point.

The potential function can simply be given as the sum of the difficulty field and the force from the rotation, so

$$U = g \cdot z - \frac{r^2 \cdot \omega^2}{2} . \qquad 10.19$$

Since both "r" and "z" are zero in the origin, zero is zero for all points in the water surface, so the above equation can be written as follows:

$$z = \frac{r^2 \cdot \omega^2}{2 \cdot g} \qquad 10.20$$

The expression obtained is a parabola equation, so the water surface is a rotational paraboloid. Apply the function to the "R" radius, then get the "z_0" value.

$$z_0 = \frac{R^2 \cdot \omega^2}{2 \cdot g}$$

The rise of the "z_0" angle is, therefore, a function of angular velocity, if we want to define the "ω" where the surface reaches the top of the vessel, then we need to know the sinking of the original resting surface. increase. This can be determined from the equality between volumes before and after rotation.

From mathematics we know that the volume of a quadratic rotational paraboloid can be calculated by taking half the volume of the cylinder with the same floor area and height (see **Example 10.1**).

Mark the parabola rise with the "b" relative to the resting surface. In the cylinder $R^2 \cdot \pi$ of the base and in the "b" height before the rotation, the air becomes "paraboloid" after rotation, because neither the water below it nor the volume of air above it changes. In our present task, the volume of air in the paraboloid:

$$V_p = \frac{1}{2} \cdot R^2 \cdot \pi \cdot z_0$$

So make the two volumes equal

$$R^2 \cdot \pi \cdot b = \frac{1}{2} \cdot R^2 \cdot \pi \cdot z_0$$

we get that

$$b = \frac{z_0}{2} = \frac{R^2 \cdot \omega^2}{4 \cdot g},$$

that is, the parabola falls as much as it rises to the resting surface. When the top of the vessel reaches the water, this situation changes, but it is still true when it is reached, so if replaced by the "b" the „h_0”, we get the desired rotational angular velocity ω_1.

$$h_0 = b = \frac{R^2 \cdot \omega_1^2}{4 \cdot g},$$

from where

$$\omega_1 = \frac{2 \cdot \sqrt{h_0 \cdot g}}{R} = \frac{2 \cdot \sqrt{0.02 \cdot 9.81}}{0.1} = 8.86 \frac{1}{s}.$$

b./
When the water surface reaches the cover, the dashed line is formed. The paraboloid still has the property that the potential is constant because the pressure is constant, p_0.
The surface equation continues to be described in *10.19*. We use the symbols of the figure, so

$$z_1 = \frac{r_1^2 \cdot \omega^2}{2 \cdot g}.$$

Neither "z_1" nor "r_1" are known, so we will need further correlation. The former also applies the same volume equation. Before the rotation, the volume of air in the cylinder above the water corresponds to the volume of air in the paraboloid after rotation, so

.

$$R^2 \cdot \pi \cdot h_0 = \frac{1}{2} \cdot r_1^2 \cdot \pi \cdot z_1$$

Using r_1^2 and replacing it with the latter equation, we get it

$$z_1 = R \cdot \omega \cdot \sqrt{\frac{h_0}{g}}.$$

If the water has reached the top of the vessel, the descent and the angular velocity are linearly related. This relationship exists until the parabola reaches the bottom of the vessel.
This property has long been used to measure speed. A glass-filled, oil-filled, outside-scale vessel was used to measure the speed of mining machines.

c. / The specified angular velocity $\omega = 20\frac{1}{s}$ is greater than the value of **a./** so the water surface is already falling into the container lid, but hopefully its bottom is not yet reached. This can be easily verified because it is only necessary to replace the formula obtained with the previous "z_1" and if this value is less than the height of the vessel, this is true.

$$z_1 = R \cdot \omega \cdot \sqrt{\frac{h_0}{g}} = 0.1 \cdot 20 \cdot \sqrt{\frac{0.02}{9.81}} = 0.09\text{m} = 90\text{mm}$$

The resulting lowering is smaller than the height of the vessel, so there is no lower incision.
The difference in pressure between a

$$\frac{p_0}{\rho} + 0 = \frac{p_A}{\rho} + U_A$$

from this equation can be calculated.
The potential of point „A” can be calculated based on the data given, taking into account, for example, the coordinate "z_A" has a negative sign.

$$U_A = -\frac{R^2 \cdot \omega^2}{2} - (H - z_1) \cdot g$$

Entering the above equation and sorting, we get the required differential pressure:

$$p_A - p_0 = \rho \cdot \left[\frac{R^2 \cdot \omega^2}{2} + (H - z_1) \cdot g \right] = 1000 \cdot \left[\frac{0.1^2 \cdot 20^2}{2} + (0.3 - 0.09) \cdot 9.81 \right] = 4060 \,\text{Pa}.$$

d./The lid of the vessel is lifted by the force from overpressure generated by rotation.
The pressure between the radii "r_1" and "R" varies according to:

$$p - p_0 = \rho \cdot \frac{\omega^2}{2} \cdot \left(r^2 - r_1^2 \right)$$

What we also get from the application of static statutes. Only integration can be used to determine the size of the force. Mark the force with "F_f", which obviously points upwards.So

$$F_f = \rho \cdot \frac{2\pi \cdot \omega^2}{2} \int_{r_1}^{R} \left(r^2 - r_1^2 \right) \cdot r \cdot dr = \rho \cdot \pi \cdot \omega^2 \left[\frac{r^4}{4} - \frac{r_1^2 \cdot r^2}{2} \right]_{r_1}^{R},$$

simplified and merged, and then replaced, the strength of the force:

$$F_f = \rho \cdot \pi \cdot \omega^2 \cdot \left(\frac{R^2 - r_1^2}{2} \right)^2 = \rho \cdot \pi \cdot 20^2 \left(\frac{0.1^2 - 0.066^2}{2} \right)^2 \cong 10\text{N}.$$

Previously, the value of "r_1" had to be calculated, for example. replacing *equation 10.20* with "z_1".

$$z_1 = \frac{r_1^2 \cdot \omega^2}{2 \cdot g},$$

from this

$$r_1 = \frac{\sqrt{2 \cdot g \cdot z_1}}{\omega} = \frac{\sqrt{2 \cdot 9.81 \cdot 0.09}}{20} = 0.066\text{m} = 66\text{mm}$$

is given.

e./ The compressive force applied to the bottom of the vessel can be calculated in exactly the same way. First, calculate the pressure at point „C” at the bottom of the vessel, simply this

$$p_C - p_0 = \rho \cdot g \cdot (H - z_0) = 1000 \cdot 9.81 \cdot (0.3 - 0.09) = 2060\text{Pa}.$$

As the pressure changes between "C" and "A", it is as follows (analogously for the lid), here only "r_c" radius is zero, so

$$p - p_C = \rho \cdot \frac{\omega^2}{2} \cdot r^2.$$

The force on the bottom then follows:

$$F_a = (p_c - p_0) \cdot R^2 \pi + \rho \cdot \frac{2\pi \cdot \omega^2}{2} \int_0^R r^2 \cdot r \cdot dr = (p_c - p_0) \cdot R^2 \pi + \rho \cdot \pi \cdot \omega^2 \left[\frac{r^4}{4} \right]_0^R$$

$$F_a = 2060 \cdot 0.1^2 \cdot \pi + 1000 \cdot \pi \cdot \omega^2 \cdot \frac{0.1^4}{4} = 96.2N.$$

Comment:

Tis force is greaterthan the weight of the water in the container, wight of the fluid plus the lifting force being exactly the same. The weight of the water is:

$$F_a - F_f = \rho \cdot g \cdot R^2 \cdot \pi \cdot (H - h_0) = 1000 \cdot 9.81 \cdot 0.1^2 \cdot \pi \cdot (0.3 - 0.02) = 86.2N$$

After discussing the basic laws of hydrostaticism, let us explore the kinematic of the fluids.

11. Kinematics and continuity

11.1. Description of fluid movement

To describe the movement of solid bodies, it is sufficient to provide the center of gravity and to rotate about three contiguous axes passing through the center of gravity. The position of the other points of the body can be obtained in any place and time, since the body does not change its shape during movement.
In the liquid, the individual particles can move freely from each other, the movement of each particle must be monitored separately. The freedom of the system is infinite.

Lagrange's description method essentially corresponds to the method used in solid bodies (French physicist ***Joseph-Louis Lagrange (1736-1813)***). Let's imagine the elemental parts of the liquid, and the movement of each part is examined separately. However, we must distinguish between the liquid parts and give them names. This is done by characterizing each liquid part at a given (initial) moment with a space vector or its coordinates. At a later time, with the vector "$\underline{r}$ " indicate the position of the particles that is defined by the "$\underline{r}_0$" spot vector and the "t" time:

$$\underline{r} = \underline{r}(\underline{r_0}, t)$$

The velocity and acceleration of the liquid part is given by the "1st" and "second" differential of "r" with fixed "$\underline{r}_0$":

$$\underline{v} = \left(\frac{\partial \underline{r}}{\partial t}\right)_s, \underline{a} = \left(\frac{\partial^2 \underline{r}}{\partial t^2}\right)_s$$

The index "s" means that the differentiation is to be performed along the constant "s" for the same particle along a particle's path.
This method should only be used in specific cases. Because of its difficulty it is generally not used to describe the movement of liquids.

Euler's description mode, which is the speed, acceleration, etc fixed in a point in space function of time, so it differs significantly from the description of the solid bodies. We will use this method later ***(Leonhard Euler (1701-1783)***, a Swiss scientist who spent most of his work in St. Petersburg), which gives the velocity of fluid parts in terms of space "r" and time "t"

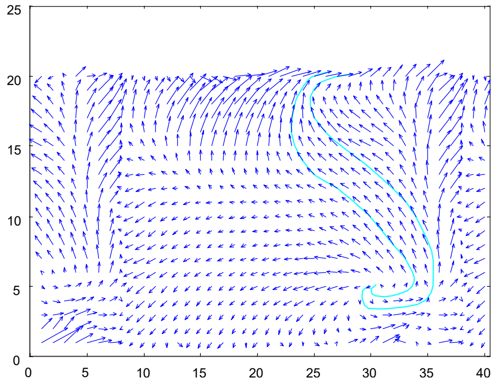
$$\underline{v} = \underline{v}(\underline{r}, t)$$

or broken into components in a rectangular coordinate system:

$$\underline{v}(\underline{r}, t) \quad = v_x(x, y, z, t) \cdot \underline{i} + v_y(x, y, z, t) \cdot \underline{j} + v_z(x, y, z, t) \cdot \underline{k}$$

The speed function can be described with **vector-vector** function. Both the dependent and the independent variable vectors.

In most cases, the velocity function is a continuous and even continuously derivable function of space and time. This method of description most often assumes continuous spatial and temporal changes in speed, consistent with modeling fluid as a continuum. In Lagrange's description mode, this clause, due to the discrete treated particles, is only indirectly met. (Numerical calculations may have two particles at the same point of the square at the same time when the calculation is not accurate enough due to numeric errors, which is of course not possible in reality.) If the flow is stationary, the velocity vector is only the function of the site, further simplifies.



Flat flow

Forrás: Vad J.and Bencze F.: Secondary Flow in Axial Flow Fans of Non-Free Vortex Operation

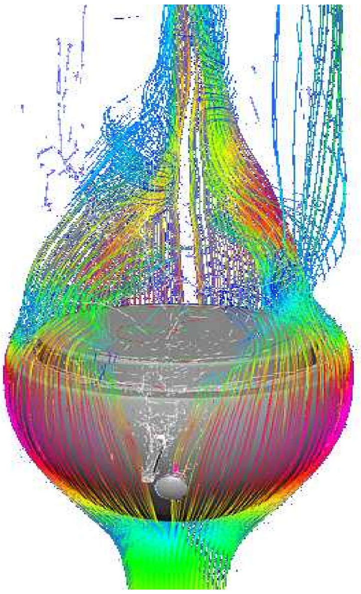
Figure 11.1

Figure 11.1 shows an even "simpler" current image that is two-dimensional plane flow and time-independent stationary flow.

The vectors at each point indicate the direction and magnitude of the velocity. Obviously, it is advisable to make a very complicated flow more visible in a way that makes the flow visible.

11.2. Strean line, path line, streak linel, steady flow

The following curves are used in the fluid space to illustrate space and time varying speed ranges.



Flow around the spherical body
Streak line structure Figure 11.2

The path line is a path that has run through a point-like liquid part. For example, the bright lines created by the position lights of the cars visible in the night scenes with long exposures, or the condensation strips of a high-flying plane.

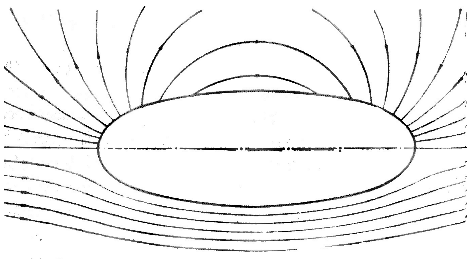
The streamline is a curve that at one point touches every point of the velocity vector $\underline{v} \times d\underline{s} = 0$, where $d\underline{s}$ is a vector of the elemental length of the streamline. The streamline at a particular moment is the envelope curve of velocity vectors, so it is usually a time-varying curve. (**Figure 11.1** shows some streamlines.)

The treack line is a line connecting fluid sections passing through one passage at a given moment. (Such a trace is eg a cigarette smoke ascent from an ashtray or a line of traces displayed by a color liquid used to display a circular pattern around the visible spherical glass shown in **Figure 11.2.**) The trajectory generated by a tracer projectile around the aircraft can be observed.

A very important feature of the flows is their time dependence, that is, their characteristics (speed, pressure, density) depend on time.

Stacioner or **steady flow** is the flow, if its characteristics do not depend on time. If the velocity is independent of time at any point of the space, the above three lines coincide because a particle always travels in the direction of the constant current line in time, so that particles passing through a point are all aligned on the same stream. Such flow is thus referred to as a $\underline{v} = \underline{v}(\underline{r})$ time-stable or steady flow.

In the steady flow, the streams can be visualized with the help of traces of smoke, painted water etc. in point sources.



Flow around the bridge pillar
Source: Gruber J.-Blahó M. Folyadékok mechanikája
Figure 11.3

The flow time resistance is generally not independent of the choice of the coordinate system. For example, the bridge bridge in the river is stationary from the bridge, the corresponding streamlines can be seen in the lower part of **Figure 11.3**. It is not steagy unsteady) from the coordinate system fixed to the river because the parts of the river beyond the pillar are still in relation to the coordinate system fixed to the river, while the parts near the pillar move away (they go to the pillar in front) and return to their place again. The corresponding streamlines can be seen in the upper part of **Figure 11.3**. This current flowing along the river, along with the pillar.

11.3 The continuity law

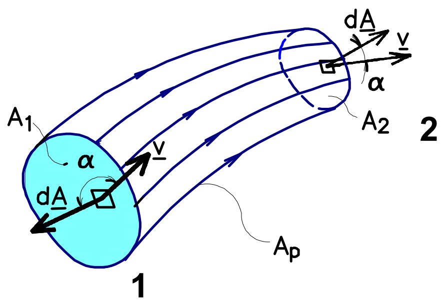
Hereinafter, we only deal with fluids in which the liquid does not disappear and does not occur. This property is called fluid continuity. For chemical reactions in the flow, at phase transformations (eg source, condensation) some of the liquid may disappear or may be formed. If, for example, vapor flows through a pipeline, the water vapor precipitating on the pipe wall disappears from the vapor phase. We do not deal with this type of flow in this note.

11.3.1 Continuity of steady flow

First, let's look at a **figure 11.4**, fixed flow at the same time. Tighten the streamlines around the perimeter of a flat surface, from which a flow tube is obtained. The wall of the stream pipe is made up of stream lines, so the liquid can not pass through it as velocity always tangent to the streams forming the wall. The mass flow into the "1" surface is controlled by a

$$q_m = \iint_{A_1} \rho \cdot \underline{v} \cdot d\underline{A} \qquad 11.1$$

gives the expression. If the density and velocity are nearly constant along the "A_1" surface, and the velocity is perpendicular to the surface, then the mass flow can be easily calculated by simply multiplying the three quantities:



Stream Tube
Figure 11.4

$$q_m = A_1 \cdot \rho_1 \cdot v_1 \qquad 11.2$$

On the "A_2" surface, the same mass flow must flow out as the liquid can not disappear. it can not be formed in the tube. So the continuity of the word states that the input and output mass flows are the same, so:

$$A_1 \cdot \rho_1 \cdot v_1 = A_2 \cdot \rho_2 \cdot v_2 \left[\frac{kg}{s}\right] \qquad 11.3$$

If the **density is constant**, the continuity of the stream can be further simplified for the current tube, and the equation of the flow flows should be written between the two cross-sections, because the density can be simplified, so the inlet and outlet volume flow rate are identical:

$$A_1 \cdot v_1 = A_2 \cdot v_2 \left[\frac{m^3}{s}\right] \qquad 11.4$$

For pipelines, use the terms *11.3* and *11.4* for steady flows.

11.3.2 Continuity law for unsteady flow

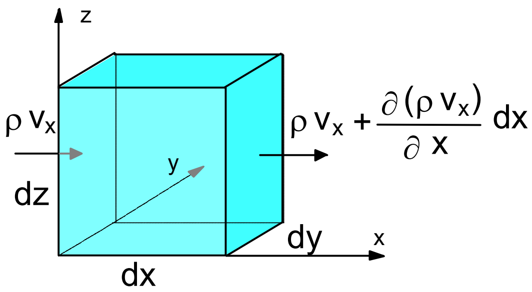


Figure 11.5

Fig. 11.5 shows an elemental flow, dx, dy and dz, which is freely permeable to the liquid in a spatial flow. Write the amount of mass flow into and out of the unit during the time unit. For the sake of simplicity, we first examine the mass flow in the direction "x".

The "v_y" and "v_z" speeds are parallel to the sheet, so the mass flow rate can come on the "$dy \cdot dz$" area with "v_x" velocity component

$$-\rho \cdot v_x \cdot dy \cdot dz$$

On the right side the media exits, but its velocity and density change. The change is taken linearly to the distance "dx", so approach the changed densities and speeds with the first two members of the Taylor line. On the right page, the

$$\rho \cdot v_x + \frac{\partial(\rho \cdot v_x)}{\partial x} \cdot dx$$

with velocity and densities. After multiplying the surface, we get the amount of mass per second that is on the right side:

$$\left[\rho \cdot v_x + \frac{\partial(\rho \cdot v_x)}{\partial x} \cdot dx\right] \cdot dy \cdot dz$$

In the "x" direction, when the inflow is negative and the outflow is positive, the difference between the inlet and outlet mass:

$$\frac{\partial(\rho \cdot v_x)}{\partial x} \cdot dx \cdot dy \cdot dz \qquad 11.5$$

Likewise, calculations can be made in "y" and "z" direction. Difference between input and output mass:

$$\frac{\partial(\rho \cdot v_y)}{\partial y} \cdot dx \cdot dy \cdot dz$$

"y" direction and

$$\frac{\partial(\rho \cdot v_z)}{\partial z} \cdot dx \cdot dy \cdot dz$$

"z" direction.

If we assume that a mass does not disappear and does not arise, it is necessary that the sum of the surplus in the three directions is equal to the decrease of the mass in the brick. The reduction of mass is obtained by the change of density and the volume of product, that is the temporal change of mass:

$$-\frac{\partial \rho}{\partial t} \cdot dx \cdot dy \cdot dz$$

("ρ" is the mean density in the brick, which is derived partly because it may be a function of the spase.) The sum of the three-way mass outflow is equal to the volume decrease in volume by time, so

$$\left[\frac{\partial(\rho \cdot v_x)}{\partial x} + \frac{\partial(\rho \cdot v_y)}{\partial y} + \frac{\partial(\rho \cdot v_z)}{\partial z}\right] \cdot dx \cdot dy \cdot dz = -\frac{\partial \rho}{\partial t} \cdot dx \cdot dy \cdot dz.$$

Shuffled with the elemental volume and one side to the members, we get the

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho \cdot v_x)}{\partial x} + \frac{\partial(\rho \cdot v_y)}{\partial y} + \frac{\partial(\rho \cdot v_z)}{\partial z} = 0 \qquad 11.6$$

continuity law in differential equation form. In the equation, divergence from the vector analysis emerged. Using this, the continuity item in differential form for the unsteady flow is as follows:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \cdot \underline{v}) = 0 \qquad 11.7$$

If the flow is steady the density at a given point does not depend on time $\frac{\partial \rho}{\partial t} = 0$,

in this case, the continuity becomes simpler form

$$\mathrm{div}(\rho \cdot \underline{v}) = 0 \qquad 11.8$$

The physical meaning of divergence is the volume source strength, if its value is everywhere zero, that is, the **vector space is free of source**.

Divergence is a scalar-vector function just like pressure distribution or temperature distribution in space.

If we simplify the task and the density is constant, it can be removed from the divergence and simplified, so

$$\mathrm{div}(\underline{v}) = 0 \qquad 11.9$$

which also applies to a constant density medium in an unsteady case.

The constant density media is always free of resources.

The volume integration of both sides of equation 11.8 can be obtained.

$$\iiint_V \operatorname{div}(\rho \cdot \underline{v}) \cdot dV = 0 , \qquad 11.10$$

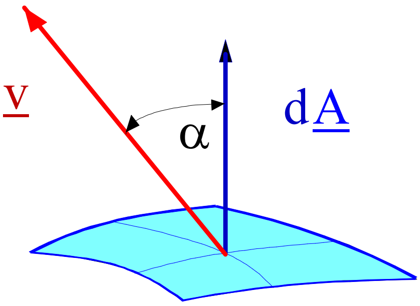
and **Gauss-Osztrogradsky** may be used, according to which the volume integration of the divergence can be converted to a closed surface integer as follows:

$$\iiint_V \operatorname{div}(\rho \cdot \underline{v}) \cdot dV = \oiint_A \rho \cdot \underline{v} \cdot d\underline{A} \qquad 11.11$$

(***Karl Friedrich Gauss 1777-1855*** is a prominent German mathematician, physicist, astronomer, teacher at the University of Göttingen. ***Mihail Vassiljevich Osztogradszkij1801-1862*** is a Russian mathematician, member of the Academy of St. Petersburg.)
If the expression *11.10* is satisfied, then equation *11.11* is also true

$$\oiint_A \rho \cdot \underline{v} \cdot d\underline{A} = 0 \qquad 11.12$$

The integrator expresses that the sum of liquid in and out of a closed space is stationary in zero at all times. Apply *equation 11.12* to the current tube as shown in **Figure 11.4**.
In the $\oiint_A \rho \cdot \underline{v} \cdot d\underline{A} = 0$ integral the closed surface "A" is to be disaggregated by the sum of three different surfaces.



Scalar product
Figure 11.6

In the $A = A_1 + A_2 + A_p$ the "1" indexed surface where the media enters the current tube, the "2" indexed surface where the medium exits the current tube and the mantle where the medium will not surely pass. The surface element "$d\underline{A}$" always appears outwardly from the surface and perpendicular to the surface. Behind the integral signal, the scalar product of the surface element and velocity vector must be formed at each point. As we know, the scalar product of two vectors means:

$$\underline{v} \cdot d\underline{A} = |\underline{v}| \cdot |d\underline{A}| \cdot \cos\alpha$$

Illustrations of **Figure 11.6**. If the angle "α" is acute, then the product is positive if the "α" obtuse angle the produst is negative and if the "α" is the right angle, then the scalar product is zero.

$$\oiint_A \rho \cdot \underline{v} \cdot d\underline{A} = \iint_{A_1} \rho \cdot \underline{v} \cdot d\underline{A} + \iint_{A_2} \rho \cdot \underline{v} \cdot d\underline{A} + \iint_{A_p} \rho \cdot \underline{v} \cdot d\underline{A} = 0$$

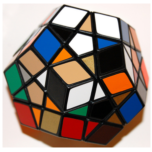
On the surface "A_1" the media enters the surface, so the two vectors are dull, so their product is negative, "A_2" the surface exits the surface where the center angle is acute, so the product is positive. The two vectors are perpendicular to the hull, because we are on streamlines, so the product is zero.
If the densities and velocities on the two end faces of the current tube are nearly constant and close to the perpendicular to the plates, the surface "A_1" and "A_2" should be simply multiplied by the density and velocity there, to get the integer value.
Thus, the above expression takes the following simple form:

$$-\rho_1 \cdot v_1 \cdot A_1 + \rho_2 \cdot v_2 \cdot A_2 + 0 = 0$$

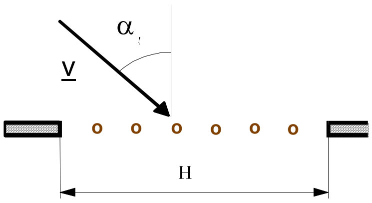
Once reorganized, we get the already deduced *expression 11.3*:

$$\underline{\rho_1 \cdot v_1 \cdot A_1 = \rho_2 \cdot v_2 \cdot A_2}$$

11.3.3 Continuity on the roof window



As an example, the flow rate of the air flowing through the roof window as shown in Figure 11.7 is determined. At the "v" velocity, the air flow through the air, the velocity vector "α" angled with the normal surface of the grid. The perpendicular dimension of the ventilation grille shown in the drawing is 1 m.



Roof window
Figure 11.7

Data: $v = 3\frac{m}{s}$; $H = 2m$; $\alpha = 60°$

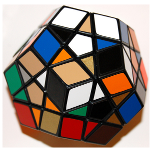
Question: *incoming volume flow rate.*

Solution:

The flow rate flow rate is the product of the flow-through surface and the perpendicular velocity component. In general, the flow rate is calculated by the surface integration of the velocity vector:

$$q = \int_{(A)} \underline{v} d\underline{A} = \int_{(A)} v \cdot \cos\alpha \cdot dA|\underline{n}| = v \cdot \cos\alpha \cdot H \cdot 1 = 3 \cdot \cos 60^0 \cdot 2 = 3\frac{m^3}{s}$$

11.3.4 Continuity in compressor

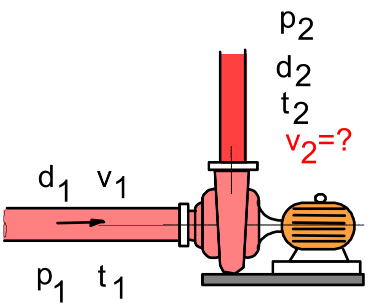


The compressor outlined in **Figure 11.8** shows the air flow of the compressor suction pipe at "v_1" velocity. The pressure and the temperature of the inlet and outlet gas are measured: p_1, t_1, p_2, t_2.

Data: $p_1 = 1bar$; $p_2 = 2bar$; $t_1 = 20°C$; $t_2 = 70°C$; $d_1 = 50mm$; $d_2 = 35mm$; $v_1 = 20\frac{m}{s}$.

Specific gas constant of air $R = 287\frac{J}{kgK}$.

Question:



Compressor
Figure 11.8

a./ Determine the compressed air velocoty (v_2).

b/ To repeat the changes in the state of gas in the first chapter, determine what polycropic exponent applies to the change of state between the intake and the outlet?

Solution:

a. According to the law of continuity, the mass flows into and out of the steady state of the compressor are the same:

$$\rho_1 v_1 \frac{d_1^2 \pi}{4} = \rho_2 v_2 \frac{d_2^2 \pi}{4}$$

To calculate the densities we need to use the equation of the ideal gases:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{10^5}{287 \cdot (273 + 20)} = 1.189\frac{kg}{m^3},$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{2 \cdot 10^5}{287 \cdot (273+70)} = 2.032 \frac{kg}{m^3}$$

In view of these, the flow rate can be calculated in the pressure pipe:

$$v_2 = v_1 \frac{\rho_1 d_1^2}{\rho_2 d_2^2} = 20 \frac{1.189 \cdot 0.05^2}{2.032 \cdot 0.035^2} = 23.89 \frac{m}{s}$$

b./ Applied from the **1. paragraph** to the polythropic state change (see *equation 11.9*) between the suction and trailing status indicators:

$$\frac{p_1}{\rho_1{}^n} = \frac{p_2}{\rho_2{}^n}$$

It is advisable to sort the pressures and densities on one side and take the logarithm of both sides of the equation and then express "n" after substitution:

$$n = \frac{\log \frac{p_2}{p_1}}{\log \frac{\rho_2}{\rho_1}} = \frac{\log 2}{\log \frac{2.032}{1.189}} = 1.29$$

The resulting polycropic exponent is smaller than the adiabatic exponent of the air ($\kappa = 1.4$), which means that the compressor is cooled to some degree or cooled from the ambient air itself.

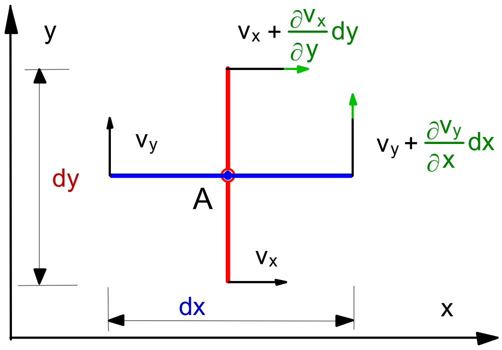
11.4 Vortex, circulation

Cyclones and anti-cyclones play an important role in weather development. On satellite recordings, it is clear that cloud systems are swirling over the Earth's surface (see **Figure 10.5**). Vortexes can also be observed on the surface of rivers. The mathematical description of whirling-free rotationless or rotational-free flows is a much simpler task than vortex or rotational rotational flows.
In order to be able to judge the difference between the two types of flow, it is first and foremost to understand the gullibility itself, both physically and mathematically.

11.4.1 Swirling in plane flow

Imagine a plane flow eg. the surface of a river. Place two rods perpendicularly to the surface of the water that are centered to each other with a pin that permits angular rotation of each other. The sticks are drifted along the surface of the water along with the flowing water, and the points of the chopper move along the water surface. (If you have a small bobbin bowl, then this condition is met by a good approximation.)
At the moment shown in **Figure 11.9**, the sticks are just perpendicular to each other and parallel to the length "dx" the length "x" and "dy" to the axis "y".

At point "A" at the point of encounter of the rods, we want to give the rotational angular velocity or, in other words, the average rotational angular velocity of the two rods, be it "ω_A".



Vortex Indicator in Plane Flow
Figure 11.9

To solve this problem, we calculate the angular velocity of the "dx" and "dy" length rods around the axis perpendicular to the plane of the drawing passing through "A", corresponding to the "z" rotation around the third coordinate axis.

The angular velocity of the "dx" length of the chopper at point A is obtained by dividing the difference of the perpendicular velocities at the two endpoints with the length of the rod. The perpendicular velocities are "y" oriented. At the left end point "v_y", at the right end point, this is slightly different. Knowing the speed of water, this could be precisely stated. In the present case, approximate with the linear function "v_y", which represents the first two members of the Taylor series as a function of "x", in this case

$$v_y + \frac{\partial v_y}{\partial x} \cdot dx .$$

The stick angle "dx" has the following angular velocity:

$$\omega_{dx} = \frac{v_y + \frac{\partial v_y}{\partial x} \cdot dx - v_y}{dx} = \frac{\partial v_y}{\partial x}$$

The "dy" length chopper angular velocity can be calculated similarly. By dividing the difference between the perpendicular points perpendicular to the chopper, the length of the chopper is obtained with the angular velocity. The perpendicular velocities are in this "x" direction and the Taylor line should be executed as "y". Another difference is that the figure "dy" rotates in clockwise direction, but in a better twisted coordinate system we find a counterclockwise rotation positive, so a negative sign must be made before the expression 11.13

$$\omega_{dy} = -\frac{v_x + \frac{\partial v_x}{\partial y} \cdot dy - v_x}{dy} = -\frac{\partial v_x}{\partial y} \qquad 11.14$$

The angular velocity around the "z" axis in point A is obtained from the arithmetic mean of the angular velocity of the two rods:

$$\omega_{zA} = \frac{1}{2} \cdot \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) = \frac{1}{2} \cdot (\text{rot}\,\underline{v})_{zA} . \qquad 11.15$$

In mathematics, the expression in the parentheses of the rotation of the "$\underline{v}$" vector space, more precisely the "z" direction of the rotation, is called.

If it is not a plane but a spatial flow, the two chopsticks can not only rotate around the "z" axis but also around the "x" and "y" axes. Similarly, the angular velocity and the other two coordinate components of the rotation, which are:

$$\omega_{xA} = \frac{1}{2} \cdot \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) = \frac{1}{2} \cdot (\text{rot}\,\underline{v})_{xA} \qquad 11.16$$

$$\omega_{yA} = \frac{1}{2}\cdot\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) = \frac{1}{2}\cdot(\text{rot}\underline{v})_{yA} \qquad 11.17$$

The term is true at any point in the space, so it is written in vector

$$\underline{\omega} = \frac{1}{2}\cdot\text{rot}\underline{v}, \qquad 11.18$$

components, or the nabla vector using the rotation vector:

$$\underline{\nabla}\text{x}\underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} = \text{rot}\underline{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\cdot\underline{i} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\cdot\underline{j} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\cdot\underline{k}$$

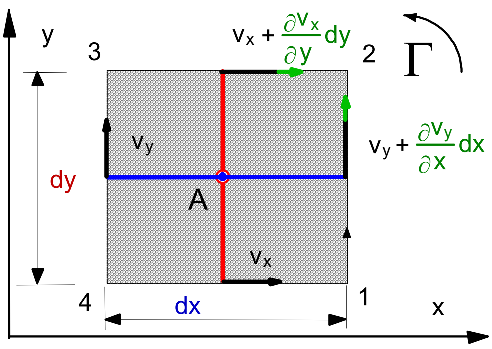
Rotation is a vector that can pick different values at different points of the space, and may even change in time at a given point of the space, in the latter case the flow is instable. 11.19

The rotation vector-vector function expresses the rotational angular velocity of the liquid particles around their own axis, or more exactly twice that of the fluid particles.

11.4.2 Circulation, Stokes theorem

Circulation has been introduced to further characterize swirl fluxes, which means the velocity encapsulated integral integrity. With mathematical expression:

$$\Gamma = \oint_g \underline{v}\cdot d\underline{s} \qquad 11.20$$



Circulation
Figure 11.10

Below the closed curve "g", the "ds" path must be multiplied by scalar at the current velocity at each point and the resulting values, which are scalar numbers (both positive and negative), must be summed over the entire closed curve length. Carry out the circulation calculation for the closed curve 1-2-3-4 in *Figure 11.10*. Let's start with "1". In section 1-2, my "ds" path is the same as "dy". The component of the speed in the direction of "dy" is precisely the

$$v_y + \frac{\partial v_y}{\partial x}\cdot dx.$$

(There is a "x" direction along the road segment, but it does not play a role in scalar product, see **Figure 11.6**). In section "2-3", my "ds" path coincides with "dx", only its control is opposed to it. The velocity is $v_x + \frac{\partial v_x}{\partial y}\cdot dy$. The direction of velocity and my path is opposite, so the term will include a negative sign in the summary. In the other two stages of the curve "g" we proceed in the same way as "2-3" and "3-4" sections, so that the circulation equation:

$$d\Gamma = \left(v_y + \frac{\partial v_y}{\partial x} \cdot dx \right) \cdot dy - \left(v_x + \frac{\partial v_x}{\partial y} \cdot dy \right) \cdot dx - v_y \cdot dy + v_x \cdot dx,$$

$1-2$ section $2-3$ $3-4$ $4-1$

then simplified:

$$d\Gamma = \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \cdot dx \cdot dy \qquad 11.21$$

The term obtained is the value of a circular value taken on an elementary closed curve. It is worth noting that the term in parenthesis is nothing more than the "z" direction of rotation. *Equation 11.19*. The other one to note is that the multiplier outside the bracket is exactly the same as the surface element,

$$dA = dx \cdot dy$$

Thus it can be written that on the elementary closed curve the speed integrates the same with the product of the "z" direction of the rotation and the surface element,

$$d\Gamma = (\text{rot}\,\underline{v})_z \cdot dA.$$

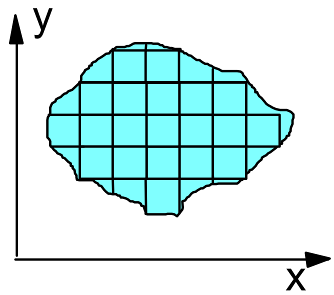


Figure 11.11

The area bounded by any closed curve can be divided into elementary rectangles, the boundary curve can be replaced with their sides by a step curve. When summing up the circles calculated on the perimeter of the elementary rectangles, the members along the common sides are eliminated by calculating circulation in each quadrilateral with the same circumcision. Only the line integer on the approximate sides of the "g" curve remains in the expression of the circulation. On the other hand, the individual elementary surfaces multiplied by the rotation of them and summing them up, also the circulation is obtained. so

$$\oint_g \underline{v} \cdot d\underline{s} = \iint_A (\text{rot}\,\underline{v})_z \cdot dA$$

It can be verified that the right side of the expression applies not only to a flat surface enclosed by the curve "g", but to any surface that fits in the curve "g" and can move out of the plane like a wire mesh stretch or soap, we blink slightly. In this case, the scalar product of the surface element and the rotation vector must be formed at integration.

Let us assume a monolithic "A" surface according to the above, which fits in the closed curve "g" ("g" direction is positive from the surface A). The rot$\underline{v}$ vector space is integrated along the "A" surface and is clear and it integrates once continuously variable velocity velocity "g" along the curve, the connection between **circulation** and a

$$\Gamma = \oint_g \underline{v} d\underline{s} = \int_A \text{rot}\,\underline{v} d\underline{A} \qquad 11.22$$

,which is referred to as **Stokes formula**. (***Georg Gabriel Stokes 1819-1903***, English physicist, mathematician, secretary of the Royal Society, one of the authors of the vector analysis, explorer of the above item.) The concept of circulation will be used later, for example. in the framework of the wing theory, when the Zsuzsnyj's theorem was derived.

11.4.3 Potential flow, potential vortex

If $\text{rot}\,\underline{v}(r)$ there is zero in the whole, single-domain, then $\underline{v}(\underline{r})$ has a potential function $\varphi(\underline{r})$ which is a scalar function in space. This is much easier to handle than a vector function because at one point only one variable should be monitored instead of three variables. Velocity Potential:

$$\varphi(\underline{r}) = \int_{\underline{r}_0}^{\underline{r}} \underline{v} \cdot d\underline{r} + \varphi(\underline{r}_0) \qquad 11.23$$

Or, forming the gradient of both sides

$$\text{grad}\varphi(\underline{r}) = \underline{v}$$

In the case of force fields (see **Chapter 10**) it is a conventional convention that the negative gradient of the field potential is the field strength vector:

$$\underline{g}(\underline{r}) = -\text{grad}U(\underline{r})$$

At the velocity potential, the negative sign is not required. Potential flows are also commonly referred to as whirling free, non-rotating streams. We will not use the velocity potentials ourselves, just for the sake of completeness. But we will look at whirling free fluids, just in the next section.

Potential vortex

In many places in nature there is a vortex flow. For example, cyclones emerging in the atmosphere of the Earth (**Figure 10.5** shows several cloud fluxes), or the water flowing out of the bathtub into the drain creates a vortex. The velocity of such vortexes can be easily approximated with the so-called potential velocity of the vortex. In the **potential vortex**, fluid particles move along concentric circular streams. The absolute value of the speed in a circle does not change, only the direction of the speed. The absolute value of the velocity decreases according to the radius, according to a hyperbolic

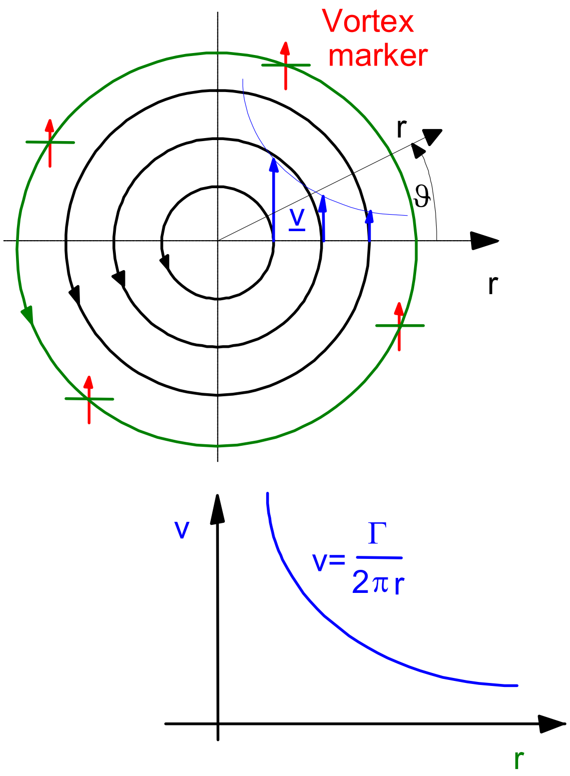
$$v = \frac{\Gamma}{2 \cdot \pi \cdot r}, \qquad 11.24$$

where $\frac{\Gamma}{2 \cdot \pi}$ a suitably chosen constant, "r" is the distance from the center point.

If you want to describe the speed range, it is best to choose a cylinder coordinate system that consists of the coordinates "r, ϑ, z". In the "ϑ" direction, which is the angle coordinate in the direction of the circumference, there is no change in velocity, the cylinder is symmetrical in flow.

The direction "z" (the direction perpendicular to the plane of the drawing) also does not change the flow, because every "z" is the same stream, we are talking about plane flow in this case. The potential velocity of the potential vortex and the streamlines are shown in **Figure 11.12**.

It can be verified that a spin in a potential vortex (see **Figure 11.12**) does not rotate around its own axis. This means that the angular velocity and thus the rotation are zero as shown in **Figure 11.9**. Let's try to look at this with mathematical tools as well. For this purpose it is advisable to write the expression of rotation in a cylinder coordinate system:



Potential vortex
Figure 11.12

$$\text{rot}\begin{bmatrix} v_r \\ v_\vartheta \\ v_z \end{bmatrix} = \begin{bmatrix} \frac{1}{r}\frac{\partial v_z}{\partial \vartheta} - \frac{\partial v_\vartheta}{\partial z} \\ \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \\ \frac{1}{r}\frac{\partial (r v_\vartheta)}{\partial r} - \frac{1}{r}\frac{\partial v_r}{\partial \vartheta} \end{bmatrix}.$$

Given that in the case of plane flow $v_r \equiv v_z \equiv 0, \frac{\partial}{\partial \vartheta} \equiv \frac{\partial}{\partial z} \equiv 0$, and $v_\varphi = v$ the above formula a

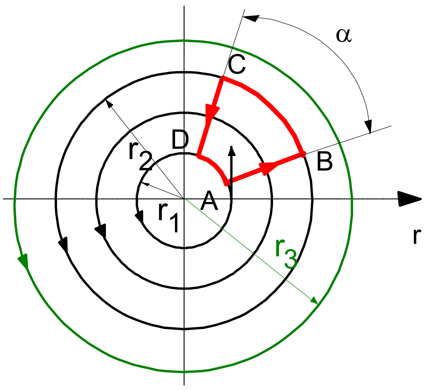
$$\text{rot}\,\underline{v} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{r}\frac{d(r \cdot v)}{dr} \end{bmatrix}$$

which has only a "z" directional component.
By deriving the derivation of the product, we get:

$$(\text{rot}\,\underline{v})_z = \frac{v}{r} + \frac{dv}{dr} \qquad 11.25$$

Using the above formula, using a given function of the speed, the rotation value is zero if the radius is not zero (the term is not interpreted):

$$(\text{rot}\,\underline{v})_z = \frac{\Gamma}{2 \cdot \pi \cdot r^2} - \frac{\Gamma}{2 \cdot \pi \cdot r^2} = 0$$



Circulation in the potential vortex
Figure 11.13

The whirlwind, therefore, does not rotate around its own axis because the flow is whirling.
If the flow is whirly free, it integrates the velocity field in the single-enclosed region on the closed curve, ie the "Γ" zero should be according to the Stokes formulae (*equation 11.22*), since on the right there is a function of a zero function on a finite surface returns zero.
Calculate the value of the circulation from a curve of "ABCD" radius and arcs in **Figure 11.13** that the velocity is tangential and constant along a circle and the velocity is perpendicular to the radius:

$$\Gamma_{ABCD} = \oint_{ABCD} \underline{v} \cdot d\underline{s} = \int_{AB} \underline{v} \cdot d\underline{s} + \int_{BC} \underline{v} \cdot d\underline{s} + \int_{CD} \underline{v} \cdot d\underline{s} + \int_{DA} \underline{v} \cdot d\underline{s} = 0 + \frac{\alpha \cdot r_2 \cdot \Gamma}{2 \cdot \pi \cdot r_2} + 0 - \frac{\alpha \cdot r_1 \cdot \Gamma}{2 \cdot \pi \cdot r_1} = 0$$

Calculate the circulation along the " r_3 " radius. The absolute value of the velocity in the radius " r_3 " is constant, so the circulation is not zero, and its value is exactly the constant value of the velocity function. (That is why Γ is the velocity function.) The circumference of the circle multiplies at the current velocity and gives the circulation:

$$\Gamma_O = 2 \cdot \pi \cdot r_3 \cdot \frac{\Gamma}{2 \cdot \pi \cdot r_3} = \Gamma$$

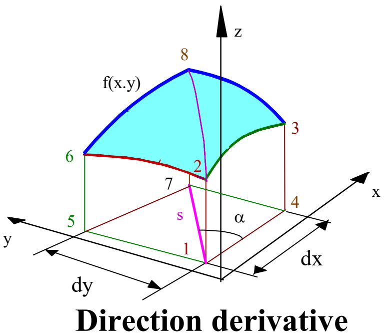
Why is not the value of circulation in the circle around the center zero when the zero value for the rotation is obtained?
The Stokes formula is true for a single-domain. In our case, in the origin, the velocity range is not interpreted, so there is a hole in the range of interpretations, so it is not a one-dimensional interfacial domain. It can be easily verified that on all closed curves that do not circumscribe the origin, the circulation is zero and the closed curves surrounding the origin of circulation are "Γ". (The electromagnetic field created by the infinite long straight conductor is quite similar in nature to the potential vortex.)

11.5 Acceleration of the fluid particle

The acceleration of a particle can be determined from the time change of the velocity, the derivative of the velocity by time being the acceleration.
To understand the acceleration of the liquid particle, let's first examine a bivariate function with its independent variables "x" and "y", which is a graph of a piece of surface in the "x, y, z" coordinate system.
Plane "1265" is perpendicular to the plane "xy" and parallel to the "y" axis and line "23" intersects the function f(x,y). The plane "1234" is perpendicular to the plane "xy", and intersects the function f(x,y) with parallel axis "x" and line "23".
Let us make a complete change of function f(x,y), which is:

$$df = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy \qquad 11.26$$

Depending on the ratio of "dx" and "dy" infinitely small quantities, "df" gives a change of function.



Direction derivative
Figure 11.14

When, for example $dy = 0$, when $df = f_3 - f_2$ and where $dx = 0$, then $df = f_6 - f_2$.
If there is a certain ratio between "dx" and "dy", we go along the line "s" shown in the figure, then there exists between the two independent variables

$$dy = tg(\alpha) \cdot dx$$

So the change of function:

$$df = f_8 - f_2 = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot tg\alpha \cdot dx$$

In this case, it is possible to interpret the derivative of the function f (x, y) in the direction „s”, which is the **direction derivative**:

$$\left(\frac{df}{dx}\right)_s = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \cdot tg\alpha = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \cdot \left(\frac{dy}{dx}\right)_s$$

If $\alpha = 0^0$ the direction derivative is the same as the partial derivative „x" in the directional derivative. If $\alpha = 90^0$ derivative is the same as the partial derivative "y", but then another reordered form of the derivative should be used, which is:

$$\left(\frac{df}{dy}\right)_s = \frac{\partial f}{\partial x} \cdot \frac{1}{tg\alpha} + \frac{\partial f}{\partial y} = \frac{\partial f}{\partial y}$$

In the Euler descriptor mode, the speed vector in the most common case and in the "x, y, z" coordinate system is the following vector vector function:

$$\underline{v}(t, \underline{r}) = v_x(t, x, y, z) \cdot \underline{i} + v_y(t, x, y, z) \cdot \underline{j} + v_z(t, x, y, z) \cdot \underline{k}$$

Three components of the speed vector depend on four independent variables.
For a simpler discussion, consider only one velocity component v_x, for example. Write the full change of velocity for the analogy of *equation 11.26* here only four independent variables:

$$dv_x = \frac{\partial v_x}{\partial t} \cdot dt + \frac{\partial v_x}{\partial x} \cdot dx + \frac{\partial v_x}{\partial y} \cdot dy + \frac{\partial v_x}{\partial z} \cdot dz \qquad 11.27$$

The relationship between the dt, dx, dy and dz independent variables is given by the elemental path length of the liquid particle having the coordinates towards the axes, respectively:

$$dx = v_x \cdot dt, \qquad dy = v_y \cdot dt, \qquad dz = v_z \cdot dt$$

This relationship corresponds to the relationship between independent variables, which is designated by the particle's "s". Replaced in *expression 11.27* and formally dotted with "dt" time, we get the derivative "s"

$$\left(\frac{dv_x}{dt}\right)_s = \frac{\partial v_x}{\partial t} + \frac{\partial v_x}{\partial x} \cdot v_x + \frac{\partial v_x}{\partial y} \cdot v_y + \frac{\partial v_x}{\partial z} \cdot v_z$$

Since the particle denotes the direction of derivation is also termed **substantive or full derivative**, and the index is usually omitted, so:

$$\frac{dv_x}{dt} = \frac{\partial v_x}{\partial t} + \frac{\partial v_x}{\partial x} \cdot v_x + \frac{\partial v_x}{\partial y} \cdot v_y + \frac{\partial v_x}{\partial z} \cdot v_z \qquad 11.28$$

Similarly, in the "y" and "z" directions, a complete change of speed components can be described:

$$\frac{dv_y}{dt} = \frac{\partial v_y}{\partial t} + \frac{\partial v_y}{\partial x} \cdot v_x + \frac{\partial v_y}{\partial y} \cdot v_y + \frac{\partial v_y}{\partial z} \cdot v_z \qquad 11.29$$

$$\frac{dv_z}{dt} = \frac{\partial v_z}{\partial t} + \frac{\partial v_z}{\partial x} \cdot v_x + \frac{\partial v_z}{\partial y} \cdot v_y + \frac{\partial v_z}{\partial z} \cdot v_z \qquad 11.30$$

Let us introduce the derivative tensor concept, which is a three-way matrix of three velocity vectors, three place-coordinate derivatives, namely nine elements:

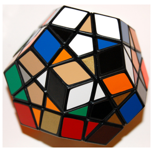
$$\underline{\underline{D}} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

Using this, you can write *11.28, 11.29* and *11.30* into a vector equation:

$$\frac{d\underline{v}}{dt} = \frac{\partial \underline{v}}{\partial t} + \underline{\underline{D}}\underline{v} \qquad 11.31$$

The acceleration of the liquid part consists of two parts, from $\frac{\partial \underline{v}}{\partial t}$ **local acceleration** and from $\underline{\underline{D}}\underline{v}$ **convective acceleration**. Local acceleration indicates whether or not the velocity changes at some point in the space. Convective acceleration shows that the velocity of the fluid changes slightly from a point of the space.
Local acceleration is only zero in the unsteady flow. That is, steady flow is always zero. The convective acceleration is not related to the time dependence of the flow, its value can be zero in the case of steady and unsteady flows. Convective acceleration occurs when the velocity and direction of the liquid space velocity or both fluctuates in the direction of movement of the liquid part (i.e. in the direction of flow). For a better understanding of local and convective acceleration, let's look at the following example.

11.5.1 Acceleration in confusor



The pipes "D_1" and "D_2" outlined in **Figure 11.1** are confused by a confetti. In the pipeline the flow is instable and the fluid density is constant. Within a cross-section, the flow rate can be taken constant along the radius.

Data: $L = 300\text{mm};\ D_1 = 400\text{mm};\ D_2 = 300\text{mm};\ v_1(t) = 8 \cdot t\left[\frac{m}{s}\right]$

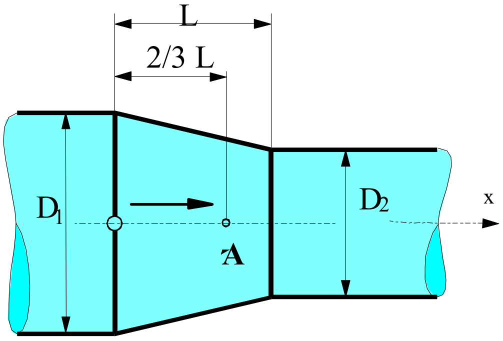
Question:
Determine the acceleration of the liquid particle passing through "A" point at the moment $t = 2\,s$!

Solution:
A full or otherwise substantive acceleration of a liquid part - in the case of Euler's description method - consists of a convective and local acceleration part:

$$\underline{a}_{subs} = \underline{a}_{loc} + \underline{a}_{conv}$$

$$\frac{d\underline{v}}{dt} = \frac{\partial \underline{v}}{\partial t} + \underline{\underline{D}}\underline{v}$$

The convective acceleration is written in coordinate form



Acceleration in confusor
Figure 11.15

$$a_{conv} = \underline{\underline{D}} \cdot \underline{v} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

we see that, after that $v_y \equiv v_z \equiv 0$, therefore $\frac{\partial v_x}{\partial y} \equiv \frac{\partial v_x}{\partial z} \equiv 0$, the convective acceleration is parallel to the axis "x" and its size is:

$$a_{conv} = \frac{\partial v_x}{\partial x} v_x$$

In the confusor symmetry axis, the local acceleration also has only axial component whose size is:

$$a_{loc} = \frac{\partial v_x}{\partial t}$$

Total acceleration of the liquid part:

$$a_{subs} = \frac{\partial v_x}{\partial t} + \frac{\partial v_x}{\partial x} v_x$$

The flow velocity $v_x(t,x)$ is dependent on the time „t” and the location coordinate "x". Given that the liquid density is constant, the volume flow rate in cross-section 1 corresponds to the volume flow rate flowing through any other cross section of the confuser so that the continuity equation can be written in the following simple form:

$$v_1(t)\frac{D_1^2 \pi}{4} = v_x(t,x) \cdot \frac{D(x)^2 \pi}{4} \qquad 11.32$$

The shape of this formula is valid for non-compressible media flowing in tubes, also for an unsteady flow (*see equation 11.4*), from which the velocity is expressed as:

$$v_x(t,x) = v_1(t)\frac{D_1^2}{D(x)^2} = 8 \cdot t \cdot \frac{D_1^2}{D(x)^2}. \qquad 11.33$$

The above formula includes the diameter depending on x coordinate, in case a conical confuser it is a linear function:

$$D(x) = D_1 + \frac{D_2 - D_1}{L} x \qquad 11.34$$

The full acceleration of the liquid part is obtained by applying the chain rule in the following form:

$$a_{subs} = \frac{\partial v_x}{\partial t} + \frac{\partial v_x}{\partial D}\frac{dD}{dx} v_x \qquad 11.35$$

Individual parts of *equation 11.35* are individually written and then specifield to "A":

$$D(x) = D_1 + \frac{D_2 - D_1}{L} \cdot x = 0.4 + \frac{0.4 - 0.3}{0.3} \cdot \left(\frac{2}{3} \cdot 0.3\right) = \frac{1}{3} m$$

$$a_{loc} = 8 \cdot \frac{D_1^2}{D(x)^2} = 8 \cdot \frac{0.4^2}{\left(\frac{1}{3}\right)^2} = 11.52 \frac{m}{s^2}$$

$$\frac{\partial v_x}{\partial D} = 8 \cdot t \cdot (-2) \frac{D_1^2}{D(x)^3} = 8 \cdot 2 \cdot (-2) \frac{0.4^2}{\left(\frac{1}{3}\right)^3} = -138.24 \frac{1}{s}$$

$$\frac{dD}{dx} = \frac{D_2 - D_1}{L} = \frac{0.3 - 0.4}{0.3} = -\frac{1}{3}$$

$$v_x(t,x) = v_1(t) \frac{D_1^2}{D(x)^2} = 8 \cdot t \cdot \frac{D_1^2}{D(x)^2} = 8 \cdot 2 \cdot \frac{0.4^2}{\left(\frac{1}{3}\right)^2} = 23.04 \frac{m}{s}.$$

Substituting the partial results in *equation 11.35*, we get the long-awaited result

$$a_{subs} = \frac{\partial v_x}{\partial t} + \frac{\partial v_x}{\partial D} \cdot \frac{dD}{dx} \cdot v_x = 11.52 + (-138.24) \cdot \left(-\frac{1}{3}\right) \cdot 23.04 = 11.52 + 1061 = 1073 \frac{m}{s^2}$$

After the example, return to the general preset of the acceleration.

The acceleration of the liquid particle can be written in another form. Divide the convective acceleration of the liquid particle into two parts

$$\underline{a}_{conv} = \underline{\underline{D}} \cdot \underline{v} = \underline{\underline{D}}^* \cdot \underline{v} + \left(\underline{\underline{D}} - \underline{\underline{D}}^*\right) \cdot \underline{v}$$

We have also added and tagged a member $\underline{\underline{D}}^* \cdot \underline{v}$ to the equation. A $\underline{\underline{D}}^*$ denotes the transposed derivative of the tensor.

With mathematical transformations it can be seen that

$$\underline{\underline{D}}^* \cdot \underline{v} = \mathrm{grad} \frac{v^2}{2} \ \textbf{és} \ \left(\underline{\underline{D}} - \underline{\underline{D}}^*\right) \underline{v} = \mathrm{rot}\,\underline{v} \times \underline{v} = -\underline{v} \times \mathrm{rot}\,\underline{v}.$$

Using these, total or substantive acceleration is equal to:

$$\frac{d\underline{v}}{dt} = \frac{\partial \underline{v}}{\partial t} + \mathrm{grad} \frac{v^2}{2} - \underline{v} \times \mathrm{rot}\,\underline{v} \qquad 11.36$$

This form of acceleration will be used in the next chapter, the Euler and Bernoulli equations.

12. Euler and Bernoulli equations

12.1 Euler equation

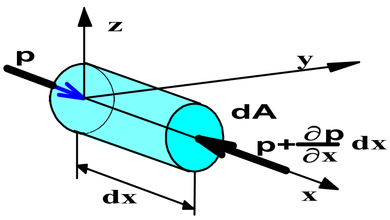
The fluid dynamics is not only about the description of fluid movement but also about the relationship between the causes (forces) and the movement of movement. The relationship between the forces and the change in the volume of movement was attributed to Newton II. (***Isaac Newton, 1642-1727)***, very significant English scientist)

$$\frac{d(m \cdot \underline{v})}{dt} = \sum \underline{F} \qquad 12.1$$

It can be used for a delimited, finite liquid part. The time change of the amount of movement of the demarcated fluid mass and its forces are written to the equation. There are usually two types of forces in the fluid part: the force acting on the mass (eg weight) and the surface force acting on the surface of the liquid part (pressure forces or friction forces). If the medium is non-frictional, the surface force has no component parallel to the surface (the sliding voltage is zero), only forces perpendicular to the surface are affected by pressure. Consider the medium to be frictionless.

Examine the elementary cylindrical drainage element shown in **Figure 12.1**. The axis of the cylinder coincides with the axis "x". The thrust acting on the back plate is positive, the pressure on the first sheet is negative. The resultant of the two forces points to negative x in the direction and value:

$$-\frac{\partial p}{\partial x} \cdot dx \cdot dA$$



Fluid forces
Figure 12.1

The x-directional component of the field strength is g_x, so the volume forces in "x" direction is

$$g_x \cdot dx \cdot dA .$$

We do not even take into account the effects of more forces or we neglect them besides the previous two forces.

The result is a change in the pulse of the liquid in the cylinder, now the motion equation can be written in the "x" direction:

$$\rho \cdot dx \cdot dA \cdot \frac{dv_x}{dt} = \rho \cdot g_x \cdot dx \cdot dA - \frac{\partial p}{\partial x} \cdot dx \cdot dA$$

Devided by cylinder volume and density, the following is obtained:

$$\frac{dv_x}{dt} = g_x - \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad 12.2$$

In the directions "y" and "z", the correlation can be derived in a completely similar way, so

$$\frac{dv_y}{dt} = g_y - \frac{1}{\rho}\frac{\partial p}{\partial y} \qquad 12.3$$

12.4

$$\frac{dv_z}{dt} = g_z - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

Combining the three equations into a vector equation

$$\frac{dv_x}{dt}\cdot \underline{i} + \frac{dv_y}{dt}\cdot \underline{j} + \frac{dv_z}{dt}\cdot \underline{k} = g_x \cdot \underline{i} + g_y \cdot \underline{j} + g_z \cdot \underline{k} - \frac{1}{\rho}\frac{\partial p}{\partial x}\cdot \underline{i} - \frac{1}{\rho}\frac{\partial p}{\partial y}\cdot \underline{j} - \frac{1}{\rho}\frac{\partial p}{\partial z}\cdot \underline{k}$$

we get it, or in other form:

$$\frac{d\underline{v}}{dt} = \underline{g} - \frac{1}{\rho}\cdot \mathrm{grad}p \qquad 12.5$$

The equation for generating fluid movement is called **Euler equation**.
With the *equations 4.28, 4.29, 4.30*, we can write total accelerations. After substitution, the "x, y and z" component equations of the Euler equation are:

$$\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z} = g_x - \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad 12.6$$

$$\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z} = g_y - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z} = g_z - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

Using the transformation of local and convective acceleration at the end of the previous chapter, the left side can be modified here using *equation 11.36*:

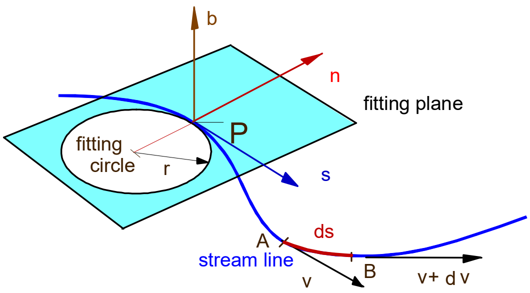
$$\frac{\partial \underline{v}}{\partial t} + \mathrm{grad}\frac{v^2}{2} - \underline{v}\times \mathrm{rot}\underline{v} = \underline{g} - \frac{1}{\rho}\cdot \mathrm{grad}p \qquad 12.7$$

The Euler equation can be used in this form as an anti-friction medium, the density of density is not required, so it is also suitable for describing the motion of a compressible medium. In the **gas dynamics chapter 19** we will use a variable density medium.
The Euler equation is a differential equation that can be solved along with the continuity (*equation 11.6*) at marginal and initial conditions. The solution, however, requires very complicated mathematical methods. Its analytic solution is only possible in exceptional, simple cases. Its numerical solution with increasing the capacity of computers is now possible for more and more tasks. In the following, we use the Euler equation for simple tasks.

12.2 Euler equation in a natural coordinate system

Investigate frictionless, stationary flow. If the flow is steady, the streamline, path line and streak line coincide. Thus, the streamline shown in **Figure 12.2** is also the particle path.
The natural coordinate system can be imagined in several ways. According to the first approach, the axes are fixed to the path, in which case the particle moves in the coordinate system.
But we can also include a coordinate system that is linked to a fluid particle and moves along with it along the path, in which case the D'Alembert principle is applied. (***Jean Le Ronald D'Alembert (1717-1783***) French scientist.]
The tangent of the path line, as this is also a streamline pointing to the velocity of the particle in one direction. Let us examine a point of the track „P”. The course is usually a spatial curve, with which we can draw the fitting plane and fitting circle in the plane which tangent in the

second order with the path line. The extension of the radius of the fitting circle through the "P" marks the axis "n" of the accompanying coordinate system, which is called a normal direction. The tangent to the path line in the direction of speed denotes the tangent coordinate, "s". The binormal direction, "b", forms a right coordinate system with the axis "s" and "n" perpendicular to the fitting plane.



Natural Coordinate System
Figure 12.2

One option is to approach the problem as a hydrostatic task, using the equally accelerating **chapter 10.6** and using the principles known in the rotating system in **chapter 10.7** (using the D'Alembert principle).

In the evenly accelerating and rotating system the fluid is in relative rest, but because of the acceleration of the coordinate system, the inertia forces of the same magnitude, but of opposite direction, must be taken into consideration for the relative balance.

The coordinate system moving along with the liquid particle accelerates along the path, on the one hand, the absolute value of the velocity changes in the direction of the path line's tangent, and on the other hand along the path's arc the direction of velocity varies. The latter can be replaced by a circular motion, the contour data is provided by the circle. There is no curvature in the binormal direction, so there is no acceleration in that direction.

Examine the acceleration of the coordinate system in the directions "s" and "n".

In the tangent direction, only the change in the absolute value of the velocity is to be considered.

Between "A" and "B" of the streamline, the speed "v" is changed to "v + dv" at the "ds" path length. Time needed to travel:

$$dt = \frac{ds}{v} \ ,$$

and acceleration

$$a_s = \frac{v + dv - v}{dt} = \frac{dv}{dt} = \frac{dv}{ds} \cdot v$$

"a_s" points to the tangent of the track.

In the normal direction, the coordinate system moves on a circular path for a moment with a perpendicular velocity of "v" and a radius of radius "r". Centripetal acceleration required to hold the circular path

$$a_{cp} = -\frac{v^2}{r}$$

The minus sign is required because the coordinate taken is countered by the acceleration.

Let's go back to the particle, which is compared to the accelerating coordinate system, but the inertia forces of the same magnitude, but the opposite ones, exactly the force fields, are the same accelerations as calculated.

Particle balance can be defined in the natural coordinate system as a hydrostatic task. Apply *equation 10.4*, which is as follows:

$$\text{grad}p = \rho \cdot \underline{g}$$

Devided the equation with the density and settle to zero, then we get it

$$0 = -\frac{1}{\rho} \cdot \text{gradp} + \underline{g}$$

The pressure gradient vector in the coordinates system "s, n and b", in which the unit vectors are designated $\underline{e}_s$, $\underline{e}_n$ és $\underline{e}_b$, is:

$$\text{gradp} = \frac{\partial p}{\partial s} \cdot \underline{e}_s + \frac{\partial p}{\partial n} \cdot \underline{e}_n + \frac{\partial p}{\partial b} \cdot \underline{e}_b$$

The field vector is composed of two parts. One part is the result of the actual working forces, we mark its components with, for example g_s, g_n and g_b, this can be the fravitation field. The other component is the inertial force section, the inertia field of the same magnitude, but opposite, in the direction of "s" and "n". Using these, we can write the equilibrium of the liquid particle in the three directions of the natural coordinate system moving along with it. This is the following:

Tangent direction $$0 = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial s} + g_s - v \cdot \frac{dv}{ds}$$

Normal direction $$0 = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial n} + g_n + \frac{v^2}{r}$$

Binormal direction $$0 = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial b} + g_b$$

The above derivation is based on the assumption that the coordinate system is moving and the particle is calm in relation to the system. The zero-shaped figure indicates that the acceleration of the particle is zero in all three directions.
If we move to a stationary coordinate system that coincides with the previously discussed motion, the particle accelerates relative to the system, so accelerations in the equations instead of the inertia forces. This coordinate transformation refers to the rearrangement of the above equations for their usual shape when acceleration members are on the left, so the **Euler equation** in the natural coordinate system is as follows:

Tangent $$v \cdot \frac{dv}{ds} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial s} + g_s \qquad 12.8$$

Normal $$\frac{v^2}{r} = \frac{1}{\rho} \cdot \frac{\partial p}{\partial n} - g_n \qquad 12.9$$

Binormal $$0 = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial b} + g_b \qquad 12.10$$

The gravity field is often negligible to the pressure forces from the pressure. In such a case, it is possible to deduce from the current image the pressure distribution or the pressure distribution to the current picture. It is very easy to apply the normal component of the Euler equation in the natural coordinate system to evaluate the pressure change. After leaving the field power, *equation 12.9* takes the following form:

$$\frac{v^2}{r} = \frac{1}{\rho} \frac{\partial p}{\partial n} \qquad 12.11$$

The expression on the left of the equation is always a non-negative number, so the right side must also be positive or zero. Outwardly from the center of the curvature, the pressure must always increase, and if the streamlines are parallel lines (no curvature), the pressure in the direction perpendicular to them will not change, e.g. formed duct flow or parallel flow to the wall.

[11]: An illustrative example from [11]:

Curved streamlines around a car

Figure 12.3

The nature of the pressure distribution on the body of a passenger car as shown in **Figure 12.3** can be determined by the above considerations. The "+" and "-" symbols indicate the overpressure of the undisturbed pressure, depression (less pressure than the outside). For example, when a bonnet and windscreen juncture meet, there is apparent overpressure on the curve of the streams, so ventilation air is introduced here.

The normal component of the Euler equation in a natural coordinate system is very useful in plane flow for circular streams (vortexes). Let's look at the next task.

12.2.1 Rotating fluid as a solid body

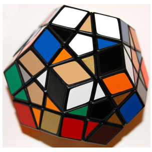


Figure 12.4 shows the velocity of the plane flow characterized by the streams with the function described by the $|\underline{v}| = \omega \cdot r$ function. Water rotates like a solid body. A horizontal section of the rotating vessel discussed in Section 10.7 (**see Figure 2.15**) is shown in **Figure 12.4.**

In the present task, we examine the phenomenon from a standing coordinate system rather than as a static task.

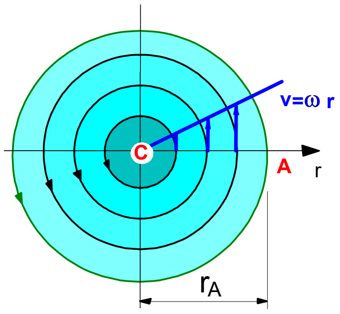
Data: $r_A = 100\,\text{mm}$; $\rho = 10^3 \frac{kg}{m^3}$; $\omega = 20 \frac{1}{s}$

Questions:

a./ What is the difference in pressure between "A" and "C"? Use the task to rewrite rotation and circulation.

b./ Calculate the value of rot $\underline{v}$ in "A".

c/ Calculate the value of circulation on the radius "r_A".



Rotating fluid as a solid body Figure 12.4

Solution:

a./ From the normal directional component of the Euler equation in the natural coordinate system, calculated from *equation 12.11*, the pressure variation

$$\frac{dp}{dr} = \rho \cdot \frac{v^2}{r} = \rho \cdot \frac{(\omega \cdot r)^2}{r} = \rho \cdot 400 \cdot r$$

The differential pressure between points A and C is obtained by integrating this with:

$$p_A - p_C = \int_0^{0.1} \frac{dp}{dr} \cdot dr = \int_0^{0.1} 400 \cdot \rho \cdot r \cdot dr = 400 \cdot 10^3 \cdot \left[\frac{r^2}{2}\right]_0^{0.1} = 400 \cdot 10^3 \cdot \frac{0.1^2}{2} = 2000\,\text{Pa}\,.$$

In **Chapter 10.7**, exactly the same result was calculated:

$$p_A - p_C = p_A - p_0 - (p_C - p_0) = 4060 - 2060 = 2000\text{Pa}$$

b. For calculating the rotation, we use *equation 11.25*, which is used to compute the rotation for circles

$$(\text{rot}\,\underline{v})_z = \frac{v}{r} + \frac{dv}{dr} = \frac{20 \cdot r}{r} + 20 = 40\frac{1}{s}$$

A rotáció nem függ a helytől értéke, tehát az adott "A" helyen is $40\frac{1}{s}$, ami éppen a statikai feladatban adott "ω"-nak a kétszerese, mint azt az előző fejezetben láttuk.

c./ The rotation does not depend on the location value, so also in the given "A" position, which is twice the "$40\frac{1}{s}$" given in the static task, as seen in the previous chapter.

c./ The velocity is constant over a circle so that the circulation can be easily written using *equation 11.20* because the perimeter of the circle should simply be multiplied by the velocity at which it is applied:

$$\Gamma = \oint \underline{v} \cdot d\underline{s} = 2 \cdot \pi \cdot r_A \cdot v_A = 2 \cdot \pi \cdot r_A \cdot 20 \cdot r_A = 2 \cdot \pi \cdot 0.1 \cdot 20 \cdot 0.1 = 1.25\frac{m^2}{s}$$

Note that any velocity distribution will increase the pressure from the center outward as the expression $\rho \cdot \frac{v^2}{r}$ is always positive. At the center of the vortex is the minimum pressure.

12.3 The Bernoulli equation

In the previous task, the differential pressure was calculated from the Euler equation between two points. The Euler equation may be integrated in a general case with two points of the fluid space along a line, without the specific knowledge of the velocity function. With some of the simplification criteria, its solution leads to a surprisingly simple context.
Let's get out of **figure 12.7** of the Euler equation

$$\frac{\partial \underline{v}}{\partial t} + \text{grad}\frac{v^2}{2} - \underline{v} \times \text{rot}\underline{v} = \underline{g} - \frac{1}{\rho} \cdot \text{grad}p$$

Apply the line integrity of the equation between the two fixed points of the flow space. (For the derivation of the baseline equation of hydrostatic, we have already received the line integer for the two members on the right side of the equation in **Section 10.1.1** with constant densities and potential force fields resulting from *equation 2.7*).

$$\int_1^2 \frac{\partial \underline{v}}{\partial t} d\underline{s} + \int_1^2 \text{grad}\frac{v^2}{2} d\underline{s} - \int_1^2 \underline{v} \times \text{rot}\underline{v} \cdot d\underline{s} = \int_1^2 \underline{g} d\underline{s} - \int_1^2 \frac{1}{\rho} \text{grad}p d\underline{s} \qquad 12.12$$

I. II. III. IV. V.

The above equation, its creator, is called the Bernoulli equation. (***Daniel Bernoulli 1700-1782 Swiss scientist***) (Historical credentials include that Bernoulli described the equation named after him, not the **figure 12.12**, but the form of **figure 12.13**, based on energetic considerations) before the Euler equation was born.

Let's look at the most common Bernoulli equation, which conditions can be made to a simpler form!

1./ The field strength vector has a potential function, so:

$$\underline{g} = -\text{grad}U$$

From mathematics we know that gradient operation and line integration are inverse operations such as **IV. term** can be added to the following simple shape:

$$-\int_1^2 \text{grad}U \cdot d\underline{s} = -[U]_1^2$$

2./ Let be the medium density constant!

In this case, the density in the **V. term** can be added to the gradient signal since it is constant, and a

like a potential function:

$$-\int_1^2 \text{grad}\frac{p}{\rho} \cdot d\underline{s} = -\left[\frac{p}{\rho}\right]_1^2$$

the **II. term** is simply simplified as above.

$$\int_1^2 \text{grad}\frac{v^2}{2} \cdot d\underline{s} = \left[\frac{v^2}{2}\right]_1^2$$

3./ The III. term $-\int_1^2 \underline{v} x \text{rot}\underline{v} \cdot d\underline{s} = \underline{0}$ **is zero** for one of the following:

- The velocity v is zero, this is the case of hydrostatic;
- the rot$\underline{v}$=0, ie. the flow is potential;

- $d\underline{s}$, $\underline{v}$ and $rot\underline{v}$ vectors in a plane, the three vectors are certainly in a plane if two of them are parallel, so in the following cases
- $d\underline{s} \parallel \underline{v}$ so integrate elong streamline;
- $d\underline{s} \parallel rot\underline{v}$, so integral elong eddy line;
- the envelope curve of the eddy current vortex vector, just like the streamline: the velocity vector envelope curve
- $\underline{v} \parallel rot\underline{v}$, this is called a Beltram flow.

4./ The flow is steady, in this case the **I. term** is zero, $\frac{\partial \underline{v}}{\partial t} = \underline{0}$.

Inserting these simplifications into *equation 12.12* gives the following:

$$\left[\frac{v^2}{2}\right]_1^2 = -\left[\frac{p}{\rho}\right]_1^2 - [U]_1^2$$

The quantities of the same index are arranged on one side in front of the **usual shape of the Bernoulli equation**:

$$\frac{v_1^2}{2} + \frac{p_1}{\rho} + U_1 = \frac{v_2^2}{2} + \frac{p_2}{\rho} + U_2 \qquad 12.13$$

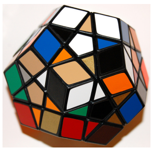
The conditions for its applicability are summarized as follows:
- the flow is steady,
- the rotation term is zero,
- flow-free or streamlined, etc.
- the force field is potent (most often the earth's gravity force)
- the density is constant,
- and of course friction is negligible.

The applications of the Bernoulli equation are discussed in a separate chapter**, chapter 13**

13. Applications of the Bernoulli equation

When applying the Bernoulli equation, the following aspects should be observed:
1./ First, you have to decide whether the conditions of the application are available. We repeat the summary of terms:
• the flow is stationary,
• the rotation member is zero, whirling-free on the flow or streamline, etc.,
• the force field is potentially (most of all, the Earth's gravity force)
• density is constant and
• friction is negligible.
2./ In the next step, a suitable coordinate system must be chosen, in which flow is well described, for example: the flow is steady and on the other hand the potentials of the field can be easily written
3./ Suitable points in the fluid space should be chosen, at least two, but in some cases, eg. if there are multiple liquids in the system, then more than two points. When choosing points, you should keep in mind the following: One should know each quantity as much as possible, and at the other point only one unknown, the requested quantity. Suitable points are: free surface, large space, spill radius, two fluid interfaces, and so on. But when using the continuity batch, there are two unknowns.
4./ After the field potential is written, apply the Bernoulli equation.

13.1 Vortex on the water surface



In the previous chapter, we investigated the potential velocity of the potential vortex. The velocity is inversely proportional to the distance from the center point. That would mean that very low radius would bring very high values. At the center of the real vortex, the velocity develops like in a into a solid body.
Figure 13.1 shows a longitudinal section and a plan view of a vortex formed on a water surface. Angular velocity $\omega = 2.55\frac{1}{s} = \text{const.}$ $r_m = 0.5\text{m}$ as a solid body rotating core or nucleus.
In the swirl the velocity change corresponds to the potential velocity of the potential swirl, ie

$$v = \frac{\Gamma}{2 \cdot \pi \cdot r}$$

Questions:
a./ Define the function of the shape of the water surface.
b./ How much circulation is the circle around the core?
c./ Calculate the thepth of zhe center!
d./ Show that there is no fracture on the surface of the core and the swirl!

Solution:

a./ In the core $v = \omega \cdot r$, in the swirl part $v = \frac{\Gamma}{2 \cdot \pi \cdot r}$ the function describes the change in the absolute value of the velocity as a function of the radius. The streamlines are everywhere concentric circles that lie in the plane perpendicular to the axis "z". The rotation value for this type (cylinder symmetric, circular stream lines and absolute value of velocity is only for the function of "r" coordinate) $(\text{rot}\,\underline{v})_z = \frac{dv}{dr} + \frac{v}{r}$ can be calculated. Applied to the function of the core and the rotating velocity, in the **core** $(\text{rot}\,\underline{v})_z = 2\cdot\omega$ and in the **swirl** $(\text{rot}\,\underline{v})_z = 0$, as shown in the previous chapters. In the swirl in a standing coordinate system, we can use the Bernoulli equation without any further action because the flow is rotation free, but we can not apply it in the core. In the core as a hydrostatic task, we can determine the shape of the surface in a co-rotation coordinate system.

a/1 Use the Bernoulli equation in the swirl in a standing coordinate system between points "1" and "2". Point "2" is in the "z = 0" position but in the infinite point where the velocity is zero Point „1" is a running point characterized by "r" and "z" coordinates in the swirl.

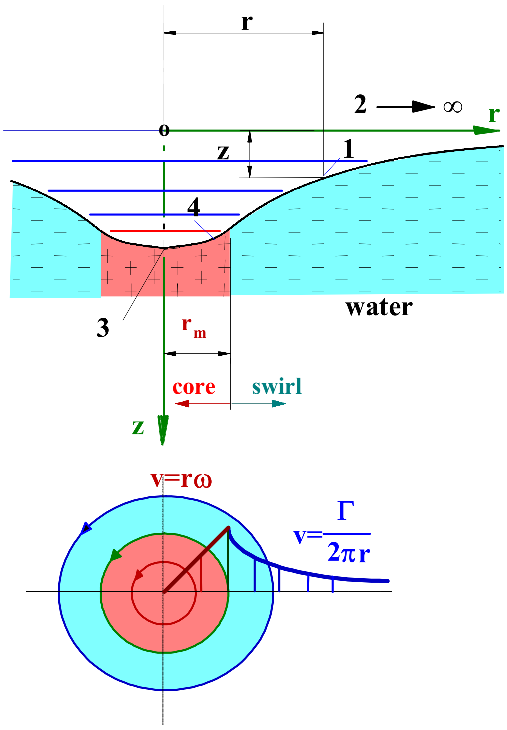
When commanding the potential of the field potential, note that the axis "z" is pointing down, but the potential must rise upwards. Thus, the potential function is in the standing coordinate system.

To avoid mistakes, write the whole form of the Bernoulli equation, *5.13 equation*

$$\frac{v_1^2}{2} + \frac{p_1}{\rho} + U_1 = \frac{v_2^2}{2} + \frac{p_2}{\rho} + U_2$$

Both points are located on the water surface, where the pressure is the ambient pressure p_0. The velocity is described by the function in both points. At point 2, "r" is infinite, so the velocity is zero.

$$\frac{1}{2}\cdot\left(\frac{\Gamma}{2\pi r}\right)^2 + \frac{p_0}{\rho} - g\cdot z = \frac{0^2}{2} + \frac{p_0}{\rho} + 0$$



Whirl on the surface of the water
Figure 13.1

Rearranged and expressed as the surface sinking "z":

$$z = \frac{\Gamma^2}{8\pi^2\cdot g\cdot r^2} \quad \text{ha } r \geq r_m \qquad 13.1$$

a/2 In the core in a rotating coordinate system, use the Bernoulli equation between the deepest point "3" and the $r < r_m$ "4" running point.

In the co-rotating system, the potential of the rotating space should also be written as described in **section 10.5** for the potential function

$$U = -g\cdot z - \frac{r^2\cdot\omega^2}{2}$$

The velocity of the liquid is zero everywhere in the nucleus when we rotate with the fluid. The Bernoulli equation simplifies the basic equation of statics:

$$\frac{p_1}{\rho} + U_1 = \frac{p_2}{\rho} + U_2.$$

Substituted, we get the following: (z_0 is the deepest point of the surface)

$$-z_0 \cdot g = -z \cdot g - \frac{r^2 \cdot \omega^2}{2} \quad ,$$

z (the second member on the right side is not a speed but a member of the potential of the rotation field)

$$z = z_0 - \frac{r^2 \cdot \omega^2}{2g} \text{ ahol } r \le r_m \qquad 13.2$$

The above *13.1* and *13.2* relationships together describe the shape of the surface. Given that "z_0" and "Γ" are unknown, we get these from two additional terms:
The first *13.3 expresses* that the velocity of the core and the swirl is at the same velocity, so:

$$r_m \cdot \omega = \frac{\Gamma}{2\pi \cdot r_m}$$

The second correlation is that the sinking of "z" at the limit of the core and the swirl is identical from the formulas *13.1* and *13.2*, :

$$\frac{\Gamma^2}{8\pi^2 \cdot g \cdot r_m{}^2} = z_0 - \frac{r_m^2 \cdot \omega^2}{2 \cdot g} \quad . \qquad 13.3$$

Reerranging the equations for "z_0" and "Γ", then entering their values into the *13.1* and *13.2* correlations, we get the following function to describe the surface:

$$z = \frac{r_m^2 \cdot \omega^2}{2 \cdot g}\left[2 - \left(\frac{r}{r_m}\right)^2\right] \text{ha } r \le r_m \text{ vagy } z = \frac{r_m^2 \cdot \omega^2}{2 \cdot g}\left(\frac{r_m}{r}\right)^2 \text{ ha } r \ge r_m \qquad 13.4$$

b./ To compute the circulation around the nucleus, we can easily express „Γ" using the relation *13.3.*

$$\underline{\Gamma = 2\pi \cdot r_m{}^2 \cdot \omega = 2\pi \cdot 0.5^2 \cdot 2.55 = 4\frac{m^2}{s}}$$

c./ From the equation describing the surface, it can be seen that the descent of the center of the vortex is as follows:

$$\underline{z_0 = \frac{r_m^2 \cdot \omega^2}{g} = \frac{0.5^2 \cdot 2.55^2}{9.81} = 0.165\text{m}}$$

d./ The fracture-free connection of the two surfaces can be verified in the place r_m by the identity of the substitution values of functions derived from functions of r. Derive the *13.4 functions* at that time

$$\frac{dz}{dr} = -2 \cdot \frac{\omega^2}{2} \cdot r \text{ if } r \le r_m \text{ or } \frac{dz}{dr} = -2 \cdot \frac{r_m^4 \cdot \omega^2}{2 \cdot r^3} \text{ if } r \ge r_m$$

It can be seen that the two values are the same at $r = r_m$, so that the water surface of the core is connecting fracture-free to the swiel surface.

13.2 Whirlwind

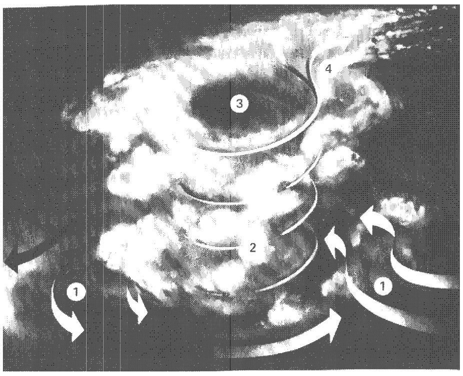
If the vortex develops in an airborne medium, e.g. whirlwind, tornado, or cyclone, the change in pressure is given in a similar manner to the above functions.

The pressure function is obtained by substitution $z = \frac{p - p_0}{\rho \cdot g}$, such as the pressure in the rotating air stream

$$p = p_0 - \rho \cdot \frac{r_m^2 \cdot \omega^2}{2} \left[2 - \left(\frac{r}{r_m} \right)^2 \right] \text{if } r \le r_m \text{ or } p = p_0 - \rho \cdot \frac{r_m^2 \cdot \omega^2}{2} \left(\frac{r_m}{r} \right)^2 \text{ if } r \ge r_m \ .$$

It can be seen that every "r" value causes depression inside the air stream and the smallest pressure is generated in the center.

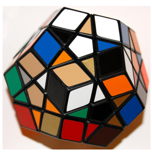
Figure 13.2 illustrates an air movement in a hurricane. (In reality, such airwaves are hundreds of miles in diameter.) The near-blurred air (1) flows from all directions to the center of the hurricane, and flows around the swirling "eye" (3) at a high speed and there is a great depression here. The rising air exits the air stream at high altitude.



Flow in the hurricane
Source: Természet ABC-je, Reader's Digest
Figure 13.2

In the eye of a hurricane (3), averaging 20 km, the air velocity is small, almost in comparison to the surrounding high-speed zone, this corresponds to the core of the vortex. Zone (2) is the limit of the core and the swirl, here the maximum wind speed is 200-300 km/h. Of course, there is a significant upward flow in the hurricane, so it's just about the same as the speed of the hurricane. Another important application of the Bernoulli equation.

13.3 Outglow from the tank



The cross section of the container can be taken as infinite in relation to the outflowing section, so the sinking velocity of the surface is negligible. During the test the phenomenon can be considered steady flow.

Data: $p_1 = 3\text{bar(absz.)}$; $p_0 = 1\text{bar}$; $h = 5\text{m}$

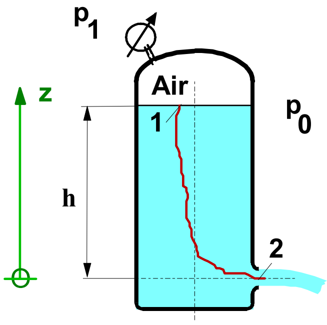
Questions:

a./ What is the velocity of the outflow if $p_1 = 3\text{bar(abs.)}$?

b./ What is the speed of the discharge when the tank is open $p_1 = p_0$?

Solution:

The flow starts from resting space and flows practically free of loss to the outlet. Point "1" is located on the surface of the liquid at rest, point "2" at the point of discharge. where the pressure is p_0. For the calculation of the potential it is advisable to select the coordinate system at the height of the "2" starting point and an upward "z" axis. Then the potentials of the field



Outflow from tank
Figure 13.3

$$U = g \cdot z$$

Applying the Bernoulli equation:

$$\frac{0^2}{2} + \frac{p_1}{\rho} + g \cdot h = \frac{v_2^2}{2} + \frac{p_0}{\rho} + 0$$

From this equation you can express the velocity you are looking for:

$$v_2 = \sqrt{2 \cdot g \cdot \left(h + \frac{p_1 - p_0}{\rho \cdot g} \right)} \qquad 13.5$$

a./ Replacing the values:

$$v_2 = \sqrt{2 \cdot 9.81 \cdot \left(5 + \frac{3 \cdot 10^5 - 1 \cdot 10^5}{1000 \cdot 9.81} \right)} = 22.31 \frac{m}{s}$$

b./ When the tank is open $p_1 = p_0$, the velocity of the discharge is a

$$v_2 = \sqrt{2 \cdot g \cdot h}$$

The term Torricelli is also referred to as ***Torricelli's*** discovery.
The flow rate in this case is exactly as if the liquid has been free from "h" height. Its positioning energy becomes completely moving energy.

13.4 Siphon

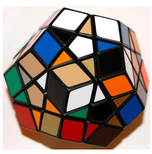
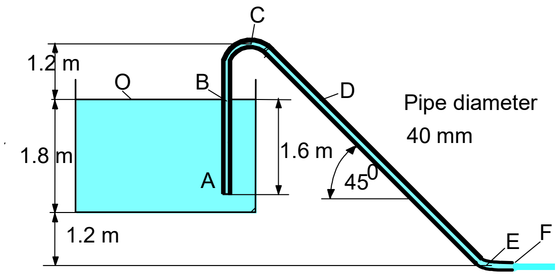


Figure 13.4 shows a siphon that sucks the water out of a pool. The surface of the pool is much larger than the pipe cross section. The geometric data is shown in the figure. Flow losses in the tube are negligible

Qestions:

a./ What is the velpcity of the outflow?
b./ Draw the pressure change along the length of the tube!



Siphon
Figure 13.4

Solution:

The steafy velocity of the pipe can be determined from the Bernoulli equation between the "O" and the "F" points.
a./ Because atmospheric pressure is at both points, the velocity can be calculated using the Torricelli formula.

$$v_F = \sqrt{2 \cdot 9.81 \cdot (1.2 + 1.8)} = 7.672 \frac{m}{s}$$

b./ Calculate the pressure at the points drawn in the diagram using the Bernoulli equation. Apply at the specified points.
To specify the potential function, you just need to pick up the "z" coordinate again and the origin on the surface of the water.
Immediately at point A there is a problem, a complicated spatial flow is formed at the point of entry into the tube, and the velocity changes very rapidly in a very short section, so it is necessary to avoid pointing to the Bernoulli equation when entering the tube. We can solve the problem by picking two points, one directly in front of the pipe "A_1" and one in the tube after entering "A_2". The distance between the two points in the figure is negligible, but in the "A_1" point the medium has no velocity, and at point "A_2" it is already in the tube velocity.
O-A1 points:

$$p_{A_1} - p_0 = \rho \cdot g \cdot 1.6 = 1000 \cdot 9.81 \cdot 1.6 = 15.69\text{kPa}$$

O-A_2 between points:

$$p_{A_2} - p_0 = \rho \cdot g \cdot 1.6 - \frac{\rho}{2} \cdot v_E^2 = 1000 \cdot 9.81 \cdot 1.6 - \frac{1000}{2} \cdot 7.672^2 = -13.73\text{kPa}$$

O-B between points. The two points are at the same height, differential pressure is caused only by the velocity difference

$$p_B - p_0 = -\frac{\rho}{2} \cdot v_E^2 = -\frac{1000}{2} \cdot 7.672^2 = -29.43\text{kPa}$$

O-C between points:

$$p_C - p_0 = -\rho \cdot g \cdot 1.2 - \frac{\rho}{2} \cdot v_E^2 = -1000 \cdot 9.81 \cdot 1.2 - \frac{1000}{2} \cdot 7.672^2 = -41.2\text{kPa}$$

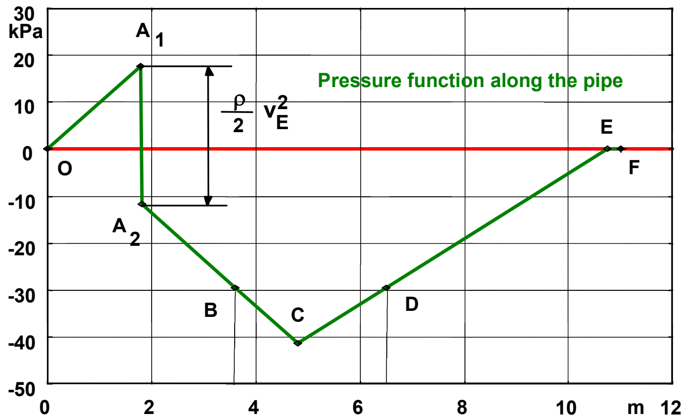
Point "D" has the same pressure as in "B"

$$p_D - p_0 = -29.43\text{kPa}$$

O-E between points:

$$p_E - p_0 = \rho \cdot g \cdot 3 - \frac{\rho}{2} \cdot v_E^2 = 1000 \cdot 9.81 \cdot 3 - \frac{1000}{2} \cdot 7.672^2 = 0\text{kPa}$$

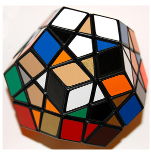
At points "E" and "F", the velocity of the height is the same, so the pressure must be the same, so that pressure "E" could be obtained without calculation.
The flow of pressure along the extended length of the tube is shown in **Figure 13.5**. Between the points "A_1" and "A_2", the medium accelerates from zero velocity to the velocity of the pipe so that the pressure decreases with the corresponding value suddenly.



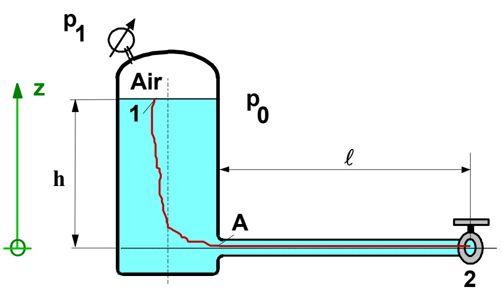
Pressure function in the pipe
Figure 3.5

Another important phenomenon from the diagram is that at point "C" the pressure drops strongly below atmospheric pressure. If the distance between the "O" and the "F" points increases, ie the outlet end of the pipe is deeper, the output velocity will increase and the pressure at C will continue to decrease.
If the pressure of "C" reaches the saturated vapor pressure at a given temperature, the fluid in the tube breaks and steam bubbles are formed that will collapse later when placed in a higher pressure. Continuous flow ceases. A pulsating throbbing flow occurs and cavitation occurs in the system (**see Chapter 1**).

13.6 Outglowl from tank in an unsteady case



In **Example 13.3**, a spill from an infinite large container was tested. Attach a relatively long tube to the tank (the length of the pipe is several times larger than its diameter) with a tap ending. The tap can be opened very quickly, for example a ball tap.
In the closed pipe end, the water is in the pipe and the pressure is constant along the pipe and is equal to the sum of the pressure of the water column in the tank and the overpressure in the container.
When the tap is suddenly opened, the pressure drops down to the atmospheric pressure behind the tap, and then the decreasing pressure falls into the other cross-sections of the tube in the form of a wave. Fluid particles in the tube are subjected to a reduction of pressure due to the acceleration force which initiates the fluid column. At the moment of opening, however, liquid is still in the tube. The fluid velocity in the tube increases gradually and reaches a maximum value, namely the steady velocity calculated in **Example 13.3** if there is no friction in the system.



Unsteady outflow from tank
Figure 3.6

Data: $p_1 = 3\text{bar(absz.)};$ $p_0 = 1\text{bar};$ $h = 5\text{m};$ $\ell = 15\text{m}$

Qestions:

a./ Determine the outglow velocity in steady state!
b./ Determine fluid velocity and acceleration as a function of time in the tube!
c./ How long does it take the steady velocity of the above data reach?
d./ Draw the pressure distribution along the pipe at $\frac{t}{\tau} = 1!$

a./ Steady solution

After a relatively long period of time, the velocity reaches the steady velocity in the tube (the sinking of the water reservoir is still negligible at this time). The tube has a negligible friction loss, so the result obtained in **Example 13.3** can also be used here. In the case of a lossless case, in a steady horizontal pipe with constant cross-section, the pressure does not change in stationary cases, so the pressure at the end of the pipe is constant. Atmospheric pressure is also displayed in the outlet section of the tank, similar to the **13.2 task**. Thus the steady velocity at the end of the pipe is equal to *equation 13.5* (with the v_{st} symbol):

$$v_{st} = v_2 = \sqrt{2 \cdot g \cdot \left(h + \frac{p_1 - p_0}{\rho \cdot g} \right)} = \sqrt{2 \cdot 9.81 \cdot \left(5 + \frac{(3-1) \cdot 10^5}{1000 \cdot 9.81} \right)} = 22.31 \frac{m}{s} \qquad 13.6$$

Express the square of the velocity from the equation, because we will need it for further solution:

$$v_{st}^2 = 2 \cdot g \cdot \left(h + \frac{p_1 - p_0}{\rho \cdot g} \right) \qquad 13.7$$

b./ Unsteafy solution

In the example, all the conditions that are fulfilled in the outlet of the steady container except that the phenomenon in the pipe is unsteady.

Therefore, we have to choose the form of the Bernoulli equation in which the condition of time resistance has not yet been determined. Writing *equation 5.12:*

$$\int_1^2 \frac{\partial \underline{v}}{\partial t} d\underline{s} + \int_1^2 \operatorname{grad} \frac{v^2}{2} d\underline{s} - \int_1^2 \underline{v} \times \operatorname{rot} \underline{v} \cdot d\underline{s} = \int_1^2 \underline{g} d\underline{s} - \int_1^2 \frac{1}{\rho} \operatorname{grad} p \cdot d\underline{s}.$$

I. II. III. IV. V.

From **II. to V.** as described in the previous chapter. The first member is unchanged 13.8

$$\int_1^2 \frac{\partial \underline{v}}{\partial t} d\underline{s} + \frac{v_2^2}{2} + \frac{p_2}{\rho} + U_2 = \frac{v_1^2}{2} + \frac{p_1}{\rho} + U_1$$

The first member of the equation integrates the local acceleration line at a given time along the path between "1" and "2".

The path in the diagram is also a streamline at each time, so the path and the acceleration are same directional, and the scalar product in the integrand can be replaced by simple scalar numerals.

The line integration of acceleration is expressed by the following considerations. If the tank is large enough, then the velocity is negligible, but then acceleration is a good approximation to zero. Therefore, the integration path is divided into two parts: "1-A" and "A-2":

$$\int_1^2 \frac{\partial \underline{v}}{\partial t} d\underline{s} = \int_1^A \frac{\partial \underline{v}}{\partial t} d\underline{s} + \int_A^2 \frac{\partial \underline{v}}{\partial t} d\underline{s}$$

The first part of the right side is 0 because in the container that is considered infinite, the acceleration of the medium can be neglected. Our integration takes the following simpler form:

$$\int_1^2 \frac{\partial v}{\partial t} d\underline{s} = \int_A^2 \frac{\partial v}{\partial t} d\underline{s} \qquad 13.9$$

In order to complete the integral, the local acceleration change should be known along the pipe length. If it stands, it follows from the continuity that in any cross section of the pipe it has to be equal to the flow rate at a given moment, otherwise the fluid would either crumble or break:

$$v_1 A_1 = v_2 A_2$$

It is also a prerequisite that the cross section of the pipe does not widen, it does not collapse. Derived from the above equation by time, only the velocity may depend on time, such as:

$$\frac{\partial v_1}{\partial t} \cdot A_1 = \frac{\partial v_2}{\partial t} \cdot A_2$$

Mark the time derivatives of the velocity with "a" at that time

$$a_1 A_1 = a_2 A_2 \qquad 13.10$$

It follows that the local acceleration of a constant density medium in a constant cross-section tube does not change along the length of the tube. Therefore, the *relation 13.9* can be transformed as follows:

$$\int_1^2 \frac{\partial v}{\partial t} d\underline{s} = \int_A^2 a ds = a \cdot \ell \qquad 13.11$$

Then write the other members of the Bernoulli equation *equation 13.8* between "1" and "2".

$$a \cdot \ell + \frac{v_2^2}{2} + \frac{p_0}{\rho} = \frac{p_1}{\rho} + g \cdot h \qquad 13.12$$

Enter the following markings $v_2 = v$ and $a = \frac{dv}{dt}$. We can use a full-time deriva tive because the velocity in the pipe depends only on time rather than on the site. Arrange the equation:

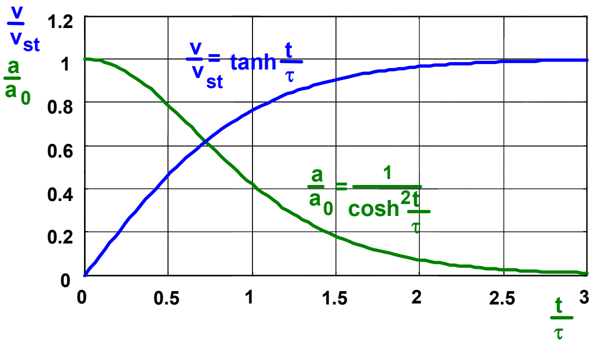
$$2 \cdot \ell \frac{dv}{dt} + v^2 = 2 \cdot \left(\frac{p_1}{\rho} - \frac{p_0}{\rho} + g \cdot h \right) \qquad 13.13$$

Note that the right side of the equation is the square of the stationary velocity "v_{st}^2" (*equation 13.7*).

By substituting and separating the following differential equation:

$$\frac{dv}{v_{st}^2 - v^2} = \frac{dt}{2 \cdot \ell} \qquad 13.14$$

At steady velocity, dimensioned and assigned the integrated



Speed and acceleration functions Figure 13.7

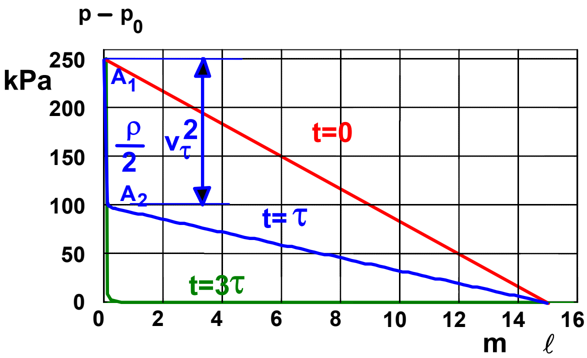
$$\int_0^{v/v_{st}} \frac{d\frac{v}{v_{st}}}{1-\left(\frac{v}{v_{st}}\right)^2} = \frac{v_{st}}{2\ell}\int_0^t dt \qquad 13.15$$

After integration, $\operatorname{artanh}\frac{v}{v_{st}} = \frac{t\cdot v_{st}}{2\cdot\ell}$ the correlation arises, for example. integral table. Enter the $\tau = \frac{2\cdot\ell}{v_{st}}$ time where it can be called the system's own time. The velocity function is then obtained by taking the "tangent hyperbolic" function on both sides.

$$v = v_{st}\cdot\tanh\frac{t}{\tau} \qquad 13.16$$

The acceleration function is the derivative of the velocity function

$$a = \frac{a_0}{\cosh^2\frac{t}{\tau}},$$



Pressure function along the length of the tube Figure 13.8

where $a_0 = \frac{v_{st}^2}{\tau}$ acceleration in the initial momentum. (the "cosh" function is the cosine hyperbolic function).

c./ From **<u>Figure 13.7</u>** it can be seen that $\frac{t}{\tau} = 3$ approximatly the medium is flowing at steady velocity in the tube. To calculate a specific time, "τ" must first determine its own time

$$\tau = \frac{2\cdot\ell}{v_{st}} = \frac{2\cdot 15}{22.31} = 1.34\text{s}$$

The three of their own time are 4 s when $\frac{v}{v_{st}} = 0.995$.

d./ In case $\frac{t}{\tau} = 1$, both velocity and acceleration can be determined from **<u>Figure 13.7</u>**. For example, you can read the values in the diagram.

The velocity: $v_\tau = 0.78\cdot v_{st} = 0.78\cdot 22.31 = 17.4\frac{m}{s}$,

The acceleration: $a_\tau = 0.4\cdot a_0 = 0.4\cdot\frac{v_{st}^2}{2\cdot\ell}$ $\quad a_\tau = 0.4\cdot\frac{22.31^2}{2\cdot 15} = 6.6\frac{m}{s^2}$.

The acceleration can be determined by other means. Write *equation 13.12*

$$a\cdot\ell + \frac{v_2^2}{2} + \frac{p_0}{\rho} = \frac{p_1}{\rho} + g\cdot h,$$

and express the acceleration:

$$a_\tau = \frac{\frac{p_1}{\rho} - \frac{p_0}{\rho} + g \cdot h - \frac{v_\tau^2}{2}}{\ell} = \frac{\frac{v_{st}^2}{2} - \frac{v_\tau^2}{2}}{\ell} = \frac{\frac{22.31^2}{2} - \frac{17.4^2}{2}}{15} = 6.5 \frac{m}{s^2}$$

The difference was due to the inaccuracy of the reading.
For the determination of the pressure function, use the Bernoulli equation (*equation 13.8*) between the points „A" and the points "2". The location of point "A" is the same as in the inlet cross section of the syphon, so that the velocity in the very short section changes very quickly due to the complicated flow in the inlet. Now place the "A_2" point in the pipe after login, and "A_1" before it enters.

$$\int_{A_1}^{2} \frac{\partial v}{\partial t} d\underline{s} + \frac{v_2^2}{2} + \frac{p_2}{\rho} + U_2 = \frac{v_A^2}{2} + \frac{p_A}{\rho} + U_1$$

The integral can be replaced by the produst of acceleration and the length, the two velocities and the two potentials being the same

$$a_\tau \cdot \ell + \frac{p_0}{\rho} = \frac{p_A}{\rho}$$

from which the pressure differece

$$(p_A - p_0)_\tau = \rho \cdot a_\tau \cdot \ell = 1000 \cdot 6.5 \cdot 15 = 97.5 \text{kPa}$$

The flow of pressure is shown in **Figure 13.8** at the time, linearly changing along the pipe.
The figure shows the $t = 0$ (red) and $t = 3 \cdot \tau$ (green), which corresponds to the status after the infinite time.

13.7 Fans, Euler turbine equation

The fans deliver air or other airborne medium from a lower pressure to a higher pressure. The fan performs work on the one hand against differential pressure and, on the other hand, increases the movement energy of the air supplied. When calculating the useful power of the fan, the density of air supplied is considered constant. This can be done as long as the density change is small. Remains below 10%. For fans this always exists. (This is called a fan.)
In the flowing air, write a Bernoulli equation along a streamline (*equation 12.13*)

$$\frac{v_1^2}{2} + \frac{p_1}{\rho} + U_1 = \frac{v_2^2}{2} + \frac{p_2}{\rho} + U_2$$

In the flow of air, the change of the position energy between two points is negligible, since the gravity acting on a given mass of air and the buoyancy generated by the surrounding air is equal if the air pressure and temperature do not deviate greatly from its surroundings. Thus, when you raise a given mass of air, you do not have to invest a job, so no potential energy is generated. From the above equation $U_1 - U_2 \approx 0$, using the condition and multiplying the density, the following equation is obtained:

$$\frac{\rho}{2} \cdot v_1^2 + p_1 = \frac{\rho}{2} \cdot v_2^2 + p_2$$

Imagine an air yet that we blow on a flat wall, see. **Figure 14.6** (next chapter). In the radius approaching the wall, the speed must be "v_1" the pressure "p_1". At point "t" in the intersection of the wall and radius axis, the air velocity is zero. This point is called a stagnation point. The pressure here is called a total pressure, which is greater than the pressure in the free yet.

$$\frac{\rho}{2}\cdot v_1^2 + p_1 = p_t$$

Ont he left side of equation

"$\frac{\rho}{2}\cdot v^2$" term the **dynamic pressure**,

"p" term the **static pressure**

"p_t", which is the sum of the previous two, we call it the **total pressure**.

The useful power of a fan that is overcome a given pressure difference:

$$P_p = q_v \cdot (p_p - p_s),$$

- p_s the suction pressure, p_p the pressure ont he pressure side, and the static pressure and the "q_v" air volume, which is constant within the cross-section.

The payload used to increase the amount of kinetic energy is increased by:

$$P_v = q_v \cdot \frac{\rho}{2}\left(v_p^2 - v_s^2\right),$$

"v_s" the suction side "v_p" is the velocity on the discharge side within the cross-section. Useful power is the sum of the two

$$P_u = P_p + P_v$$

Replacing the previous terms, the useful power:

$$P_u = q_v \cdot \left[\left(p_p + \frac{\rho}{2}v_p^2\right) - \left(p_s + \frac{\rho}{2}v_s^2\right)\right] = q_v \cdot (p_{pt} - p_{st}) \qquad 13.17$$

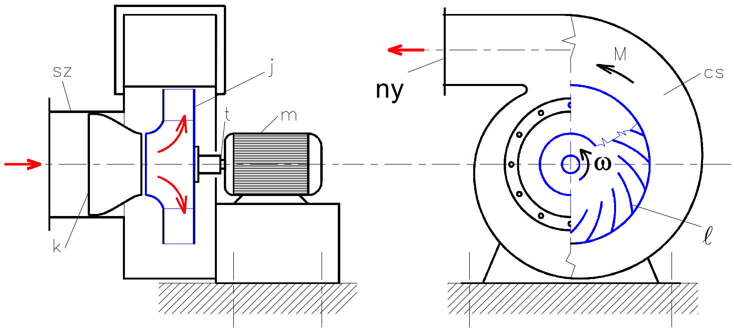
or

$$P_u = q_v \cdot \Delta p_t \qquad 13.18$$

where "Δp_t" the total pressure increase.

Figure 13.9 shows a sketch of a radial fan. The air enters the machine on the suction nozzle (**sz**) and drives a stading confuser (**k**) to the rotating impeller (**j**) which is fixed to the axis (**t**) of the motor (**m**). The medium turns radially and passes through the impeller (**l**) blades. The motor (**M**) exerts momentum on the impeller (ω) rotating at angular velocity. As a result of this torque, the fluid passing through the impeller differs from the direction of rotation. Enters the spiral casing (**cs**) and then leaves the machine through the (**ny**) pressure side. The energy coming from the engine passes through the fan impeller to the flowing medium. In order to understand the operation of the fan, the processes in the impeller must be understood first.

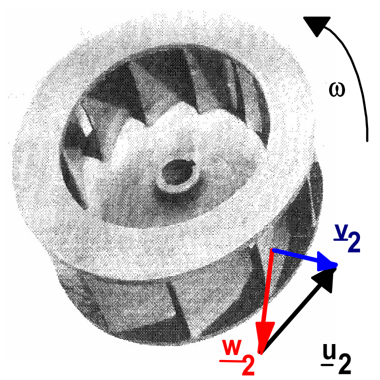
Figure 13.10 shows a radial backward curved blade fan impeller. **Figure 13.11** shows a radial backward curved blade pump impeller. The basic structure of pumps and fans' impellers is no different.Impellers for pumps with higher forces and better efficiency are usually made in molded versions and profiled shovels. Ideal, loss-free, the Bernoulli equation can also be used to determine the overall pressure increase. Let's look at the radial fan impeller. Its schema is shown in **Figure 13.12**. Select a shovel from the impeller. Assume that there are so many blades in the impeller that the flow is fully symmetrical. In parallel with the blades the medium can flow, so the blade can be considered as a streamline.



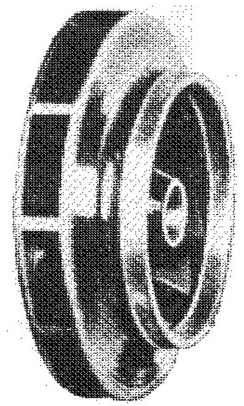
Radial flow fan
Figure 13.9

(Only eight blades are shown in the figure.)
The "1" point in front of the blades, at the entry point "2" is at the exit of the blades.
The absolute "$\underline{v}$", "$\underline{w}$" relative and "$\underline{u}$" conveyor (circumferential) velocity vectors were plotted against a blade entering and leaving the edge.
When drawing up, care must be taken to have the following relationship between the peripheral velocity

$$\frac{u_1}{r_1} = \frac{u_2}{r_2} = \omega,$$



Radial fan impeller
kamleithner budapest Kft.
E-Pool VEZÉRKÉPVISELET
Figure 13.10



Radial pump impeller
Figure 13.11

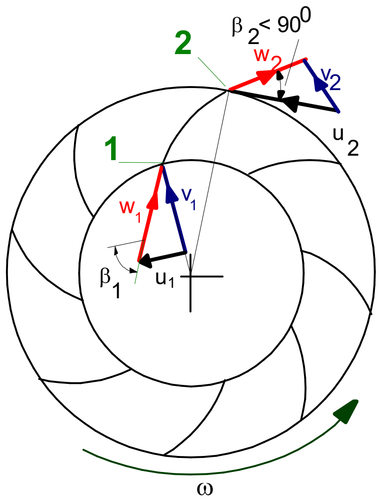
which is a condition for rotation as a solid body, and also to ensure that the velocity are perpendicular to the radius of the given point.
Write the Bernoulli equation between the "1" at the entrance and the "2" points at the exit in the wheel co-rotating system. The flow is steady but it is non-whirling.

Let's start with the Bernoulli equation (*equation 12.12*).
To replace the velocity, now the relative velocity, "w", is to be replaced.

$$\int_1^2 \frac{\partial \underline{w}}{\partial t} d\underline{s} + \int_1^2 \text{grad}\frac{w^2}{2} d\underline{s} - \int_1^2 \underline{w} \times \text{rot}\,\underline{w} \cdot d\underline{s} = \int_1^2 \underline{g} d\underline{s} - \int_1^2 \frac{1}{\rho} \text{grad}\,p d\underline{s} \qquad 13.19$$

I. II. III. IV. V.

I. integral zero, becouse the relative velocity **w** is steady. The **II.term** from the definition $\frac{w_2^2}{2}-\frac{w_1^2}{2}$. **Section III.** integral is also zero because "1" and "2" points are on the same streamline, so they are integrated onto the streamline. The **V. member** stands $-\frac{p_2-p_1}{\rho}$ as a result.



Velocity triangles
Figure 13.12

The **IV. Integral** exercise requires some consideration. After our coordinate system is rotating, the centrifugal force is activated. Earth's gravity force can be neglected with a much larger centrifugal force. If a mass point is relative to the system in a rotating system, then the Coriolis force is acted upon, which per unit of mass

$$\underline{g}_{Cor} = 2\underline{w}\times\underline{\omega} \qquad 13.20$$

where $\underline{\omega}$ is the angular velocity of the coordinate system and is equal to the angular velocity of the wheel. (***G. G. Coriolis 1792-1843*** French Physicist) Considering that the potential of the centrifugal field, can be simplified to:

$$\int_1^2 \underline{g}d\underline{s} = \left(\frac{r_1^2\omega}{2}-\frac{r_2^2\omega}{2}\right)+\int_1^2 2\underline{w}\times\underline{\omega}d\underline{s} \qquad 13.21$$

From *equation 13.21* we can see that the Coriolis field line integrates zero if it is integrated on the streamline because $\underline{w}\|d\underline{s}$.

Without a doubt we can note that if we do not integrate in a streamline, but absolute flow is irrotational, then the member with the Coriolis force space is the Bernoulli equation **III. term** together with its integrity.

The Bernoulli equation, taking into account the simplifications, is as follows:

$$\frac{w_1^2}{2}+\frac{p_1}{\rho}-\frac{r_1^2\omega^2}{2}=\frac{w_2^2}{2}+\frac{p_2}{\rho}-\frac{r_2^2\omega^2}{2} \qquad 13.22$$

Give the relative velocity as the difference between the absolute and the conveyor velocity vectors (see **Figure 13.12**), $\underline{w}=\underline{v}-\underline{u}$ which results in a square gain,

$$w^2 = v^2+u^2-2\underline{u}\cdot\underline{v}. \qquad 13.23$$

Replace this expression with "1" and "2" indices in *equation 13.22*:

$$\frac{v_1^2}{2}+\frac{u_1^2}{2}-\underline{v}_1\underline{u}_1-\frac{r_1^2\omega^2}{2}+\frac{p_1}{\rho}=\frac{v_2^2}{2}+\frac{u_2^2}{2}-\underline{v}_2\underline{u}_2-\frac{r_2^2\omega^2}{2}+\frac{p_2}{\rho} \qquad 13.24$$

We know that $u_1 = r_1\omega$ and $u_2 = r_2\omega$ by substituting simplifying the above equation and by multiplying the density, we get the following:

$$\left(p_1+\frac{\rho}{2}v_1^2\right)-\rho\cdot\underline{v}_1\underline{u}_1=\left(p_2+\frac{\rho}{2}v_2^2\right)-\rho\cdot\underline{v}_2\underline{u}_2 \qquad 13.25$$

According to *equation 13.17*, $p+\frac{\rho}{2}v^2=p_ö$ the sum of static and dynamic pressure is the total pressure in the absolute system. Next, introduce the following mark $\underline{v}_2\,\underline{u}_2=v_{2u}\,u_2$, where the v_{2u} vector is the projection of the $\underline{v}_2$ to the circumferential velocity. Entered into equation

$$\left(p_2+\frac{\rho}{2}v_2^2\right)-\left(p_1+\frac{\rho}{2}v_1^2\right)=\rho\cdot\left(v_{2u}\cdot u_2-v_{1u}\cdot u_1\right) \qquad 13.26$$

expression is obtained.

At this point, the Euler turbine equation can be written in the following form:

$$\Delta p_{öid}=\rho(v_{2u}u_2-v_{1u}u_1), \qquad 13.27$$

which states that the increase in the total pressure equals the expression on the right side of the equation. The right side can be converted

$$\Delta p_{öid}=\rho\cdot 2\cdot\pi\cdot(v_{2u}\cdot r_2-v_{1u}\cdot r_1)\cdot\frac{\omega}{2\cdot\pi}=\rho\cdot\Gamma\cdot n,$$

where "Γ" is a circulation generated by the impeller, also known as the spur, and "n" is the speed of the wheel. The impeller increases the pressure by increasing the circulation. $2\cdot\pi\cdot v_{2u}\cdot r_2$ is the result of a circulation calculations performed on the outer circumference of the impeller. For example, in **chapter 11.4**, calculate the circulation in a potential vortex along a circular stream. At entry, most of the time, there is no peripheral velocity of the medium $v_{1u}=0$ at this time $\Gamma=2\cdot\pi\cdot v_{2u}\cdot r_2$.

The Euler turbine equation applies not only to radial but also to axial flow machines.

In the case of pumps, it is also possible to write between the suction and discharge nozzles, and then the Earth's gravity force field is also taken into account, and the elevations are also written on the left side of *equation 13.26* and, in the case of pumps, instead of pressures, preferences are used. Then the equation is "ρg" so that the Euler turbine equation for pumps are as follows:

$$\left(\frac{p_2}{\rho\cdot g}+\frac{v_2^2}{2\cdot g}+z_2\right)-\left(\frac{p_1}{\rho\cdot g}+\frac{v_1^2}{2\cdot g}+z_1\right)=\frac{v_{2u}\cdot u_2-v_{1u}\cdot u_1}{g}=H_{tid} \qquad 13.28$$

The left-hand side of the equation is also referred to as the total ideal elevation height (full ideal or total ideal head).

Returning to the fans if the air does not rotate in the pipe or the fan is sucking the air from the free atmosphere, the absolute velocity direction is exactlyradial. **Figure 13.12** shows exactly that state. Now there is no peripheral component of the input absolute velocity, so the *equation 13.27* can be simplified

$$\Delta p_{tid}=\rho\cdot v_{2u}u_2 \qquad 13.29$$

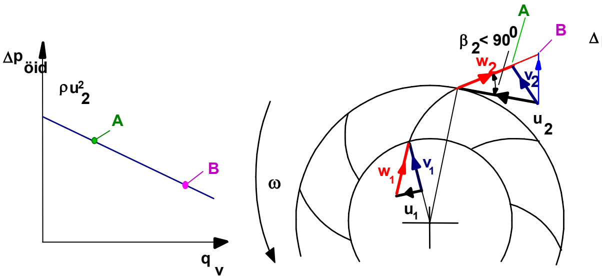
13.8 Theoretical and realistic curves of radial fans

13.8.1 Ideal curves for radial fans

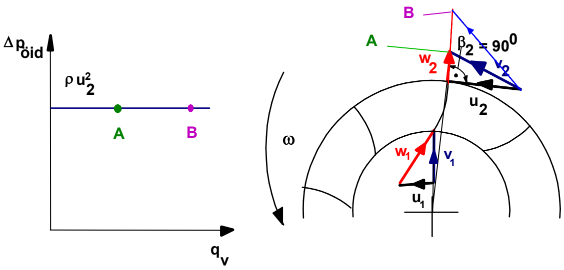
Indicate the impeller width with "b_1" and "b_2" (perpendicular to the plane of the drawing). The radial velocity is "1" and "2" in both "v_{r1}" and "v_{r2}". Use the continuity equation to enter and exit cross sections:

$$v_{r2} = \frac{q_v}{2\pi \cdot r_2 \cdot b_2} \qquad v_{r1} = v_1 = \frac{q_v}{2\pi \cdot r_1 \cdot b_1}$$

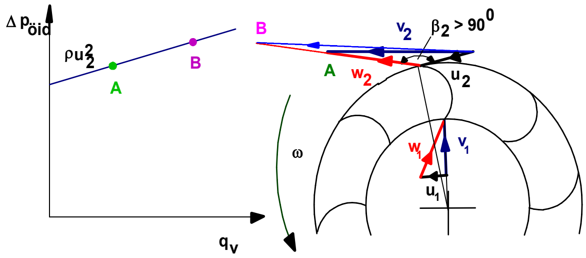
From the exit velocity triangle $v_{2u} = u_2 - w_2 \cdot \cos\beta_2$ and $w_2 \cdot \sin\beta_2 = v_{r2}$, thus the peripheral component of the output speed:



The ideal curve of the impeller with backword curved blades Figure 13.13



The ideal curve of the impeller with radial curved blades Figure 13.14



The ideal curve of the impeller with forward curved blades Figure 13.15

$$v_{2u} = u_2 - v_{r2} \cdot ctg\beta_2 = u_2 - \frac{q_v \cdot ctg\beta_2}{2\pi \cdot r_2 \cdot b_2}$$

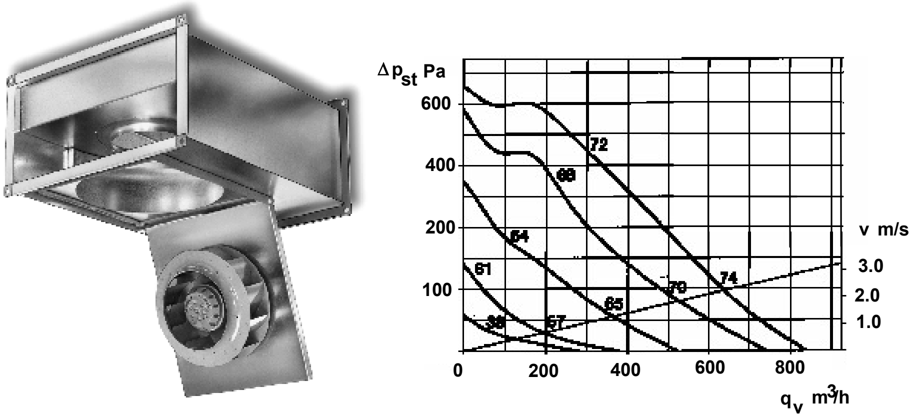
This expression is replaced and rearranged in *equation 13.29* and is used to calculate the ideal curve:

$$\Delta p_{tid} = \rho \cdot \left(u_2^2 - \frac{u_2}{2\pi \cdot r_2 \cdot b_2} \cdot ctg\beta_2 \cdot q_v \right) \qquad 13.30$$

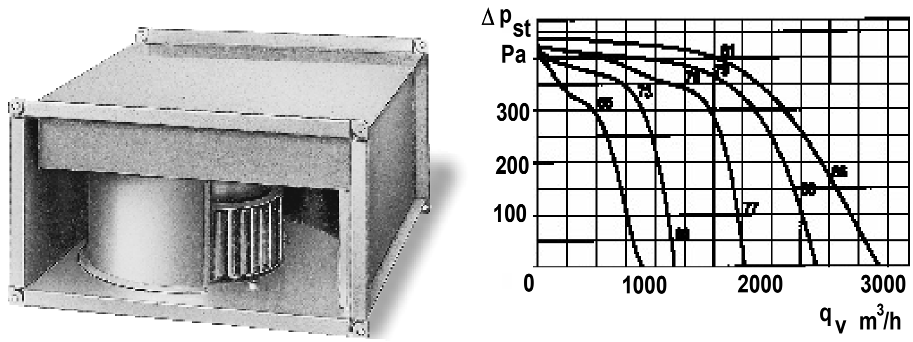
Accordingly, ideal pressure increase is a linear function of the delivered volume flow rate.

If the angle of exit is less than ninety degrees, $ctg\beta_2 > 0$ then we are talking about a backword curved blasted fan. The ideal curve of backword curved blasted fan is shown in **Figure 13.13.** If the angle of exit is ninety degrees $ctg\beta_2 = 0$, then it is called a radial blade fan. The ideal curve of radial blade fan is shown in **Figure 13.14**. The curve is horizontal straight line. And finally, if the exit angle is greater than ninety degrees $ctg\beta_2 < 0$, then we call it a forward curved blade fan. The ideal curve of the forward curved blade fan is shown in **Figure 13.15.** Increasing flow rates also increase pressure.

13.8.2 The actual characteristics of the fans



Backword blade radial flow fan and realistic curves at different speeds
Figure 13.16



Forword blade radial flow fan and realistic curves at different speeds
Figure 13.17

kamleithner budapest Kft.
E-Pool VEZÉRKÉPVISELET

The real curve deviates greatly from the ideal curve, due to the flow losses in the impeller and the spiral hause. **Figures 13.16 and 13.17** show the fans taken from the catalog and their measured curves.
Instead of the total pressure increase, the so-called static pressure increase (Δp_{st}) is represented by the catalog, which draws the compression dynamic pressure from the set pressure increase. By substituting the total pressure expression we get:

$$\Delta p_{st} = \Delta p_{ö} - \frac{\rho}{2}\cdot v_2^2 = \left[\left(\frac{\rho}{2}\cdot v_2^2 + p_2\right) - \left(\frac{\rho}{2}\cdot v_1^2 + p_1\right)\right] - \frac{\rho}{2}\cdot v_2^2 = p_2 - \left(\frac{\rho}{2}\cdot v_1^2 + p_1\right)$$

That is, the static pressure increase is the difference between the pressurized static pressure and the total suction pressure. In most cases, the fan user is interested in this differential pressure.
The pressure increase at the zero flow rate is approx. 40-70% of the ideal value. The tendency of the backward curved fan pattern is similar to that of the ideal curve due to decreasing pressure on increasing volume flow. The backward curved fan's efficiency is better for the forward and radial types.
The curve of the forvard curved fan is already different from the ideal, not just the numerical value. Generally, it only rises to a very small extent, and then an increasing volume flow is also associated with decreasing pressure. The efficiency is generally worse than the backward curved fan type. However, it is a great advantage that the same parameters can be realized in a

smaller size than in a backward curved fan version. Foreward curved pumps are rearly manufactured due to poor efficiency and relatively unstable working points.

13.9 Rotating "S" shaped tube as a simple pump

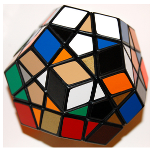


Figure 13.18 shows the "S" shaped tube around the vertical axis with a constant "ω" angular velocity. The tube in this case acts as a simple pump.

Data: $H = 1\text{m}$; $R = 0.5\text{m}$; $D = 0.1\text{m}$; $\omega = 20\frac{1}{s}$

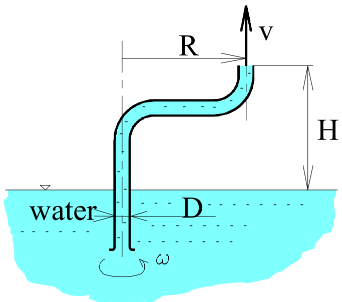
Questions:

a./ What is the volume flow rate at the end of the pipe?

b./ What is the power required to rotate the tube?

I. Solution:

a./ To start the flow, fill the tube and then start rotating. Considering a standing system, the phenomenon is unsteady, so - at least within the rotating tube - a rotating coordinate system must be used. If we assume a vortex free flow in a coordinate system relative to the Earth, the Bernoulli equation is written in the rotating system by the Coriolis field and the rotation member pronouncing each other.



Simple pump
Figure 13.18

We also need to consider the potential of the centrifugal field:

$$\left[\frac{p}{\rho} + \frac{w^2}{2} + gz - \frac{r^2\omega^2}{2}\right]_0^2 = 0$$

From a rotating system, we can see that any "0" point on the water surface can be seen to rotate "$-\omega r_0$" at a peripheral velocity.

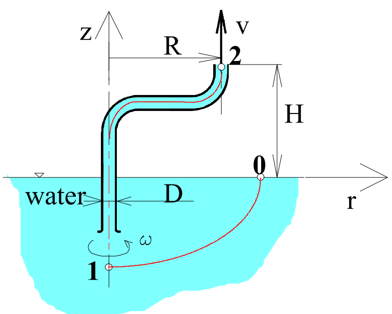


Figure 13.19

Replacing the flow characteristics of the points "0" and "2" in the equation above, we get:

$$\frac{p_0}{\rho} + \frac{(r_0\omega)^2}{2} - \frac{r_0^2\omega^2}{2} = \frac{p_0}{\rho} + \frac{w_2^2}{2} + gH - \frac{R^2\omega^2}{2}$$

The solution does not seem to depend on the "r_0", because the second and third members of the left side are pronounced. Expressing the unknown "w_2":

$$w_2 = \sqrt{2\left(\frac{R^2\omega^2}{2} - g \cdot H\right)} = \sqrt{2\left(\frac{0.5^2 20^2}{2} - 9.81 \cdot 1\right)} = 8.96\frac{m}{s}$$

we have got the value. From which volume flow rate:

$$q_v = \frac{D^2 \cdot \pi}{4} \cdot w_2 = \frac{0.1^2 \cdot \pi}{4} \cdot 8.96 = 0.0704\frac{m^3}{s}$$

b./ We need to look at energy investment in a standing system. The rotating tube (apart from the friction work) increases the fluid's movement and potebtial energy. Change of motion and potential energy of a unit mass of fluid:

$$\frac{w_2^2+(R\omega)^2}{2}+gH$$

the absolute velocity zero in the "**0**" index, the absolute velocity at "**2**" can be written with two perpendicular components "w_2 " and "$R\,\omega$".
Performance is obtained by multiplying the energy of a unit of mass of liquid and the mass flow rate.

$$P=q_v\cdot\rho\cdot\left(\frac{w_2^2+(R\cdot\omega)^2}{2}+g\cdot H\right)=0.0704\cdot 10^3\cdot\left(\frac{8.96^2+(0.5\cdot 20)^2}{2}+9.81\cdot 1\right)=7.04\text{kW}$$

II. Solution:
Only the **a./** solution varies.
The Bernoulli equation is written in a coordinate system between **0** and **1** points, because the flow here is steady and then the system is rotating in the range of **1-2**. Since the **1** point was specially picked up on the axis, therefore, both the rotating potential and the conveyor velocita disappear in the rotating system at this point.
The Bernoulli equation in the **0-1** system:

$$\frac{p_0}{\rho}=\frac{p_1}{\rho}-gz_1 \qquad 13.31$$

The Bernoulli equation in rotating systems up to **1-2**:

$$\frac{p_1}{\rho}-gz_1=\frac{p_0}{\rho}+\frac{w_2^2}{2}+gH-\frac{R^2\omega^2}{2}$$

Summing up the two equations, the unknown index **1** members are dropped and "w_2 " can be expressed.
III. Solution:
a./ Use the valid form of the Euler turbine equation pump (see *equation 13.28*)

$$\left(\frac{p_2}{\rho\cdot g}+\frac{v_2^2}{2\cdot g}+H\right)-\left(\frac{p_1}{\rho\cdot g}+\frac{v_1^2}{2\cdot g}+z_1\right)=\frac{(v_{2u}\cdot u_2-v_{1u}\cdot u_1)}{g}$$

At point "**1**", the velocity is zero and the pressure and height at point "**1**" in *equation 13.31* can be specified by atmospheric pressure, so that is our equation.

$$\left(\frac{p_0}{\rho\cdot g}+\frac{v_2^2}{2\cdot g}+H\right)-\left(\frac{p_0}{\rho\cdot g}\right)=\frac{v_{2u}\cdot u_2}{g}$$

The peripheral component of the output absolute velocity is precisely the peripheral velocity at the top end of the pipe. (The tube is like a radial pump impeller). Replaced and simplified:

$$\frac{v_2^2}{2\cdot g}+H=\frac{u_2^2}{g} \qquad 13.32$$

Let us use that the relative velocity at point "2" is perpendicular to the peripheral velocity, so that the relative velocity can be calculated as Pythagorean.

$$v_2^2=w_2^2+u_2^2$$

Replace this with *equation 13.32* and express w_2.

$$w_2 = \sqrt{2\left(\frac{u_2^2}{2} - g \cdot H\right)}$$

Which is the same as in the two previous cases.
b./ Energy investment can be calculated according to *equation 13.18*, which is:

$$P_h = q_v \cdot \Delta p_{pöid},$$

where the volume flow can be calculated as above, and the ideal total pressure increase can be derived from *equation 13.29*, that is, the Euler turbine equation

$$\Delta p_{šid} = \rho \cdot v_{2u} u_2$$

The peripheral component of the outgoing absolute velocity is exactly the same u_2, so the power output is:

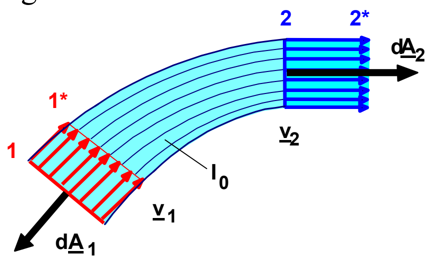
$$P = q_v \cdot \rho \cdot u_2^2 = 0.0704 \cdot 10^3 \cdot (0.5 \cdot 20)^2 = 7.04 \text{kW}$$

14. Momentum equation and its applications

In this chapter, we also use the dynamic equation of the fluid movement as in the fifth chapter. There, the equation of motion was described in differential form, which resulted in the Euler equation. The momentum is used but in the form of an integral equation. The relationship between the forces and the change in the volume of momentum was attributed to Newton II. law describes:

$$\frac{d(m \cdot \underline{v})}{dt} = \sum \underline{F} \qquad 14.1$$

It can also be applied to liquids by delimiting a liquid part with a **control surface** that changes the time of movement and the forces acting on it in the equation.
Let us examine a piece of the current tube as shown in **Figure 14.1**. Let the flow be steady, so the current tube always remains in the same space in time, the input and output velocities are not changed at the same time.



Momentum change
Figure14.1

At a given "t" moment between the surfaces "1" and "2", the liquid contained in the current tube of the current tube has a given impulse. After the "dt" time, the liquid enters the "1 *" and "2 *" cross-section, where it also has a certain impulse. We want to determine the impulse change between the two states.
Since the flow is steady, the impulse of the fluid between the cross sections "1" and "2" is the same in the „t” and „t + dt”, even though the particles of the fluid are different points of the space. Mark the liquid impulse in this compartment $\underline{I}_0$.
At the "t" moment, the liquid part of the test portion between the "1" to the "2" cross-section impulses

$$\underline{v}_1 \cdot dm_1 + \underline{I}_0 .$$

At the time of "t + dt" the impulse of fluid between the cross sections "1 * -2 *":

$$\underline{I}_0 + \underline{v}_2 \cdot dm_2 .$$

On the left side of the control surface, the same amount of mass flows as the mass flow on the right side.
Express the inflow mass with the velocity vector and the surface vector as well as the density.

$$dm_1 = -\rho_1 \cdot d\underline{A}_1 \cdot \underline{v}_1 \cdot dt$$

The minus sign is needed because the direction of velocity and surface is opposite, so the scalar product is negative. Write the outflow mass in the cross-section "2":

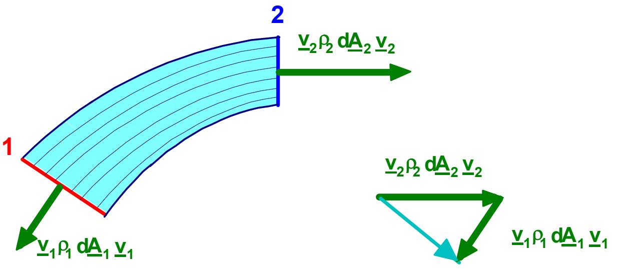
$$dm_2 = \rho_2 \cdot d\underline{A}_2 \cdot \underline{v}_2 \cdot dt$$

Momentum during "dt" time is equal to the momentum in "t + dt", which is reduced by the momentum in "t". $\underline{I}_0$ is lost when it is expired and then:

$$d(m \cdot \underline{v}) = dm_2 \cdot \underline{v}_2 - dm_1 \cdot \underline{v}_1 = (\rho_2 \cdot d\underline{A}_2 \cdot \underline{v}_2 \cdot dt) \cdot \underline{v}_2 - (-\rho_1 \cdot d\underline{A}_1 \cdot \underline{v}_1 \cdot dt) \cdot \underline{v}_1$$

By dividing the "dt" time, we get the change of the momentum, which is:

$$\frac{d(m \cdot \underline{v})}{dt} = (\rho_2 \cdot d\underline{A}_2 \cdot \underline{v}_2) \cdot \underline{v}_2 + (\rho_1 \cdot d\underline{A}_1 \cdot \underline{v}_1) \cdot \underline{v}_1$$



Momentum change vectors
Figure 14.2

Momentum change is equal to the sum of forces acting on the designated control surface. The forces are considered constant during the elemental movement. Generally, fluid forces have two types of forces: force acting on the mass (eg weight) and surface forces acting on the surface of the liquid part (pressure forces or friction forces). If the medium is non-frictional, the surface force has no component parallel to the surface (the sliding stress is zero), only the force from the pressure perpendicular to the surface is affected. We neglect the friction forces. The weight force acting on the examined liquid part is $d\underline{G}$.

Generally, the forces from the pressure are calculated in three parts:

• Part one is the result of compressive forces acting on the surfaces on which the fluid crosses,

• the other part is the result of compressive forces awakening along the wall of the current tube,

• The third part is the result of compressive forces acting on surfaces with solid wall, which is discussed in a separate chapter section when the solid body enters the control surface.

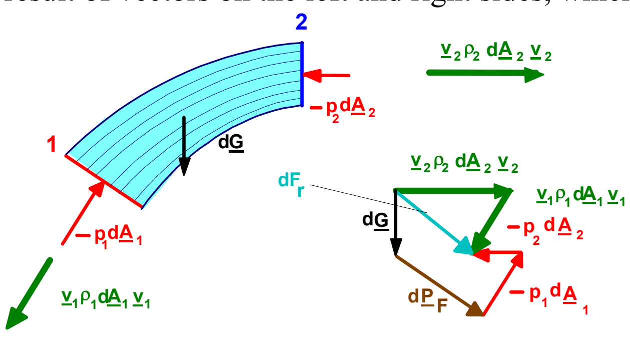
The result of the forces arising from the pressure on the wall of the current tube (which may also originate from a solid wall which is outside the control surface) denote it $d\underline{P}_F$. All the forces from the pressure result:

$$d\underline{P}_F - p_1 \cdot d\underline{A}_1 - p_2 \cdot d\underline{A}_2 ,$$

where negative signs indicate that the forces are opposite to the surface normal outward. Equalizing the impulse change with the forces, we get the following:

$$\underline{v}_2 \cdot \rho_2 \cdot (\underline{v}_2 \cdot d\underline{A}_2) + \underline{v}_1 \cdot \rho_1 \cdot (\underline{v}_1 \cdot d\underline{A}_1) = d\underline{G} + d\underline{P}_F - p_1 \cdot d\underline{A}_1 - p_2 \cdot d\underline{A}_2$$

The summary is to be made according to the directions indicated in **Figure 14.3**. A means the result of vectors on the left and right sides, which is naturally the same.



Momentum change vectors
Figure 14.3

The fluid section bounded by an optional surface can be assembled from elementary current tubes. The forces on the common surface of the elemental current tubes and the momentum change vectors are pronounced on the basis of the action reaction principle.

Thus, on the optional control surface "A", the momentum can be written as follows:

$$\oint\!\!\oint_A \underline{v}\rho(\underline{v}d\underline{A}) = \iiint_V \rho \cdot \underline{g}dV - \oint\!\!\oint_A p d\underline{A} \qquad 14.2$$

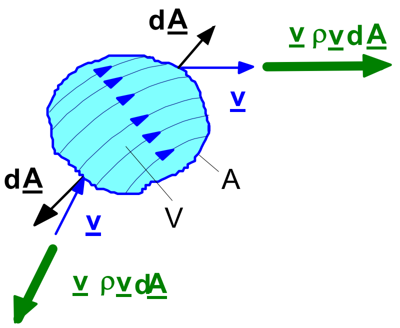


Figure 14.4

The equation applies to frictionless steady flow. The great advantage of the **momentum equation** over the other flow equations is that it is a **vector equation**. Apart from the magnitude of the forces, they also provide the direction of their forces. For the sake of completeness, we write the momentum equation to the unsteady, friction flow, which is:

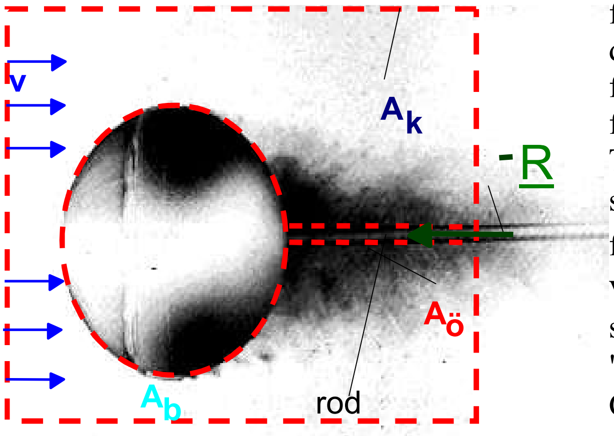
$$\frac{\partial}{\partial t}\iiint_V \underline{g}\,dV + \oiint_A \underline{v}\rho\left(\underline{v}\,d\underline{A}\right) = \iiint_V \rho \cdot \underline{g}\,dV - \oiint_A p\,d\underline{A} + \oiint_A d\underline{S}$$

Most momentum equations are applied in cases where the liquid is in contact with a solid body, or even partially or completely extends into the control surface. According to the above considerations, the liquid must be in the control surface, and no solid body is assumed not in the surface on ly ont the border of the surface.

14.1 Solid body in the control surface

Next, take a control surface within **a solid body is located**. The flow is **steady and there is no friction**.

Figure 14.5 shows a flow around a sphere. The flow of the sphere acts horizontally. To create a steady state, the ball should be held in place against the flow. We can provide this with a rod, which is visible on the right side of the figure. (The weight of the sphere and the vertical forces awakening in the rod are equalized, not drawn in the figure.) The force transmitted from the flow, is balanced by the horizontal force "-R" awake in the rod.



Solid body in the control surface
Figure 14.5

Take the control surface by closing out the solid body with the „A_b” "inner" surface from the "V" volume, which then keeps the volume between "A_k" and "A_b". The "A_k" surface is a cylinder with a horizontal axis and "A_b" a sphere.

Combine them to meet the requirement for the closed surface, with the "$A_ö$" surface that is the cylinder surrounding the support rod. This will drill the volume between the two surfaces.

The normal vector of closed surface is always outwards from the surface. This can be done by pointing surface elements vectors outward from the "A_k" surface and pointing toward the center of the "A_b" surface. $A_ö$ surface normal "vectors" point towards the inside of the cylinder surface. Thus, from the outer surface eg. "$A_ö$" on the top surface of the "$A_ö$" but inside the tube, we can get to the inner surface, then we can return to the "$A_ö$" surface, and we are moving around the surface vectors in the pipe on the outer surface and midway from the surface.

Write the form of momentum *equation 14.2* according to the drawn control surface. This can be done now because we have excluded the solid bodies, the sphere and the supporting rod from the somewhat complicated closed surface. The enclosed surface-integrated three surfaces in the equation can be aggregated to the sum of integers on "A_k" and "A_b" interfaces, and to integrals on the "$A_ö$" interface. Thus, the equation is:

$$\iint_{A_k} \underline{v}\cdot\rho\cdot(\underline{v}\cdot d\underline{A}) + \iint_{A_b} \underline{v}\cdot\rho\cdot(\underline{v}\cdot d\underline{A}) + \iint_{A_ö} \underline{v}\cdot\rho\cdot(\underline{v}\cdot d\underline{A}) =$$

outside surface insife surface interface surface

$$= \iiint_{V_F} \rho\cdot\underline{g}\cdot dV - \iint_{A_k} p\cdot d\underline{A} - \iint_{A_b} p\cdot d\underline{A} - \iint_{A_ö} p\cdot d\underline{A}$$

fluid volume outside incide interface surface

The second and third integers of the left side are zero, because there is no flow through the surface of the solid body and thus through the control surface, so the product is zero. The last integral of the right side, that is, the resultant force from the pressure on the support rod is also zero, assuming that the flow is symmetrical, the forces awakening on the opposing surfaces of the rod are pronounced.
The third member of the right side sums up the force from the pressure on the surface of the sphere and the surface normal to the center of the sphere is contradicted by the elemental compressive forces that press the fluid. The force generated by the integration is the force transmitted to the liquid by the solid wall, which is the same size but opposite as the force acting on the solid body from the liquid to the vector. **"R" is therefore the force acting by the liquid to the solid body**. This force must be balanced with the rod "-R". Simplifying the above equation we get the following:

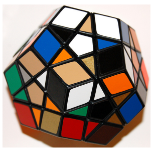
$$\oiint_{A_k} \underline{v}\rho(\underline{v}d\underline{A}) = \iiint_{V_F} \rho\cdot\underline{g}dV - \oiint_{A_k} pd\underline{A} - \underline{R} \qquad 14.4$$

According to the equation, it is sufficient to treat the outer surface as a control surface, without the need to exclude the solid body, but -R force must be taken into account when the equation is written.
We can go to another way for this equation.
If the surface „A_k„ is opaque or is treated as a black box, the inner surface is not visible, the size and shape of the inner surface need not be known, but the momentum equation can still be applied. The effect of the solid body inside is replaced by a concentrated force acting through the rod. The rod is cut by the "A_k" surface. On the surface of the cut, which is the cross section of the rod, the pressure corresponds to the pressure in the rod. The cross section of the rod can be considered infinite, with infinite high pressures, but the product of the two is just "-R". This is a concept of well-known concentrated force. (*Equation 14.4* is exactly the same as this condition, because only the "A_k" interface can be closed at this time.)
Thus, the momentum law will take the following form if there is a steady, non-friction-free flow within the control surface and a solid body inside the control surface:

$$\oiint_{A} \underline{v}\rho(\underline{v}d\underline{A}) = \iiint_{V} \rho\cdot\underline{g}dV - \oiint_{A} pd\underline{A} - \underline{R} \qquad 14.5$$

The "A" surface now corresponds to the "A_k" surface. The negative sign before the $\underline{R}$ vector is needed because the momentum load should include the force acting on the fluid from the solid body.
The first member of the right side represents the combined weight of the liquid in volume and the solid body in *equation 14.5*, unlike *equation 14.4*, where the volume occupied by the solid body was excluded when the integral was written.
Of course, the calculation of weight can be separated separately, only to calculate the weight of the solid body. This problem does not usually occur when solving problems because the weight of the liquid in the surface is neglected by other forces.

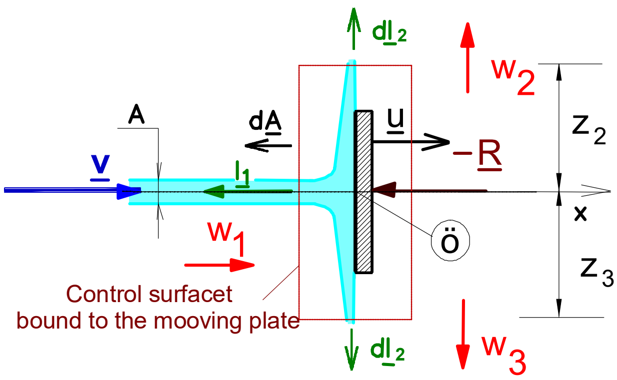
14.2 Force acting on a flat plate



Apply a horizontal free jet, with "A" cross-section, "v" absolute velocity to a vertical plane (**Figure 14.6**). The flat plate is large enough to distract the jet perpendicularly to the original direction. The flat plate moves horizontally with "u" velocity. What is the direction and magnitude of the force acting the water jet on the plate?
The task can be easily solved using the momentum equation. With the momentum law can be answered without knowing the details of the flow. Let's look at the case when the plate moves at "u" velocity. The relative velocity of the water jets arriving at the plate is the next:

$$\underline{w}_1 = \underline{v}_1 - \underline{u},$$

in the coordinate system bound to the mooving plate.



Water jet acting on a moving flat plate
Figure 14.6

I. First step determine the relative velocity of the water jets leaving the plate in the coordinate system bound to the plate, starting from the Bernoulli equation, "velocitiew:

$$\frac{p_1}{\rho} + \frac{w_1^2}{2} + g \cdot z_1 = \frac{p_2}{\rho} + \frac{w_2^2}{2} + g \cdot z_2$$

$$\frac{p_1}{\rho} + \frac{w_1^2}{2} + g \cdot z_1 = \frac{p_3}{\rho} + \frac{w_3^2}{2} + g \cdot z_3$$

If we neglect the difficulty field in the small area of the plate $z_1 = z_2 = z_3$, we can assume a hypothesis. Each jet of the fluid passes at atmospheric pressure, therefore $p_1 = p_2 = p_3 = p_0$. Taking into account these two circumstances, we find that the incoming and outgoing jets have the same relative fluid velocity as: $w_1 = w_2 = w_3$. The phenomenon can be regarded as an axy-symmetric with this simplification.
When applying the momentum law we use this approximation many times.

II. As a second step add a control surface, which should be drawn to it
• the phenomenon should be steady in the control surface (eg the solid body does not leave the surface)

• If you are looking for a force acting on solid body, insert the body into the control surface,
• where flow is present, the control surface must be perpendicular to the flow rate or be parallel to it and
• the control surface can be removed from the flowing liquid if it is to simplify the task.

III. As a third step write the momentum law

$$\oint\!\!\!\oint_A \underline{v}\rho(\underline{v}d\underline{A}) = \iiint_V \rho \cdot \underline{g}dV - \oint\!\!\!\oint_A pd\underline{A} - \underline{R}$$

(*Equation 14.5*) and determine which members should be considered when solving the given task.
Since we use the coordinate system bound to the plate, the absolute velocity "$\underline{v}$" is replaced by the relative velocity "$\underline{w}$".
Only steady and frictionless flow is assumed. If the plate is standing, the flow is steady, as the direction and velocity of the jet will not change over time. If the plate moves, the flow in the stationary system is unsteady: At a particular point of the space, the velocity depends on the position of the moving plate. However, if the coordinate system and the control surface are fixed to the moving olate then the flow becomes steady. The first member of the right side, that is, the weight is neglected, so our equation is:

$$\oint\!\!\!\oint_A \underline{w}\rho(\underline{w}d\underline{A}) = -\oint\!\!\!\oint_A pd\underline{A} - \underline{R} \qquad 14.6$$

The integer argument on the left only differs from the zero on the "A" surface, where the liquid flows in the surface or flows out. These sub-integrals are commonly referred to as vectors $\underline{I}_1, \underline{I}_2, \ldots, \underline{I}_n$. $\underline{I}_1, \underline{I}_2, \ldots, \underline{I}_n$ surface momentums flow through in unit time „A" over a period of time and are thus referred simply momentum vectors. With this mark, the integral on the left side of the equation, which expresses a change in the volume of fluid momentum, can be written as the sum of concentrated momentum vectors:

$$\underline{I}_1 + \underline{I}_2 + \ldots + \underline{I}_n = \sum_{i=1}^{n} \underline{I} = \oint\!\!\!\oint_A \underline{w} \cdot \rho \cdot \underline{w} \cdot d\underline{A} \qquad 14.7$$

Similarly, the calculation of forces from the pressure can be performed by summing up the partial integrals:

$$\sum \underline{P} = -\oint\!\!\!\oint_A p \cdot d\underline{A} \qquad 14.8$$

With these markings, the above figure of the momentum law:

$$\sum \underline{I} = \sum \underline{P} - \underline{R} \qquad 14.9$$

i.e. the addition of vectors instead of complicated surface integrals. The control surface on the figure is subjected to atmospheric pressure everywhere, so the pressure forces are zero. This is natural in the area outside the liquid jet. In the liquid jets, as we have seen before (Euler equation in a natural coordinate system), the pressure does not change perpendicular to the parallel straight stream lines, i.e. a constant pressure in a free yet and equal to the external pressure.
With the foregoing considerations, *equation 14.9* is further simplified:

$$\sum \underline{I} = -\underline{R} \qquad 14.10$$

IV. As a fourth step it is defined as the whole integral of the integer multiple surface area for the complete enclosed control surface "A".

$\underline{I}_1 = \iint\limits_{A_1} \underline{w}_1 \cdot \rho \cdot \left(\underline{w}_1 d\underline{A}\right)$ hence the size of the vector $\left|\underline{I}_1\right| = w_1^2 \rho A_1$ and $\underline{w}_1$ $\underline{I}_1$ is opposite to $d\underline{A}$, pointing outward from the control surface. In general, the impulse current vectors point outward from the control surface and are always parallel to the velocity.

The elemental momentum vectors of the circulating fluid flow from the plate result in a zero vector, so there are no more momentum vectors in our present task.

The forces from the pressure are zero, because at each point there is an environmental pressure. Due to the neglect of friction, only forces perpendicular to the plate may wake up, so the direction of ”R” may be just that.

V. Fifth step or vector illustration is made based on *equation 14.9* as shown in **Figure 14.3**, or a coordinate system is recorded, and the equilibrium forces are written as coordinate directions. Taking into account that the vector is the same or opposite to the positive direction of the "x" axis, the absolute values of the individual vectors (in this case vector 1) must be written either by the positive or negative sign on the corresponding side of *equation 14.10*:

"x" balance:

$$-I_1 = -R_x \text{ azaz} - \rho w_1^2 A_1 = -R_x = -R$$

i.e. the force transmitted from the liquid to the standing body is

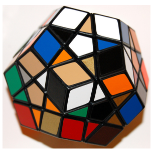
$$R = \rho \cdot w_1^2 \cdot A_1 . \qquad 14.11$$

Using the expression $w_1 = v_1 - u$:

$$R = \rho (v_1 - u)^2 A_1$$

If the plate moves against the yet, then the relative velocity $v_1 + u$, if the sheet is standing, then the "u" velocity is dropped from the expression.

14.3 Pelton turbine



In 1880, US engineer **Lester Pelton** invented a special or impuls-type water turbine known as the Pelton turbine. It is especially suitable for the use of energy from the high-altitude water. Small Pelton turbines are used eg. for winding irrigation equipment, to move the winding drum.

Figure 14.6 shows a Pelton turbine impeller. The Pelton turbine transforms the moving energy of the high-velocity jet into mechanical performance. Pressure change before and after the blade is not in the yet (discount water turbine). In calculating, we approximate that the blades that are in contact with the water jet are always perpendicular to the water jet. (In reality, the rotation of the blades slightly reduces the peripheral force.) The turbine "D" rotating wheel rotates at an „ω”angular velocity.

Data: $D = 1100\text{mm};\ \omega = \frac{\pi}{2} \cdot 50 \frac{1}{s};\ \vartheta = 80^0;\ v_1 = 120 \frac{m}{s};\ A_1 = 10\text{cm}^2$

Pelton turbine
Figure 14.6

Qestions:

a./ Determine the force acting on a turbine blade.
b./ Determine the average circumference force of the turbine.
c./ Determine the time function of force R_{kx} in the direction of the peripheral velocity when the number of turbine blades is $z = 18$!
d./ How much is the turbine's performance in a given operating state and how does the performance vary from peripheral speed?

The solution will be presented in the following four chapters.

14.3.1 A force acting on a blade

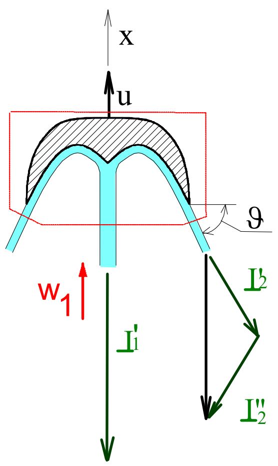
a./ A force acting on a turbine blade moving at a "u" velocity can be determined by the momentum law. The solution is exactly the same as for a moving flat plate. The difference is just that the jet of water coming out from the plade also has an effect. In the coordinate system that moves along with the blade, the "x" axis points upward, drawing a control surface that circles the blade perpendicular to the water jets.
There is an atmospheric pressure everywhere along the control surface, so the forces from the pressure drop out. The phenomenon is steady in the coordinate system bound to the blade, and the weight is neglected. Thus, *equation 14.6* will be composed of the following members:

$$\oint\!\!\oint_A \underline{w}\rho(\underline{w}\,d\underline{A}) = -\underline{R}$$

On the left side of the equation there are three momentum vectors, which are shown in *Figure 14.7*:

$$\underline{I}_1' + \underline{I}_2' + \underline{I}_2'' = -\underline{R}$$

Components in the "x" direction:



A force acting on a blade
Figure 14.7

$$-I_1' - \left(I_2' + I_2''\right)\sin\vartheta = -R_{1x},$$

where impulse vectors are in the relative system

$$I_1' = \rho A_1 w_1^2 \quad \text{and}$$

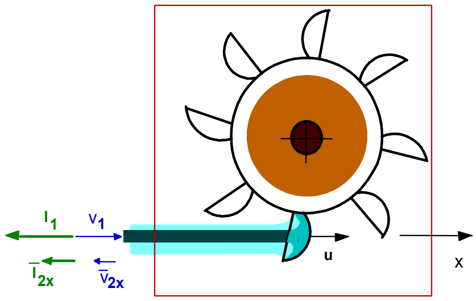
$$I_2' = I_2'' = \frac{1}{2}\rho A_1 w_1^2 .$$

By replacing the "x" equation, we get the force acting on a blade:

$$R_{1x} = \rho A_1 w_1^2 \left(1 + \frac{2\cdot\sin\vartheta}{2}\right) =$$

$$= 10^3 \cdot 10^{-3} \cdot 76.8^2 \cdot \left(1 + \frac{2\cdot\sin 80^0}{2}\right) = 11706\,\text{N}$$

14.3.2 Average force acting on the wheel



The force on the wheel
Figure 14.8

b./ To calculate the average force on the impeller, put the whole wheel into a control surface. The velocity of the jet entering the control surface at high velocity is the same as the one used for calculating a single blade. The velocity of the water jets leaving the blades varies slightly in time and space, depending on which position the blade is left in. The fluctuation periodically changes with the time of changing blades. In **Figure 14.8**, the mean "x" direction of the jet of water flowing from the control surface and the average momentum that can be calculated from it is drawn. The absolute velocity of the discharge water jet is much smaller than the inlet velocity, so the cross-section of the jet is much larger than the inlet cross section. (We have neglect the vertical velocity component caused by gravity.)
The "x" equation of the momentum law is very simple when we know the momentum vectors:

$$-I_1 - \overline{I}_{2x} = -\overline{R}_{kx} \qquad 14.13$$

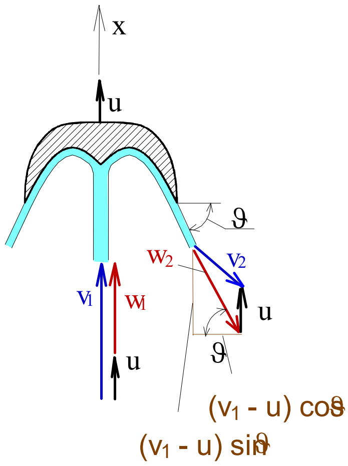


Figure 14.9

The momentum vectors in the absolute system:

$$I_1 = \left(\rho \cdot A_1 \cdot v_1\right) \cdot v_1 \qquad 14.14$$

The amount in parentheses is the amount of mass flowing into the wheel in the unit time.
The x-directional component of the output momentum can be obtained by multiplying the amount of mass flowing out of the time unit (which is the same as the input mass) with the absolute velocity of the output jet, more precisely with its "x" component „v_{2x}". So

$$\overline{I}_{2x} = \left(\rho \cdot A_1 \cdot v_1\right) \cdot \overline{v_{2x}} \qquad 14.15$$

To determine the "v_2" velocity, draw the velocity triangles to the jets that lie beneath the rotation axis of the turbine and the jets that leave the blade (see **Figure 14.9**).
First calculate the perimeter velocity:

$$u = \frac{D}{2} \cdot \omega = \frac{1.1}{2} \cdot \frac{\pi}{2} \cdot 50 = 43.2 \frac{m}{s}$$

The absolute values of the relative velocities that reach the blade and leave the blade are the same:

$$w_1 = w_2 = v_1 - u = 120 - 43.2 = 76.8 \frac{m}{s}$$

Based on the exit velocity triangle, the "x" direction of the absolute velocity of the exit:

$$v_{2x} = \left(v_1 - u\right) \cdot \sin\vartheta - u = \left(120 - 43.2\right) \cdot \sin(80^0) - 43.2 = 32.43 \frac{m}{s} \qquad 14.16$$

It is assumed that the absolute velocity that escapes from a blade does not differ greatly from the "x" directional component of the average absolute velocity leaving the wheel. If the difference is significant, then a downward factor can be taken into account in the following equation.
Replacing the *equation 14.13* we get the result:

$$-\left(\rho \cdot A_1 \cdot v_1\right)\cdot v_1 - \left(\rho \cdot A_1 \cdot v_1\right)\cdot \overline{v_{2x}} = -\overline{R}_{kx}$$

$$\overline{R}_{kx} = \left(\rho \cdot A_1 \cdot v_1\right)\cdot\left(v_1 + v_{2x}\right) = 10^3 \cdot 10^{-3} \cdot 120 \cdot (120 + 32.43) = 18292N \qquad 14.17$$

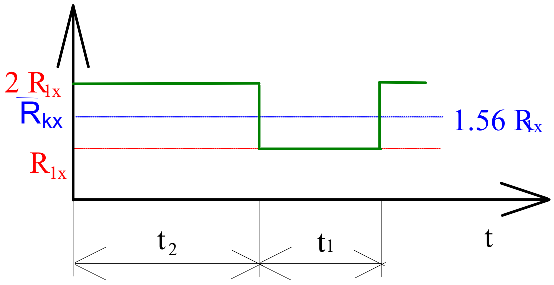
The average force on the wheel does not match the force acting on a blade, as the jet can simultaneously affect several blades.

14.3.3 Changes in peripheral force

c./ Ratio of peripheral force and the force acting on a blade:

$$\frac{\overline{R}_{kx}}{R_{1x}} = \frac{18292}{11706} = 1.56$$

The number of blades operating at the same time is more than one. This means where there are "1" or "2" blades, the phenomenon is repeated 18 times per revolution. In those moments when you insert a new blade into the jet, the number of blades running increases by 1 from 1 to 2. The duration of the operation of two balades is „t_2” for one blade is „t_1".



Change of peripheral force
Figure 14.10

Calculate "t_1" and "t_2" times based on the one-blade and average peripheral forces:

$$2 \cdot R_{1x} \cdot t_2 + R_{1x} \cdot t_1 = \overline{R}_{kx} \cdot (t_1 + t_2),$$

and

$$t_2 + t_1 = T.$$

$$T = \frac{2\pi}{\omega z} = \frac{2 \cdot \pi}{\frac{\pi}{2} \cdot 50 \cdot 18} = 4.44ms$$

$$t_2 = T \cdot \left(\frac{\overline{R}_{kx}}{R_{1x}} - 1\right) = 4.44 \cdot (1.56 - 1) = 2.49ms,$$

$$t_1 = 1.95ms$$

14.3.4 Calculation of power

d./

d/1 Solution

The turbine's performance can be obtained from the peripheral force and the peripheral velocity:

$$P_e = R_{kx} \cdot u = 18292 \cdot 43.2 = 790kW \qquad 14.18$$

The dependence of power on the peripheral velocity can be written by substituting equation *14.17 in equation 14.18*

$$P_e = R_{kx} \cdot u = \left(\rho \cdot A_1 \cdot v_1\right)\cdot\left(v_1 + \overline{v}_{2x}\right)\cdot u \qquad 14.19$$

In the resulting expression $\overline{v}_{2x}$, this depends on the peripheral velocity obtained from *equation 14.16*

$$P_e = R_{kx} \cdot u = \left(\rho \cdot A_1 \cdot v_1\right)\cdot\left(v_1 + (v_1 - u)\cdot \sin\vartheta - u\right)\cdot u$$

14.20

$$\underline{P_e = (\rho \cdot A_1 \cdot v_1) \cdot (1 + \sin\vartheta) \cdot (v_1 - u) \cdot u}$$

The resulting expression makes it easy to see if $u = 0$ and when $u = v_1$ the power is zero. Performance changes parabolic and is maximized at $u = \frac{v_1}{2}$. Maximum Power:

$$P_{max} = (\rho \cdot A_1 \cdot v_1) \cdot \frac{(1 + \sin\vartheta)}{2} \cdot \frac{v_1^2}{2}$$

If $\sin\vartheta \approx 1$ it is, then at the maximum power the total energy of the water is used by the turbine. We control the extreme value as a useful exercise for the reader.

d/2 Solution:

It is assumed that there are no losses in the flow, so the water energy lost by the water is turned to the wheel drive. In the previous chapter, the concept of rotating "S" shaped tube is also used here. Moving energy lost by unit mass of water:

$$\frac{v_1^2}{2} - \frac{v_2^2}{2},$$

where "v_1" the entry "v_2" is the absolute velocity of the output water jet. For the definition of "v_2" velocity, we use the expression of *equation 14.16* and the horizontal component "v_2" can be given as shown in **Figure 14.9**

$$v_{2x} = (v_1 - u) \cdot \sin\vartheta - u$$

and

$$v_{2y} = (v_1 - u) \cdot \cos\vartheta.$$

The outflow absolute velocity squared based on these:

$$v_2^2 = (v_1 - u)^2 - 2 \cdot u \cdot (v_1 - u) \cdot \sin\vartheta + u^2 =$$

$$= (120 - 43.2)^2 - 2 \cdot 43.2 \cdot (120 - 43.2) \cdot \sin(80^0) + 43.2^2 = 35.06^2 \frac{m}{s}$$

Mass flow rate:

$$q_m = v_1 \rho A_1 = 120 \frac{kg}{s}$$

Thus, the theoretical performance of the turbine:

$$\underline{P_e = q_m \left(\frac{v_1^2}{2} - \frac{v_2^2}{2} \right) = 120 \cdot \left(\frac{120^2}{2} - \frac{35.06^2}{2} \right) = 790\,\text{kW}}$$

Of course, without replacing the numbers, the term "v_2" can be replaced in the *expression 14.20*. We also trust this reader as a useful exercise.

d/3 Solution:

The Euler turbine equation can also be used to calculate power in the same way as for a rotating "S" shaped tube.

When using the equation, make sure that the points "1" and "2" are replaced because it is a turbine and not a pump. The turbine is driven by the energy change of the medium.
Equation 6.28 with indexes swapped is as follows:

$$H_e = \left(\frac{p_1}{\rho \cdot g} + \frac{v_1^2}{2 \cdot g} + z_1\right) - \left(\frac{p_2}{\rho \cdot g} + \frac{v_2^2}{2 \cdot g} + z_2\right) = \frac{(v_{1u} \cdot u_1 - v_{2u} \cdot u_2)}{g}$$

The point "1" is the entry, the "2" index is in the exit jet.
A H_e is called a theoretical total head or with a foreign word, which is a disponable drop (this is the theoretical tozal head for pumps).
The pressure is everywhere atmospheric, the heights are the same, the perimeter velocity are the same (u), so the theoretical total head of the turbine:

$$H_e = \frac{v_1^2}{2 \cdot g} - \frac{v_2^2}{2 \cdot g} = \frac{(v_{1u} - v_{2u}) \cdot u}{g}$$

The quantities on the right side of the equation can be easily read out from **Figure 14.9**

$$v_{1u} = v_1 \qquad \textbf{and} \qquad v_{2u} = -v_{2x}$$

There is a negative sign in the second term because the velocity "u" is opposite to "v_2" velocity.

$$H_e = \frac{(v_{1u} - v_{2u}) \cdot u}{g} = \frac{[v_1 + (v_1 - u) \cdot \sin\vartheta - u] \cdot u}{g} = \frac{(v_1 - u) \cdot (1 + \sin\vartheta) \cdot u}{g}$$

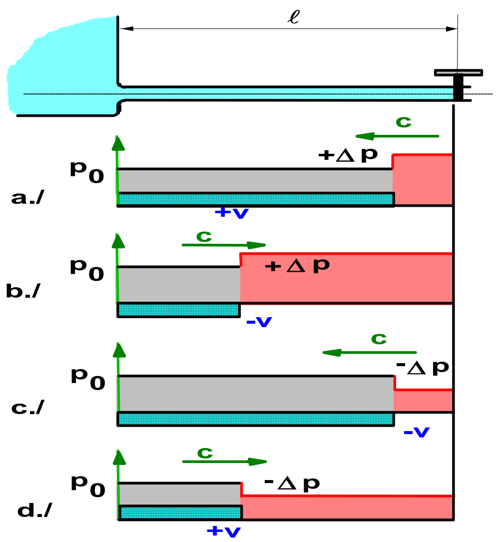
Theoretical power can be calculated as follows:

$$P_e = q_v \cdot \rho \cdot g \cdot H_e = (A_1 \cdot v_1) \cdot \rho \cdot g \cdot \frac{(v_1 - u) \cdot (1 + \sin\vartheta) \cdot u}{g}$$

The expression is the same as *equation 14.20*.

14.4 Closing the pipeline suddenly

In **section 13.6**, the time course of the velocity and acceleration of fluid flowing through a relatively long tube from a container was examined at a sudden opening of the pin at the end of the tube. In our present example, we analyze what happens when we suddenly close the fluid path with the end of the tube. If the shutdown would be in an infinite short time, and neither the pipe wall nor the fluid would have any elasticity, then, in principle, infinite high pressure would be generated at the closing point. **This is not possible, so in the present example we can not neglect the compressibility of water and the elasticity of the pipe wall.**



14.4.1 Pressure swings on sudden closing

Figure 14.11

At a sudden closing, a pressure increase wave starts at the closing position at "c" speed. The magnitude and velocity of the pressure increase depend on the material of the tube, its geometrical size and the speed and material of the flowing fluid.

From the point of closure, more and more liquid particle stops and the wall of the tube expands and the fluid is compressed. The current energy of the water accumulates in the form of a potential energy (see **Figure 14.11/a**).

In the next phase, the infinite bulky container is reflected in the wave counterforce. The energy accumulated in the inflated pipe wall and in the fluid tends to flow back into the tank (see **Figure 14.11/b**).

At the end of the phase, the fluid flows through the whole tube at the "v" velocity into the tank.

In the third phase, the liquid particles stop again at the sealing site and a depression wave starts at the sealed end. The fluid plug that flows outwards sucks the tube (see **Figure 14.11/c**).

In the fourth phase, the tube under the depression sucks the fluid out of the tank (see **Figure 14.11/d**), then the process starts from the first phase. Internal friction in the fluid attenuates the phenomenon without the process being stopped.

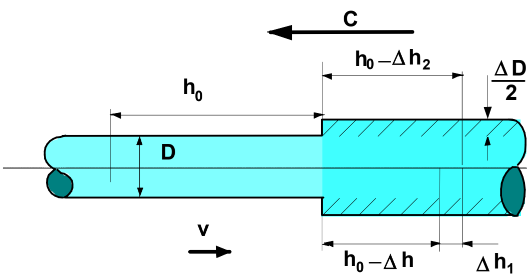


Figure 14.12

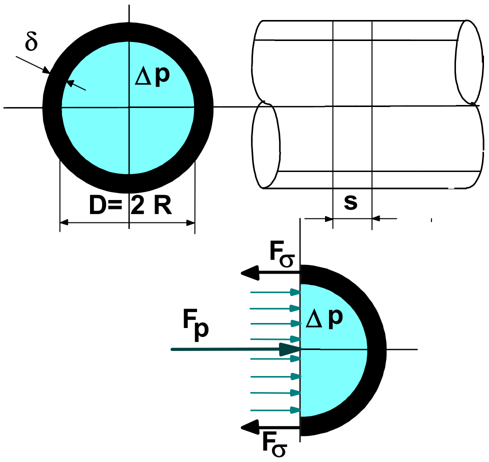
Let us examine in more detail from the first phase the wavefront neighborhood (see **Figure 14.12**). The theory of the phenomenon was developed by an ***Italian scientist from Allievi***.

On the left side of **Figure 14.12**, the water is still flowing at an unobtrusive "v" velocity, and the water's velocity drops to a sudden zero, and the pressure increases with the "Δp" value. The water is slightly compressed and the pipe wall expands.

14.4.2 Shortening the water column

In the undisturbed flow, select a water column length "h_0". At a higher pressure due to the expansion of the pipe and due to compression, the water column occupies only one "$h_0-\Delta h$" length. Assume that the "Δh_1" fluctuations due to the compression of the fluid and the extension of the pipe wall "Δh_2" can be independently calculated. By using the $\varepsilon=\frac{\sigma}{E}$ Hooke law, which is well known in terms of strength, the compression of water can be very simple:

$$\frac{\Delta h_1}{h_0}=\frac{\Delta p}{E_v}.$$



Tension in the pipe wall
Fugure 14.13

The water column shortening due to the expansion of the flexible pipe wall should be "Δh_2" shortened. Thus, the volume of shortening occurs in the increased ring cross section, i.e.,

$$\frac{D^2\cdot\pi}{4}\cdot\Delta h_2=\frac{(D+\Delta D)^2-D^2}{4}\pi(h_0-\Delta h_2).$$

Inserted on the right and simplified:

$$(D+\Delta D)^2\cdot\Delta h_2=2\cdot\Delta D\cdot\left(D+\frac{\Delta D}{2}\right)\cdot h_0$$

In parentheses, besides "D", the relative shortening of the fluid column due to the "ΔD" collapse is negligible

$$\frac{\Delta h_2}{h_0}=2\cdot\frac{\Delta D}{D} \qquad 14.22$$

The tencion in the pipe wall can be given as shown in **Figure 14.13**. We examine the "s" width of the tube. The pipe can be cut along the longitudinal axis of the tube. Write your balance:

$$F_p=2\cdot F_\sigma$$

The force from the pressure and the forces from the tension are written:

$$\Delta p\cdot D\cdot s=2\cdot\Delta\sigma_t\cdot\delta\cdot s$$

From which extra tendion in the pipe wall:

$$\Delta\sigma_t=\frac{\Delta p\cdot D}{2\cdot\delta}. \qquad 14.23$$

(The term refers to thin-walled pipes and is called a boiler form.)
Thus, the specific extension of the pipe wall and the proportional diameter of the pipe can be specified again using the Hooke Act:

$$\frac{\Delta D}{D}=\frac{\Delta\sigma_t}{E_{cs}}=\frac{\Delta p\cdot D}{2\cdot\delta\cdot E_{cs}} \qquad 14.24$$

Substituting the *equation 14.22*, we get the relative shortening due to the expansion of the pipe wall.

$$\frac{\Delta h_2}{h_0}=\frac{\Delta p}{\frac{\delta}{D}\cdot E_{cs}} \qquad 14.25$$

The specific shortening of the water column, the collapse of the water and the expansion of the pipe wall:

$$\frac{\Delta h}{h_0}=\frac{\Delta h_1+\Delta h_2}{h_0}=\frac{\Delta p}{E_v}+\frac{\Delta p}{\frac{\delta}{D}\cdot E_{cs}}=\frac{\Delta p}{E_r} \qquad 14.26$$

The "E_r", the so-called "reduced" elastic modulus, whose value is from the above equation

$$\frac{1}{E_r}=\frac{1}{E_v}+\frac{1}{\frac{\delta}{D}\cdot E_{cs}}. \qquad 14.27$$

Flexibility modulus
Table 14.1

Water	$2.1\cdot 10^9$ Pa
Steel	$2\cdot 10^{11}$ Pa
Cast iron	$1\cdot 10^{11}$ Pa

In practice, the elasticity modulus values for most of the occurrences of substances occur in **Table 14.1**.

14.4.3 Calculation of pressure increase

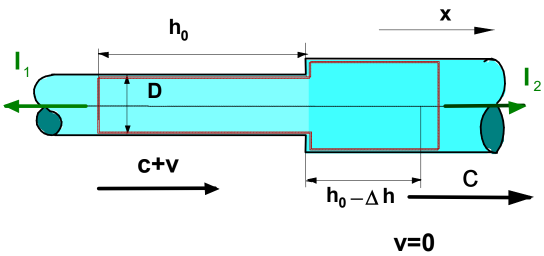


Figure 14.14

In **Figure 14.11/a**, the pressure wave speed is "c" from right to left. If we are moving with the wave, we can add a control surface as shown in **Figure 14.14**, where we can see a steady flow.
Write the continuity equation on the left and right cross section of the control surface. Apply the dimensions of **Figure 14.12**.
The left cross section has a "c + v" speed, and the diameter D the dencity is "ρ". At the right cross section, the "c" speed will be in the "$D+\Delta D$" diameter and "$\rho+\Delta\rho$" densities.

$$(c+v)\cdot\frac{D^2\cdot\pi}{4}\cdot\rho=c\cdot\frac{(D+\Delta D)^2\cdot\pi}{4}\cdot(\rho+\Delta\rho) \qquad 14.28$$

Az impulzustételt alkalmazva a **14.14 ábrába** berajzolt ellenőrző felületre, amely közvetlenül a csőfal mellett halad.
Using the momentum law applied to the control surface of **Fig. 14.14**, which passes directly to the pipe wall.

$$-(c+v)\cdot\frac{D^2\cdot\pi}{4}\cdot\rho\cdot(c+v)+c\cdot\frac{(D+\Delta D)^2\cdot\pi}{4}\cdot(\rho+\Delta\rho)\cdot c=$$

$$=\frac{\left[(D+\Delta D)^2-D^2\right]\cdot\pi}{4}\cdot\Delta p-\frac{(D+\Delta D)^2\cdot\pi}{4}\cdot\Delta p$$

When applying forces from the pressure, only the forces from overpressure were taken into account. This proves to be very useful when solving many tasks.
The first part of the right side is the pressing force on the increased ring surface.
In the second part of the left, replace the mass flow from the continuity, and make the operations on the right and simplify, so

$$-(c+v)\cdot D^2\cdot\rho\cdot(c+v)+(c+v)\cdot D^2\cdot\rho\cdot c=-D^2\cdot\Delta p$$

Devided by the square of the diameter and expressing the pressure increase, we get the following:

$$\Delta p=\rho\cdot(c+v)\cdot v \qquad 14.29$$

The flow velocity of the fluid is several orders of magnitude smaller than "c", the velocity of the wave propagation, therefore the pressure increase:

$$\Delta p = \rho \cdot c \cdot v \qquad 14.30$$

are used.

14.4.4 Speed of wave propagation

The control surface in **Figure 14.14** has a "h_0" lengths liquid column with "c+v" speed "Δt" over time. In the same „Δt" time with "c" speed, only "$h_0 - \Delta h$" lengths of fluid columns will exit. Writing "Δt" time on both sides:

$$\frac{h_0}{c+v} = \frac{h_0 - \Delta h}{c}$$

Explain $\frac{\Delta h}{h_0}$ its value from the above equation:

$$\frac{c}{c+v} = \frac{h_0 - \Delta h}{h_0} \qquad \frac{c+v}{c+v} - \frac{v}{c+v} = \frac{h_0}{h_0} - \frac{\Delta h}{h_0}$$

$$\frac{\Delta h}{h_0} = \frac{v}{c+v} \qquad 14.31$$

Compare the resulting expression with *equation 14.26*:

$$\frac{\Delta h}{h_0} = \frac{\Delta p}{E_r}$$

To replace „Δp" in the *equation 14.29*, we get the following:

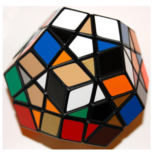
$$\frac{v}{c+v} = \frac{\rho \cdot (c+v) \cdot v}{E_r},$$

of which the speed "+" wave speed is expressed

$$c + v = \sqrt{\frac{E_r}{\rho}} \qquad 14.32$$

At wave speed, the flow velocity is usually negligible, so its standard prescribing is based on the rate of wave propagation speed

$$c = \sqrt{\frac{E_r}{\rho_{víz}}}. \qquad 14.33$$



In a plumbing system at the end of a $\ell = 200\,\mathrm{m}$ long straight section, a quick closing closure was installed. How much pressure buildup is generated in the lead made of cast iron when it is suddenly closed. Diameter of pipe is $D = 250\mathrm{mm}$, wall thickness is $\delta = 10\mathrm{mm}$. Water flow velocity is $v = 1.8\frac{m}{s}$.

Solutions:

First, calculate the reduced modulus of elasticity from e*quation 14.27*, the data is taken from t*able 14.1*:

$$\frac{1}{E_r} = \frac{1}{E_v} + \frac{1}{\frac{\delta}{D} \cdot E_{cs}} = \frac{1}{2.1 \cdot 10^9} + \frac{1}{\frac{10}{250} \cdot 10^{11}} = 7.26 \cdot 10^{-10} \frac{1}{Pa}$$

$$E_r = 1.377 \cdot 10^9 \, Pa$$

It can be seen that the resulting modulus of elasticity is less than the elastic modulus of both the tube and the water. *Equation 14.33* is the wave rate

$$c = \sqrt{\frac{E_r}{\rho_{víz}}} = \sqrt{\frac{1.377 \cdot 10^9}{10^3}} = 1173 \frac{m}{s}$$

and finally the pressure increase according to *equation 14.30*:

$$\underline{\Delta p = \rho \cdot c \cdot v = 10^3 \cdot 1173 \cdot 1.8 = 21.1 \cdot 10^5 \, Pa = 21.1 bar}$$

The sudden close in pressure increase is significant because the approx. Almost twice as big as 10 bar in the base pressure is added to the base pressure.
Closing is considered a sudden closing when the valve is fully closed before returning from its location. In this case, the wave is there and its reflection:

$$t = \frac{2 \cdot \ell}{c} = \frac{2 \cdot 200}{1173} = 0.341 \, s$$

If the valve is closed in less than this time, it will be a sudden shutdown and not at a slower closing. At slower closure, the pressure increase is lower than the calculated above.
Normal gate valves can only be locked in minutes. However, for example, the ball valves in the household water supply system can easily be used to produce a sudden closing and opening. Suddenly when opening, a similar phenomenon occurs in the system than at closing time.
Sudden closure and opening up in the human vasculature occurs at every heart beat. The blood flowing from the heart produces a pulsating wave of shock. The modulus of elasticity of the vascular wall is several orders of magnitude smaller than water or blood. Thus, the resulting modulus of elasticity, and thus the wave speed and pressure increase, are much smaller than in the previous example. Everyone can measure wave speed on their own, for example, on the cervical and ankle bodies, pulse the pulse at once, the ankle being ca. After 0.1-0.2 seconds, the drum is detected. It can be inferred that the speed of the wave is in the vascular system $10 \div 15 \frac{m}{s}$.

14.5 Calculation of wing grid

The basis for the design of the flow machines is Zsuzsny's wing theory, an extension of this, the theory of wing grilles. (***Nikolai Jegorovich Zhukovsky***, Russian scientist from 1847-1921) Grids in fans or turbines can be formed into infinite planar grids. The calculations here can also be used in rotating grids.

Figure 14.15 shows a section of an endless wing grid. The flow is considered to be a plane flow. In the "z" direction a unit of long wings are examined.

Data: $v_1 = 30\frac{m}{s}$; $\alpha_1 = 40^0$; $\alpha_2 = 25^0$; $t = 0.3m$; $\rho = 1.2\frac{kg}{m^3}$

Questions:

a./ What is the force acting on a blade as a vector $\underline{R}$?

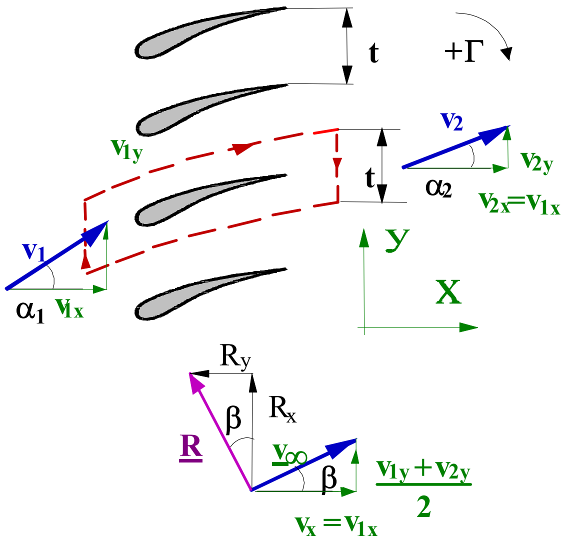
b./ Prove that, the $\underline{R} \perp \underline{v}_\infty$!

c./ What is the bladder circulation and how can it be interpreted if $t \to \infty$?

Solutions:

In **Figure 14.15** is used the control surface for the solution. Lowered from the bottom and the top by the same set of "t" divisions. Along the two streams, the pressure and velocity change in the same way. On the right and left, a vertical "t" height line closes the surface.
Enter the following markings:

$$v_{1x} = v_1 \cdot \cos\alpha_1;\; v_{1y} = v_1 \cdot \sin\alpha_1;\; v_{2x} = v_2 \cdot \cos\alpha_2;\; v_{2y} = v_2 \cdot \sin\alpha_2$$



Wing grid
Figure14.15

Aoolie the continuity law:

$$v_{1x} \cdot t = v_{2x} \cdot t,$$

so

$$v_{1x} = v_{2x}$$

Write the Bernoulli equation between the two endpoints of one stream line:

$$p_1 + \frac{\rho}{2} \cdot v_1^2 = p_2 + \frac{\rho}{2} \cdot v_2^2$$

Express the differential pressure and apply the Pythagoras law to the velocities:

$$p_1 - p_2 = \frac{\rho}{2} \cdot \left(v_{2x}^2 + v_{2y}^2 - v_{1x}^2 - v_{1y}^2\right)$$

$$p_1 - p_2 = -\rho \cdot \left(v_{1y} - v_{2y}\right) \cdot \frac{v_{1y} + v_{2y}}{2}$$

Write the momentum law in the specified coordinate system:

"x" direction: $0 = (p_1 - p_2) \cdot t - R_x$

"y" direction: $-\rho \cdot v_{1x} \cdot t \cdot v_{1y} + \rho \cdot v_{1x} \cdot t \cdot v_{2y} = -R_y$

where "$\rho \cdot v_{1x} \cdot t$" expresses both the inflow and outflow mass flow.

The pressures from the two streams are pronounced by each other. (Although they create a torque, but we are not dealing with it at this moment. For aircraft, the torques have an important role in the stability of the aircraft.)

Place the preceding expression on the two force components and replace the pressure from the other equations, so that the two force components are as follows:

$$R_x = -\rho \cdot \left(v_{1y} - v_{2y}\right) \cdot t \cdot \frac{v_{1y} + v_{2y}}{2}$$

$$R_y = \rho \cdot (v_{1y} - v_{2y}) \cdot t \cdot v_{1x}$$

Enter the following expression:

$$v_\infty = \sqrt{v_{1x}^2 + \left(\frac{v_{1y} + v_{2y}}{2}\right)^2}$$

The $\underline{v}_\infty$ arithmetic mean of a $\underline{v}_1$ and $\underline{v}_2$ vectors. It also has a physical meaning: far and behind the grid, this direction and magnitude of velocity.

$\Gamma = (v_{1y} - v_{2y}) \cdot t$ is on the enclosed closed curve clockwise with a positive circulation, the same velocity range on the two stream lines must be integrated with the opposite sign and therefore will be lost during integration.

(Circulation as it is known is the velocity integrated in a closed curve. Until now, the positive circumcision was counter-clockwise, and we were moved by Zhukovsky, and only here, for the opposite sign.)

Replaced by forces:

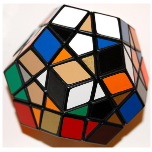
$$R_x = -\rho \cdot \Gamma \cdot \left(\frac{v_{1y} + v_{2y}}{2}\right)$$

$$R_y = \rho \cdot \Gamma \cdot v_{1x}\,.$$

So:

$$\left|\underline{R}\right| = \sqrt{\left[-\rho \cdot \Gamma \cdot \left(\frac{v_{1y} + v_{2y}}{2}\right)\right]^2 + \left[\rho \cdot \Gamma \cdot v_{1x}\right]^2} = \rho \cdot \Gamma \cdot v_\infty \qquad 14.34$$

This is the ***Zhukovsky* item**, which also applies to unique wings.



a./ For the given task, the first "v_2" velocity is calculated using the continuity:

$$v_2 = \frac{v_1 \cdot \cos\alpha_1}{\cos\alpha_2} = \frac{30 \cdot \cos 40^0}{\cos 25^0} = 25.35 \frac{m}{s}$$

The exit velocity is less than the inlet, the flow slows down, so this type of wing grille is called a slowing grid. As the velocity decreases, the pressure increases, which is advantageous, for example, in the design of certain types of fans or pumps.

With the given data:

$$\Gamma = (v_{1y} - v_{2y}) \cdot t = (v_1 \cdot \sin\alpha_1 - v_2 \cdot \sin\alpha_2) \cdot t = \left(30 \cdot \sin 40^0 - 25.35 \cdot \sin 25^0\right) \cdot 0.3$$

$$\Gamma = 2.57 \frac{m^2}{s}$$

$$v_\infty = \sqrt{v_{1x}^2 + \left(\frac{v_{1y} + v_{2y}}{2}\right)^2} = \sqrt{\left(30 \cdot \cos 40^0\right)^2 + \left(\frac{30 \cdot \sin 40^0 + 25.35 \cdot \sin 25^0}{2}\right)^2} = 27.44 \frac{m}{s}$$

And finally, the size of the force:

$$\left|\underline{R}\right| = \rho \cdot \Gamma \cdot v_\infty = 1.2 \cdot 2.57 \cdot 27.44 = 84.6 N$$

Direction of force:

$$\cos\beta = \frac{R_y}{R} = \frac{\rho \cdot \Gamma \cdot v_{1x}}{\rho \cdot \Gamma \cdot v_\infty} = \frac{30 \cdot \cos 40^0}{27.44} = 0.837 \qquad \beta = 33.2^0$$

Moves upward from the "y" axis to the left "β" at the "y" axis.

b./ The force and velocity vectors are perpendicular when the scalar product is zero. Write the two vectors and their scalar product:

$$\underline{R} = -\rho \cdot \Gamma \cdot \frac{v_{1y} + v_{2y}}{2} \cdot \underline{i} + \rho \cdot \Gamma \cdot v_{1x} \cdot \underline{j} \qquad \text{és} \qquad \underline{v}_{\infty} = v_{1x} \cdot \underline{i} + \frac{v_{1y} + v_{2y}}{2} \cdot \underline{j}$$

$$\underline{R} \cdot \underline{v}_{\infty} = -\rho \cdot \Gamma \cdot v_{1x} \cdot \frac{v_{1y} + v_{2y}}{2} + \rho \cdot \Gamma \cdot v_{1x} \cdot \frac{v_{1y} + v_{2y}}{2} = 0$$

c./

In the term $\Gamma = (v_{1y} - v_{2y}) \cdot t$, if, $t \to \infty$ then $v_{1y} \to v_{2y}$, but their product remains constant.

Similarly to potential vortex, when circulating around the center circle, then:

$$\Gamma = \frac{\Gamma}{2 \cdot \pi \cdot r} \cdot 2 \cdot \pi \cdot r$$

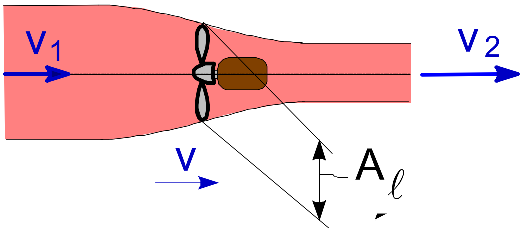
irrespective of the radius of the circles, the circulation remains constant.

Items 14.34 also apply in cases where "Γ" circulation is not a wing but creates anything else (e.g. rotating rolls, spin-ping pong or tennis balls).

It is worth noting that circulating around a wing is generated by friction on the wing surface. Farther away from the wing, the effect of friction decreases, so we could use the Bernoulli equation there.

14.6 Propulsion theory of propeller

The propeller is intended for propelling a plane, helicopter or other vehicle. The required thrust is provided by the air by its acceleration. An airplane propelled in the free airspace is moving at "v_1" velocity. The propeller further accelerates the intake air in the direction of progress, thereby providing traction power to the aircraft. **Figure 14.16** illustrates the flow pipe that delimits the air passing through the propeller in the airplane-bound system.



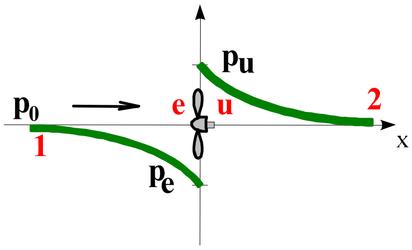
Propeller
Figure14.16

Determine the force on the propeller!

It can be considered that the propeller increases the pressure of the air flowing through it suddenly. The air that passes through the propeller and leaves the propeller steadily accelerates with a flow that can be regarded as frictionless.

The Bernoulli equation can not be applied directly between "1" and "2" because we need to pass through a solid body through the propeller, so use the Bernoulli equation for the "1-e" and "u-2" sections shown in **Figure 14.17**:

$$p_0 - p_e = \frac{\rho}{2}\left(v_e^2 - v_1^2\right) \quad p_u - p_0 = \frac{\rho}{2}\left(v_2^2 - v_u^2\right)$$



Change pressure near the propeller
Figure 14.17

At points "e" and "u", the flow velocity is approximately the same and the same as that of the air flow through the propeller. Adding the two equations above gives the following:

$$p_u - p_e = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) \qquad 14.35$$

From which the tractive force can be determined:

$$F_v = A_\ell\left(p_u - p_e\right) = A_\ell \frac{\rho}{2}\left(v_2^2 - v_1^2\right) \qquad 14.36$$

To determine the average velocity of the air passing through the propeller, the momentum law must be applied.
Select the control surface for a large cylinder along which the pressure can be considered constant. Move the control surface with the velocity of the aircraft so the flow is steady. The force acting on the propeller can be calculated from the outputs of the outgoing and intake momentum current vectors:

$$\underline{I}_{be} + \underline{I}_s + \underline{I}_{gy} + \underline{I}_p = -\underline{R}$$

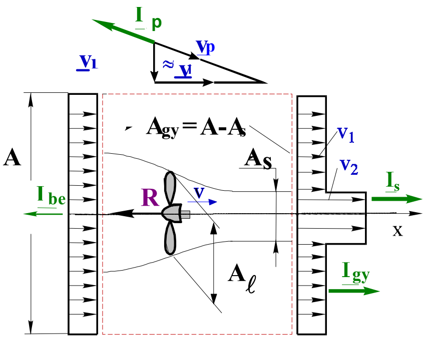


Figure 14.18

The figure illustrates the cross section of the current stream pipe in which the air passing through the propeller is flowing. In this current pipe, the air velocity up from "v_1" velocity to "v_2" velocity and leaves the control surface "A_s" in a radial cross section.
As a continuity, the sum of the mass flows in and out of the control surface is zero and therefore the mass of the floating mass flow can be calculated by the following relation:

$$q_{mp} = \left(v_2 - v_1\right)\rho A_s$$

The streamlines passing through the mantle intersect the control surface at a very low angle, so when calculating the momentum vector, the component of the local velocity axis is well approximated to the velocity "v_1". so

$$\underline{I}_p = \int_{A_p} \underline{v}_p \rho \underline{v}_p d\underline{A} \cong \underline{v}_1 \int_{A_p} \rho \underline{v}_p d\underline{A} = \underline{v}_1 q_{mp}.$$

Because of the rotational symmetry, the vectors are parallel to the axis "x"

$$I_{be} = v_1^2 \rho A, \quad I_{gy} = v_1^2 \rho\left(A - A_s\right)$$

$$I_s = v_2^2 \rho A_s, \quad I_p = v_1\left(v_2 - v_1\right)\rho A_s$$

$$-v_1^2 \rho A + v_2^2 \rho A_s + v_1^2 \rho\left(A - A_s\right) - v_1 A_s \rho\left(v_2 - v_1\right) = -R$$

$$\rho A_s v_2\left(v_2 - v_1\right) = -R$$

So the tractive force points to a negative coordinate direction $F_v = |R|$:
According to the continuity applicable to the current tube $\rho A_s v_2 = \rho A_\ell v$:

$$F_v = \rho A_s v_2\left(v_2 - v_1\right) = \rho A_\ell v\left(v_2 - v_1\right) \qquad 14.37$$

Equalizing the result of the momentum law and the Bernoulli equation previously stated:

$$F_v = \rho A_\ell v(v_2 - v_1) = \frac{\rho}{2} A_\ell (v_2^2 - v_1^2) = \rho A_\ell \frac{v_2 + v_1}{2}(v_2 - v_1) \quad 14.38$$

From this air velocity through the propeller:

$$v = \frac{v_2 + v_1}{2}, \quad 14.39$$

that is, the mean velocity of the propulsion on the propeller is equal to the propeller jet, far ahead of the propeller and far beyond the arithmetic mean of the velocities behind the propeller.
In the drainage, the propeller was idealized, velocities were taken everywhere along the axis and ignored that the propeller rotated the air jet and did not even consider that the air jet absorbs a certain amount of air from the surrounding air, which is a decrease in velocity increases the amount of air flow. (The latter does not cause an error because the pulse does not change. Internal force does not change the momentum of the system.)
The propeller mounted on the aircraft provides a " v_1 " velocity.

$$P_u = F_v \cdot v_1 \quad 14.40$$

The power required to drive the ideal propeller can be obtained when the performance delivered by the propeller to the air is determined. For example, in the same way as the Euler turbine equation (*equation 12.18)*:

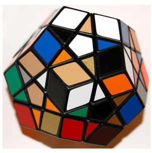
$$P_{tid} = q_v \cdot \Delta p_ö = v \cdot A_\ell \cdot (p_u - p_e) \quad 14.41$$

The total pressure change is the same as the static pressure on the propeller, since the velocity is the same as in "e" and "u". The term is equal to the pulling force $A_\ell \cdot (p_u - p_e)$ acting on the propeller, so

$$P_{tid} = v \cdot F_v$$

The so-called **propulsion efficiency** of the ideal propeller means the following:

$$\eta_{pr} = \frac{P_u}{P_{tid}} = \frac{F_v \cdot v_1}{F_v \cdot v} = \frac{v_1}{v} = \frac{v_1}{\frac{v_1 + v_2}{2}} = \frac{2}{1 + \frac{v_2}{v_1}} \quad 14.42$$



The propulsion efficiency is ideal, in case of loss, give the drive efficiency.

Data: $A_\ell = 1 m^2$; $v_1 = 90 \frac{m}{s}$; $v_2 = 150 \frac{m}{s}$; $\rho = 1.2 \frac{kg}{m^3}$

Qestions:

a./ Calculate the tractive force (or propulsion force of a propeller) on the propeller "A_l" when accelerating to the airborne air velocity ”v_2” in the coordinate system fixed to the "v_1" moving airplane!

b./ Determine the propulsion efficiency of the propeller!

Solution:

a./ Calculate the tractive force from *equation 14.36:*

$$F_v = A_\ell \cdot \frac{\rho}{2} \cdot \left(v_2^2 - v_1^2\right) = 1 \cdot \frac{1.2}{2} \cdot \left(150^2 - 90^2\right) = 8640\text{N}$$

b./ Calculate propulsion efficiency from equation 14.42:

$$\eta_{pr} = \frac{P_u}{P_{tid}} = \frac{2}{1 + \frac{v_2}{v_1}} = \frac{2}{1 + \frac{150}{90}} = 0.75$$

14.7 The ideal wind generator

With the windmill, the wind energy can be transformed into useful energy, for example. electricity. Wind as a renewable source of energy is a good source of energy in economically feasible locations in certain parts of the Earth. For example, in Holland, wind mills have been used for pumping water and for other purposes for centuries. The wind turbine shaft on the photo is approx. 65 m high and the wind turbine diameter reaches 44 m. Its rated output is 600 kW. (Examination of wind energy utilization in Hungary is underway.) The wind turbine is a reversed working propeller. The wind generator provides work for the slowdown of air. **Figure 14.19** shows a wind generator with blades, generator and wind direction. In the ideal Wind Generator theory, we can use the interconnections derived from the propeller, but note that the speed before the wind turbine is greater than that of the wind turbine. The beam passing through the wind generator is shown in **Figure 14.20**. Determine the useful wind power of the ideal wind generator. The mass flow of air flowing through the wind generator surface a

$$q_m = \rho \cdot A_{sz} \cdot v$$

where the "v" velocity is the average axial velocity in the wind generator plane.

Wind generator in Kulcs

http://www.alternativenergia.hu/kulcsi-szeleromu-video/6903

Figure 14.19

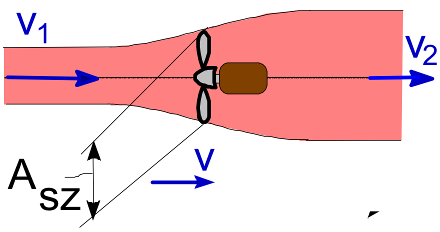
If we consider that this velocity can be calculated as the arithmetic mean of the wind generator before and after the wind generator, then the flow rate

$$q_m = \rho \cdot A_{sz} \cdot \frac{v_1 + v_2}{2}$$

The ideal wind generator fully utilizes the air delivered by the air. In the present case, calculate enough energy to calculate transformed energy because the pressure is equal to "1" and "2", so the ideal power is:

$$P_{id} = q_m \cdot \frac{1}{2} \cdot \left(v_1^2 - v_2^2\right) = \frac{1}{2} \cdot \rho \cdot A_{sz} \cdot \frac{v_1 + v_2}{2} \cdot \left(v_1^2 - v_2^2\right)$$

When given at wind velocity „v_1”, it is questionable how slow the wind is to get the most energy?



Wind generator
Figure 14.20

Continuing with the previous thought, if the wind energy lost by the wind is utilized by the wheel, then the wind must be stopped behind the wheel. But in this case the air will not leave behind the wind generator. (It has to be "shoved" from there.) So, in the exit crossection, it is necessary to leave the energy to leave the air.

Examine the above expression by changing the "v_2" velocity when you get maximum performance at a given „v_1” velocity. So look for the zero position of the derivative of the performance.

$$\frac{\partial P_{id}}{\partial v_2} = \frac{1}{4} \cdot \rho \cdot A_{sz} \cdot \frac{\partial \left[v_1^3 + v_1^2 \cdot v_2 - v_1 \cdot v_2^2 - v_2^3\right]}{\partial v_2} = \frac{1}{4} \cdot \rho \cdot A_{sz} \cdot \left(v_1^2 - 2 \cdot v_1 \cdot v_2 - 3 \cdot v_2^2\right).$$

The expression in parenthesis has an extreme value at the zero point. Rearrange the term in parenthesis and equal to zero:

$$3 \cdot \left(\frac{v_2}{v_1}\right)^2 - 2 \cdot \left(\frac{v_2}{v_1}\right) - 1 = 0,$$

from which

$$\frac{v_2}{v_1} = \frac{-2 \pm \sqrt{4 + 4 \cdot 3}}{2 \cdot 3} = \left\langle \begin{array}{c} \frac{1}{3} \\ -1 \end{array} \right.$$

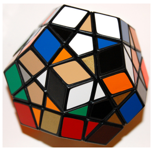
For us, only the positive root has a physical meaning. We still need to look at whether or not the maximum has the term or minimum. This can be determined on the basis of the second derivative if the second derivative for that value is negative, then there is really a maximum of performance.

$$\frac{\partial P^2{}_{id}}{\partial v_2^2} = \frac{1}{4} \cdot \rho \cdot A_{sz} \cdot \left(-2 \cdot v_1 - 6 \cdot v_2\right) = \frac{1}{4} \cdot \rho \cdot A_{sz} \cdot \left(-12 \cdot v_2\right) < 0$$

In that case, this is the case. Let's look at what the maximum performance is when the wind turbine brakes $\frac{1}{3}$ the velicity of the prevailing wind. Replacing the exit velocity in place of the $\frac{1}{3}$ inlet velocity, with the maximum ideal power:

$$P_{max} = \frac{1}{2} \cdot \rho \cdot A_{sz} \cdot \frac{v_1 + \frac{v_1}{3}}{2} \cdot \left[v_1^2 - \left(\frac{v_1}{3} \right)^2 \right] = \frac{16}{27} \cdot \frac{\rho}{2} \cdot A_{sz} \cdot v_1^3$$

According to the term, only the part of the wind power passing through the given cross-section can ideally be utilized $\frac{16}{27}$. This term is Betz's formula. The real wind generator provides about 65-80% of this performance due to various losses.



As an example, you should choose a ca. 2 m diameter wind turbine at $v_1 = 10 \frac{m}{s}$ wind velocity

$$P_{max} = \frac{16}{27} \cdot \frac{\rho}{2} \cdot A_{sz} \cdot v_1^3 = \frac{16}{27} \cdot \frac{1.2}{2} \cdot \frac{2^2 \cdot \pi}{4} \cdot 10^3 = 1117 W$$

deliver ideal performance. This term is Betz's formula.

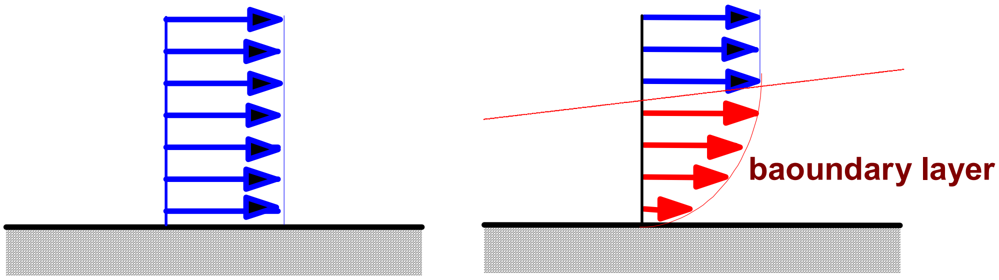
Comment:

In some parts of Hungary, some areas may argue for the economical utilization of wind energy. In most cases, the wind velocity can be measured in a few m/s at the height of the average position of the wind turbine, which must be at least 8 to 15 m in order to exclude the interfering effects of buildings and trees. The wind farm in the photo is far larger than approx. 50-60 m high and the wind turbine diameter reaches 50 m. The maximum ideal power output of such a wind turbine in the above example is 698 kW. 65-80% of the nominal power, taking into account the flow and electrical losses, so that approx. 450 kW nominal power is obtained.

15. Friction fluids

15.1 Viscosity

In the discussion so far, we have neglected the friction inside the fluid. This greatly simplified the mathematical description of motion. The results of neglecting friction give good approximation of reality for low internal friction fluids and when velocity increases in the direction of flow (confusing flow). It is not negligible to friction in high internal friction liquids such as oil flow in bearing. Also in low internal friction fluids, friction changes the flow when the velocity decreases in the direction of flow (diffuser flow).

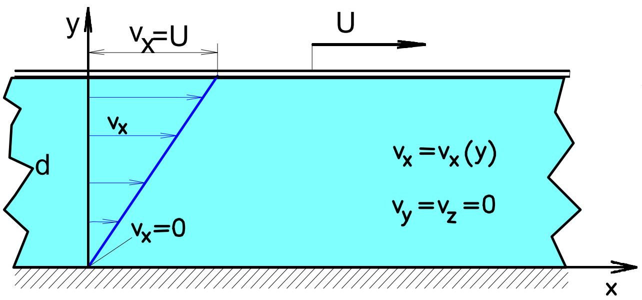


Ideal Fluid **Real Fluid**

Figure 15.1

Near the solid wall, in all friction fluids, the nature of the flow changes fundamentally. The ideal fluid also has a finite velocity near the wall, so only the velocity perpendicular to the wall is zero. **And the viscous liquid adheres to the wall, so the speed of the solid wall is always zero.** Faster than the wall reaches the external flow rate. This layer of flow is the **boundary layer**, which is of great importance in the formation of flow.

Liquid friction definition is derived from Newton.
Examine fluid flowing between two flat sheets. The bottom plane plate stands and the top moves parallel to the bottom with the "U" speed (see **Figure 15.2**). The liquid adheres to both the lower and the top sheets, so at the top of the "U" speed at the bottom there is fluid flow at the nozzle. There is a parallel flow between the plates, so there is only a "x" speed in the liquid. The speed between the two sheets is linear.
To move the flat plate, it is necessary to apply some force, the force of this force per unit surface is the sliding voltage.



Flow between two flat plates
Figure 15.2

The relationship between velocity and the shear stress was discovered by Newton, which states that

$$\tau = \mu \cdot \frac{dv_x}{dy} . \qquad 15.1$$

"μ" is a proportionality factor depending on the properties of the fluid, **dynamic viscosity**, whose unit of measure can be expressed after expression from the equation as follows:

$$[\mu]=[\tau]\left[\frac{1}{\frac{dv_x}{dy}}\right]=\frac{kgm}{s^2m^2}\frac{1}{\frac{m/s}{m}}=\frac{kg}{ms}.$$

In flow regulation, we often use the **kinematic viscosity**, which is the ratio of dynamic viscosity and density:

$$\nu=\frac{\mu}{\rho} \qquad 15.2$$

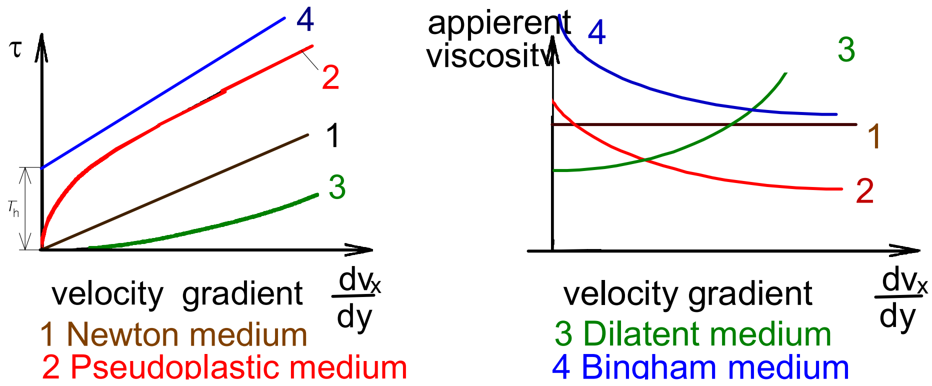
$$[\nu]=\frac{kg}{m\cdot s}\cdot\frac{m^3}{kg}=\frac{m^2}{s}.$$

Some other unusual viscosity viscosities as well as dynamic and kinematic viscosity of water and air are given in **Table 15.1.** Dynamic viscosity depends on temperature, dependence on pressure is low. Only depend on the low pressure of the gases.
The dynamic viscosity of different materials as a function of temperature is depicted in **Appendix 1.**
Liquids that behave according to Newton's Viscosity Act (*equation 15.1*) are called **Newtonian fluids**.

Table 15.1 Kinematic and dynamic viscosity units and values

	Dynamic viscosity	Kinematic viscosity
SI	$\frac{N\cdot s}{m^2}$, $Pa\cdot s$, **or** $\frac{kg}{m\cdot s}$	$\frac{m^2}{s}$
cgs unit	**poise =** $\frac{dyn\cdot s}{cm^2}=\frac{g}{cm\cdot s}$ **centipoise=poise/100**	**stokes =** $\frac{cm^2}{s}$ **centistokes = stokes/100**
Water $4\,^0C$	$130\cdot10^{-5}\ \frac{kg}{m\cdot s}$	$1.3\cdot10^{-6}\ \frac{m^2}{s}$
Air $0^0\,C$; 1bar	$1.8\cdot10^{-5}\ \frac{kg}{m\cdot s}$	$14.4\cdot10^{-6}\ \frac{m^2}{s}$

Newtonian fluids are the most commonly used media in engineering practice: water, air, oils and gases.
However, there are many liquids in which the shear stress is not directly proportional to deformation velocity. These fluids are called non-Newtonian fluids. Non-Newtonian media are concerned with the science of rheology.



Rheological curves of different media

Figure 15.3

Section **15.3. Figure** shows a rheological curve of some media showing the relationship between the shear stress "τ" in the liquid and the deformation velocity $\frac{dv_x}{dy}$. On the right hand diagram, apparent viscosity was drawn on the various materials. These materials have both **time-independent rheology curves**.

The curve "**1**" refers to a Newtonian medium with a constant viscosity. Curves "**2**" and "**3**" are the so-called power function medium where the relationship between the shear stress and deformation velocity is described by a power function:

$$\tau = k \cdot \left(\frac{dv_x}{dx}\right)^n .$$

If $n < 1$ the **pseudoplastic medium** "**2**" is referred to as the curve character resembles the "**4**" medium that is referred to as a plastic or Bingham medium. These media generally contain long chain molecules. Pending the "arrangement" of these molecules, the rate of deformation velocity increases with a high variation in sliding voltage, and is later smaller. Such materials include blood plasma, polyethylene and water-mixed clay.

The so-called "**3**" with the power exponent is $n > 1$. it is typical of **dilating media** that its low velocity deformation has relatively low shear stress, coupled with a rapidly increasing shear stress for an increasing deformation velocity. Such media, for example. sludges containing mineral powders. At low speeds, the liquid between the particles serves as a lubricant between the solid particles moving together. At higher speeds, solid particles increasingly clash together to produce greater internal friction.

"**4**" refers to the **Bingham medium** or **pseudoplastic fluid**, at which, once a specific "τ_h" boundary shear stress is reached, the layers of the medium begin slipping on each other. As long as the shear stress does not reach the limit shear stress, the material acts as an elastic, deformable body, deforms, but retains its original shape after the tension has ceased. Above the boundary shear stress the material flows, and after the tension ceases, it will not recover its original shape. (Actually, the steel behaves beyond such a force as such.)

$$\tau = \tau_h + \mu_\infty \frac{dv_x}{dy},$$

where "μ_∞" is a constant depending on the nature of the medium. Typically, plastic fluids are characterized by a cross-linking structure which causes the media to flow after τ_h the collapse. Plastic medium eg. oil paints, toothpaste, asphalt, chocolate, butter and fats.
For some materials, apparent viscosity also depends on other factors, for example, **from time** or from the rate of preventive deformation. These include "memory", dilatation, etc. materials. Nylon, some gels, more polymers belong to this group of substances.

15.2 Motion equation of friction medium

In the description of fluid movement, the effect of internal friction has not been taken into consideration so far. In the motion equations, only the forces from the pressure as in the ***Euler equation*** (see *Equations 12.5 and 12.6*) and the force acting on the bounded fluid part in the **momentum equation** load as well as the force acting on the surface and weight force as volume force were taken into account.
Consideration of friction in the motion equation may be complicated during the derivation which is not described in this note.
Assuming a Newtonian medium and constant density, the equation of motion is **Navier-Stokes equation**:
(***French Claude-Louis Navier and George Gabriel Stokes 1819-1903*** were independently conducted.)

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}=g_x-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right) \qquad 15.3$$

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}=g_y-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)$$

$$\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}=g_z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)$$

A négy ismeretlen (v_x, v_y, v_z és p) meghatározásához szükséges negyedik egyenlet a folytonosság tétele, amely $\rho=$ áll. esetén a
For four unknowns (v_x, v_y, v_z and p) is necceserry fourth equation for determining the continuity is the fourth on at constant dencity

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0, \qquad 15.4$$

in the form. The above complicated equation system has been solved analytically in various simplified cases. In the theory of plain bearings, it was possible to find analytical or semi-analytical solutions for several bearing types in a steady case. In the boundary layer theory, a simplified equation system was created from the Navier-Stokes equation known as boundary equation in the flow.
Recently, it has also been developed for unsteady, turbulent, three-dimensional flows.
In vectoriL form, the Navier-Stokes equation is:

$$\frac{d\underline{v}}{dt}=\frac{\partial\underline{v}}{\partial t}+\mathrm{grad}\frac{v^2}{2}-\underline{v}\times\mathrm{rot}\underline{v}=\underline{g}-\frac{1}{\rho}\mathrm{grad}p+\nu\Delta\underline{v} \qquad 15.5$$

It can be seen that the Navier-Stokes equation differs from the Euler equation derived from the frictionless case in the last member of the right side (*see equation 12.6*).

15.3 Solution of Navier-Stokes equation between two planes

The flow and velocity distribution between the steady and movable plate shown in **Figure 15.2** are determined from the N-S equation. Of course, we can anticipate its solution because the linear velocity profile in the diagram is the result.
From the Navier-Stokes equations and the continuity equation, the following simplifications can be made for the case given in the task:

- steady flow is tested
- let it be $v_x = u$

$v_y = v_z = 0$; and all derivatives therefore are zero

- and any change in the "z" and "x" directions of each variable should be zero, so assuming a plane flow, assuming the same pressure and velocity in every "x".
- the effect of the gravity field can be neglected; $g_x = g_y = g_z = 0$

Following these simplifications, we get the following (since all depends on "y", so straight "d" can be used instead of the partial derivative):

$$0 = \nu \cdot \frac{d^2u}{dy^2} \qquad 15.6$$

$$0 = \frac{dp}{dy} \qquad 15.7$$

According to the second equation, the pressure does not change perpendicular to the stream lines. The first equation divided by "ν" and integrates twice with the following:

$$u = C_1 \cdot y + C_2$$

It is necessary to provide two boundary conditions for an axact solution from the law of adhesion at $y = 0$

$$u = 0$$

and at $y = d$

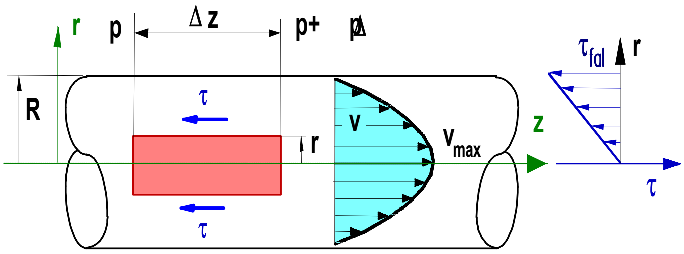
$$u = U.$$

By replacing them, we get the velocity change between a standing and a moving plane:

$$\underline{u = \frac{U}{d} \cdot y}$$

15.3 Solution in cylindrical tube

We have always calculated average velocities in the pipelines, either the continuity or the Bernoulli equation. In actual flows, the velocity of the pipes in the pipes varies along the radius of the tube. In long straight pipes, the velocity profile remains constant and does not change along the axis, cylindrical symmetrical and does not rotate around the axis of the tube, this is called a pipe flow. The following chapter deals with laminar and turbulent flows. In the next example, suppose laminar flow!



Laminar velocity profile in tube
Figure 15.4

The N-S equation must be written in cylindrical coordinate form (r,φ,z, curvelnear coordinate system). In the cylinder coordinate system, the completely common Navier-Stokes equation and continuity look like this:

$$\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\varphi}{r}\frac{\partial v_r}{\partial \varphi}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\varphi^2}{r}=g_r-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \varphi^2}+\frac{1}{r}\frac{\partial v_r}{\partial r}+\frac{\partial^2 v_r}{\partial z^2}-\frac{1}{r^2}\frac{\partial v_\varphi}{\partial \varphi}-\frac{v_r}{r^2}\right)$$

$$\frac{\partial v_\varphi}{\partial t}+v_r\frac{\partial v_\varphi}{\partial r}+\frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial \varphi}+v_z\frac{\partial v_\varphi}{\partial z}-\frac{v_r v_\varphi}{r}=g_\varphi-\frac{1}{\rho r}\frac{\partial p}{\partial \varphi}+\nu\left(\frac{\partial^2 v_\varphi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 v_\varphi}{\partial \varphi^2}+\frac{1}{r}\frac{\partial v_\varphi}{\partial r}+\frac{\partial^2 v_\varphi}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \varphi}-\frac{v_\varphi}{r^2}\right)$$

$$\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\varphi}{r}\frac{\partial v_z}{\partial \varphi}+v_z\frac{\partial v_z}{\partial z}=g_z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \varphi^2}+\frac{\partial^2 v_z}{\partial z^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}\right)$$

$$\frac{\partial v_r}{\partial r}+\frac{1}{r}\cdot\frac{\partial v_\varphi}{\partial \varphi}+\frac{\partial v_z}{\partial z}+\frac{v_r}{r}=0 \qquad 15.8$$

The equation system is very complicated, but after a few short thoughts it can be simplified to a more friendly form.
Simplified condition is that $v_r = v_\varphi = 0$ there is nothing in the direction of "φ", that is, the cylinder is symmetrical, and the difficulty is neglected. After that, there are far fewer equations, and there are far fewer members left:

$$0=\frac{1}{\rho}\frac{\partial p}{\partial r}$$

$$v_z\cdot\frac{\partial v_z}{\partial z}=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial z}+\nu\cdot\left(\frac{\partial^2 v_z}{\partial r^2}+\frac{\partial^2 v_z}{\partial z^2}+\frac{1}{r}\cdot\frac{\partial v_z}{\partial r}\right) \qquad 15.9$$

The upper equation gives us so much information that the pressure is constant along one cross-section.
The second equation can be further simplified if we know that the velocity does not change along the axis of the tube, so only the radius-dependent members remain in the equation. The pressure depends only on the "z" coordinate and there is only one speed, so the index "z" can be omitted. Then the equation is:

15.10

$$0=-\frac{1}{\rho}\cdot\frac{dp}{dz}+\nu\cdot\left(\frac{d^2v}{dr^2}+\frac{1}{r}\cdot\frac{dv}{dr}\right)$$

The pressure drop along the pipe length is constant. The equation can be converted to the following form:

$$\frac{r}{\rho\cdot\nu}\cdot\frac{dp}{dz}=\frac{d\left(r\cdot\frac{dv}{dr}\right)}{dr}$$

Integrating the equation twice, it results:

$$\frac{dv}{dr}=\frac{r}{2\cdot\rho\cdot\nu}\cdot\frac{dp}{dz}+C_1 \qquad 15.11$$

The constant value of "C_1" is zero, because at axis r=0, the change in speed must be zero.

$$v=\frac{r^2}{4\cdot\rho\cdot\nu}\cdot\frac{dp}{dz}+C_2 \qquad 15.12$$

If $r=R$, the velocity on the pipe wall is $v=0$, the liquid will stick to the pipe wall. By substituting this boundary condition, we get the integrating constants, and then "v"

$$v=-\frac{1}{4\mu}\cdot\frac{dp}{dz}\cdot\left[R^2-r^2\right]$$

function. The velocity distribution varies according to the second degree rotation paraboloid, the change of pressure under "z" is returned after the next thought path.
To calculate the velocity function, we can also get a different way of thinking.
Take a cylinder with hights "Δz" with a radius "r" concentric with the axis of the tube and write down the balance of the forces acting on it (**see Figure 15.4**). The fluid cylinder does not accelerate, so the forces acting on it must balance each other. The force from the difference between the pressure on the base and the top plate of the cylinder is influenced by the force generated by the shear force generated on cylinder sleeve.

$$r^2\cdot\pi\cdot p-r^2\cdot\pi\cdot(p+\Delta p)+2\cdot r\cdot\pi\cdot\Delta\ell\cdot\tau=0 \qquad 15.13$$

Replaced and simplified from $2\cdot\tau\cdot\Delta z=r\cdot\Delta p$ which the shear stress distribution is:

$$\tau=\frac{1}{2}\cdot r\cdot\frac{\Delta p}{\Delta z} \qquad 15.14$$

Newton's Viscosity law *equation 15.1* in this case is simply rewritable in a cylindrical symmetric case (the other two coordinates are not so simply transcribed):

$$\tau=\mu\cdot\frac{dv}{dz} \qquad 15.15$$

Equalizing the *equations 15.14 and 15.15* the resalt is the next:

$$\frac{1}{2}\cdot r\cdot\frac{\Delta p}{\Delta z}\tau=\mu\cdot\frac{dv_x}{dy} \qquad 15.16$$

which fully complies with *equation 15.11*. The differential equation is solved by separating the variables:

$$\int dv=\frac{1}{2\mu}\frac{\Delta p}{\Delta z}\int rdr$$

$$v = \frac{1}{4\mu} \cdot \frac{\Delta p}{\Delta z} \cdot r^2 + C \qquad 15.17$$

If $r = R$, that is, on the pipe wall the velocity $v = 0$. By substituting this boundary condition, we get the integrating constants, and then "v"

$$v = -\frac{1}{4\mu} \cdot \frac{\Delta p}{\Delta z} \cdot \left[R^2 - r^2\right] \qquad 15.18$$

It can be seen that the direction of flow with the positive direction of the "z" axis is the same, i.e. that $\frac{\Delta p}{\Delta z} < 0$ the pressure decreases in the direction of the increasing "z" coordinates. (In the direction of flow, the force from decreasing pressure moves the medium against the frictional forces.) Introduce the concept of pressure friction loss „$\Delta p'$", which here is a reduction of friction due to friction which is generally treated as a positive quantity, so it is possible to write: $\frac{dp}{dz} = \frac{\Delta p}{\Delta z} = -\frac{\Delta p'}{\ell}$ where "ℓ" the pipe length at which pressure drop occurs.

With this, *the terms 15.14 and 15.18* can be written in the following form:

$$\tau = -\frac{\Delta p'}{2 \cdot \ell} r, \qquad 15.19$$

$$v_z = \frac{\Delta p'}{4 \cdot \mu \cdot \ell} \left[R^2 - r^2\right] \qquad 15.20$$

Figure 15.4 depicts the changes in the share stress and velocity in the radial direction. It can be seen that the negative and absolute values of "τ" are increased by the radius.

At maximum velocity $v_{max} = \frac{\Delta p' R^2}{4\mu\ell}$ is given at $r = 0$. Secondary parabolic shape velocity distribution so average velocity is $\bar{v} = \frac{v_{max}}{2}$.

How to write:

$$\bar{v} = \frac{\Delta p'}{8 \cdot \mu \cdot \ell} R^2,$$

respectively. the pressure loss is expressed as:

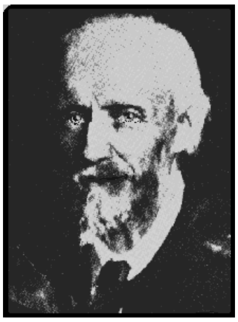
$$\Delta p' = \frac{8 \cdot \mu \cdot \bar{v} \cdot \ell}{R^2}. \qquad 15.21$$

The sliding voltage takes the value $\tau_{fal} = -\frac{\Delta p' \cdot R}{2 \cdot \ell}$ at the wall *15.19* to the wall.

16. Flow of viscous fluids in pipelines

In this chapter, we have a very important problem that we face in the engineering practice, we are dealing with various types of flow in the pipelines. Pipeline systems occur in almost every engineering work. Flow phenomena in pipes have been studied by many in a theoretical and practical approach. The fluid flowing through the tube shows, under certain conditions, a calm, **laminar or laminar flow**, and in other conditions the flow shows a chaotic flow in space and time, **turbulent flow** in a foreign word.

16.1. Laminar and turbulent flow



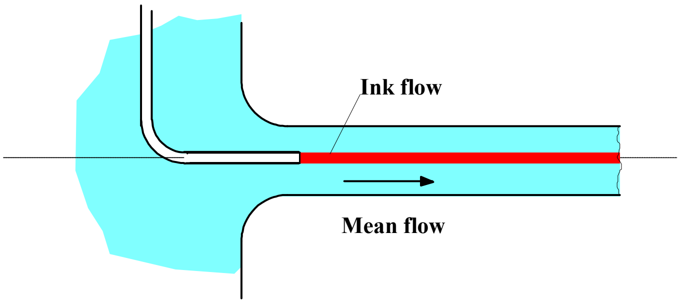
At the end of the ninethen century, he carried out his basic experiments (***Osborn Reynolds 1842-1912***) to reveal the specificities of the flows in the pipeline. **Figure 16.1** shows a pipe from a large container. The velocity of the flowing water can be controlled in the pipe.

In the shaft of the glass pipe, another liquid that has been stained through a thinner tube, e.g. red ink.

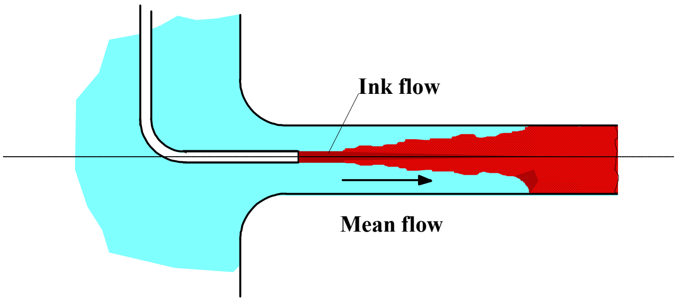
At low velocity of the main stream, the dyed liquid stream flows in the tube axis in a definite, virtually constant cross-section and does not mix with the main flow. The main flow flows in well separated layers. This type of flow is called laminar or layered flow. The velocity vector at any point of the tube does not change in time, the flow is steady. **Figure 16.1/a** shows a laminar flow.

Increasing the rate of the main flow, the intermittent disturbance of the dyed liquid fiber, the dyed liquid begins to wavy motion, but it is still a well-separated stream of current from the main stream. To further increase the main flow rate, the snapping colored flow tube breaks out and mixes with the main flow. At a certain distance the two fluids are completely mixed and the main flow becomes evenly red (**Figure 16.1/b**).

Looking at a point of the main flow at a time of change of velocity, we can observe the following:



a./Laminar flow



b./ Turbulent flow
Figure 16.1

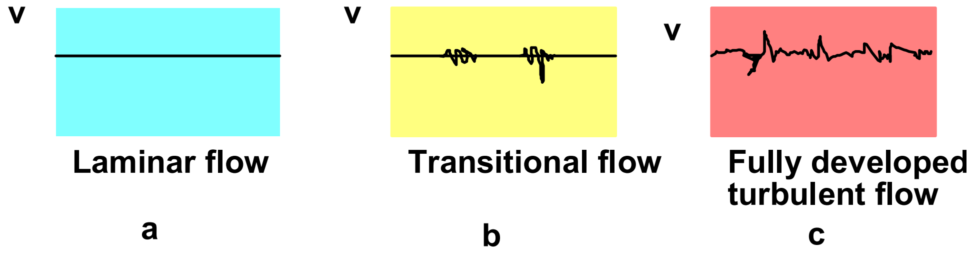
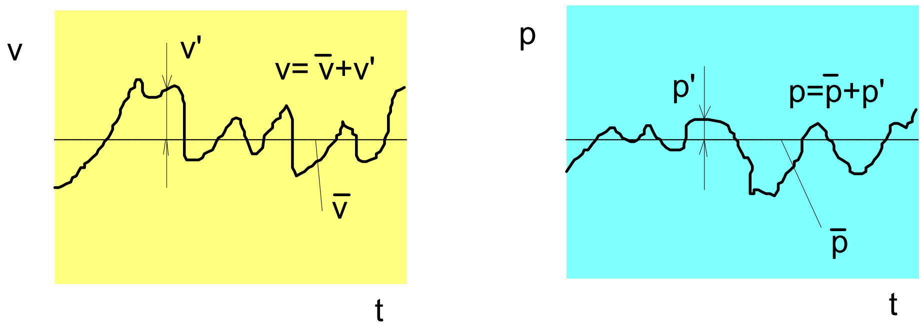


Figure 16.2.

In the laminar flow the velocity does not change in time **Figure 16.2/a**. When the ink pad waves, at a selected point in the main stream, the velocity changes at times, but returns to the steady state, **Figure 16.2/b**.
As the velocity is so large in the mainstream that the ink is mixed in the entire cross-section, at a point in the main stream the velocity changes completely stochastic. No periodicity can be discovered in time change. The flow becomes instable, this is called complete or emerging turbulence in **Figure 16.2/c.**
Looking at the velocity in the turbulent flow, we not only find that velocity fluctuates in time but also in space, which means that at each point a component perpendicular to the axis of the tube is also formed. This perpendicular component blends the ink into the mainstream.
In his experiments, Reynolds has come to the conclusion that the emergence of turbulent and laminar flow basically depends on a dimensional number

$$Re = \frac{v \cdot d \cdot \rho}{\mu}, \qquad 16.1$$

which is called Reynolds number, where
"v" is the average velocity measured in the tube,
"d" is the diameter of the tube,
"ρ" is the density of medium and
"μ" is the dynamic viscosity.
If the pipe, whereby the flow is exposed to vibrations and disturbances, it occurs around $Re \cong 2300$ **laminar-turbulent transformation**. If the flow is sufficiently undisturbed and the inner wall of the tube is perfectly smooth then a laminar flow can be achieved with a significantly higher Reynolds number value. However, this condition is very unstable and the flow is swirling to the turbulence as a result of the slightest disturbance.



Velocity and pressure change in time at turbulent flow
Figuren 16.3

Most industrial turbulent flows occur in most cases. The rate of fluctuation is usually not high, only a few percent, so most of the time the pressure or velocity is well characterized by an average flow.
Average values:

$$\bar{v} = \frac{1}{T}\int_T v \cdot dt \qquad \bar{p} = \frac{1}{T}\int_T p \cdot dt \qquad 16.2$$

The instantaneous value of the velocity at a given time can be given as the sum of the mean value and the deviation, so.

$$v = \bar{v} + v' \qquad p = \bar{p} + p' \qquad 16.3$$

Of course, there is a fluctuation of velocity and pressure that their time average is zero.

$$0 = \frac{1}{T}\int_T v' dt \qquad 0 = \frac{1}{T}\int_T p' dt \qquad 16.4$$

The integration time "T" should be sufficiently large compared to the average period of fluctuation.
To characterize the **degree of turbulence**, the square root of the squared of the fluctuation square is compared to the average velocity.

$$Tu = \frac{\sqrt{\overline{(v')^2}}}{\bar{v}} \qquad 16.5$$

Typical values of turbulence degrees are 0.1-20%, turbulent lateral movements are 1-10000 Hz frequencies in high velocity flow.

16.2 Darcy's formula

In the previous chapter, we investigated the solution of the Navier-Stokes equation in circular cross-section pipes. We have come to the conclusion that, in the direction of flow, the pressure decreases. This pressure drop drives the medium against friction on the pipe wall. Pressure reduction can be approached from another side. For the horizontal straight pipe section shown in **Figure 16.4**, apply the Bernoulli equation between points „1” and „2”. The equation will not be valid in the form so far, because the same velocity, at the same height, should have the same pressure in the lossless flow, instead of "2" the pressure is less than "1". In order to restore equality, in the direction of flow, to the right side of the equation, the proportional to the dissipation must be taken into account, and this **is called a pressur friction loss or simlier pressure loss**:

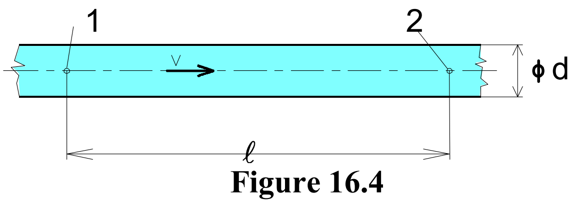


Figure 16.4

$$\rho\frac{\bar{v}_1^2}{2} + p_1 + \rho U_1 = \rho\frac{\bar{v}_2^2}{2} + p_2 + \rho U_2 + \Delta p' \qquad 16.6$$

The *16.6 relationship* is called a **Bernoulli-equation with pressure loss**. (Though Bernoulli had nothing to do with the lossy member).
The pressure loss $\Delta p'$ for straight pipes

$$\Delta p' = \frac{\rho}{2} \cdot \bar{v}^2 \cdot \frac{\ell}{d} \cdot \lambda \qquad 16.7$$

where "λ" is called a dimensionless quantity and a pipe friction coefficient. *Equation 16.7* has come into the literature as a Darcy-Weisbach equation. (***Julius Weisbach, German Professor***, published in 1850 as the first modern hydrodynamics book). In the previous chapter, we investigated the flow of pipes in the Navier-Stokes equation. In the second solution, examining the equilibrium of the circular cylinder from the flow, the pressure drop in the shear stress was given the *relation 8.21*.

$$\Delta p' = \frac{8 \cdot \mu \cdot \bar{v} \cdot \ell}{R^2} \qquad 16.8$$

Instead of the radius of the tube, use the diameter in the expression at so

$$\Delta p' = \frac{32 \cdot \mu \cdot \bar{v} \cdot \ell}{d^2}$$

we get an equation. Let this term equate with *equation 16.7*, then we get:

$$\frac{\rho}{2} \cdot \bar{v}^2 \cdot \frac{\ell}{d} \cdot \lambda = \frac{32 \cdot \mu \cdot \bar{v} \cdot \ell}{d^2}. \qquad 16.9$$

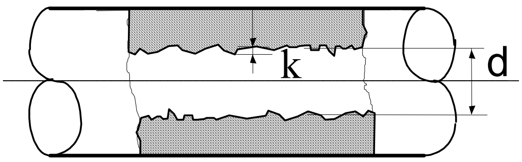
Errange the term to "λ":

$$\lambda = \frac{64}{\frac{\bar{v} \cdot d \cdot \rho}{\mu}} = \frac{64}{Re}. \qquad 16.10$$

As a result, we have found that **in the laminar flow the "λ" friction coefficient changes with the Reynolds number inversely proportionally**. The laminar-turbulent transition value is around $Re \cong 2300$. Therefore, the *relation 16.10* applies to the domain if $\mathbf{Re \le 2300}$. How does the "λ" coefficient depend on the Reynolds number for circular piping and turbulent flow?

16.2.1 Turbulent solution

The laminar solution suggests that in turbulent cases, the relationship between average velocity and pressure loss is determined from the turbulent velocity profile and the friction coefficient "λ" is obtained. Unfortunately, the situation is not that simple, because in turbulent cases, the Navier-Stokes equation requires an instable solution due to the turbulent side movements, and the roughness of the tube also strongly influences the friction coefficient.
Several theories and semipirical theories were born to determine the pipe friction factor.
Let's first get acquainted with the concept of **wall roughness** and the **laminar base layer**.



Rough pipe wall
Figure 16.5

The pipe wall is not smooth because of its manufacture and corrosion, but has a rough surface. By forming the relationship between the average roughness and the inner tube diameter, the relative roughness is obtained, or the reciprocal of its ratio is preferable $\frac{d}{k}$ to its value, or it is likened to the radius of the tube $\frac{r}{k}$. Small values have relatively higher roughness.

In the case of turbulent flow and smooth (hydraulically) tubes, there is a so-called viscous or laminar base near the wall. This thickness is indicated by y_v. The thickness of the base layer and the diameter of the pipe diameter can be determined by the following expression:

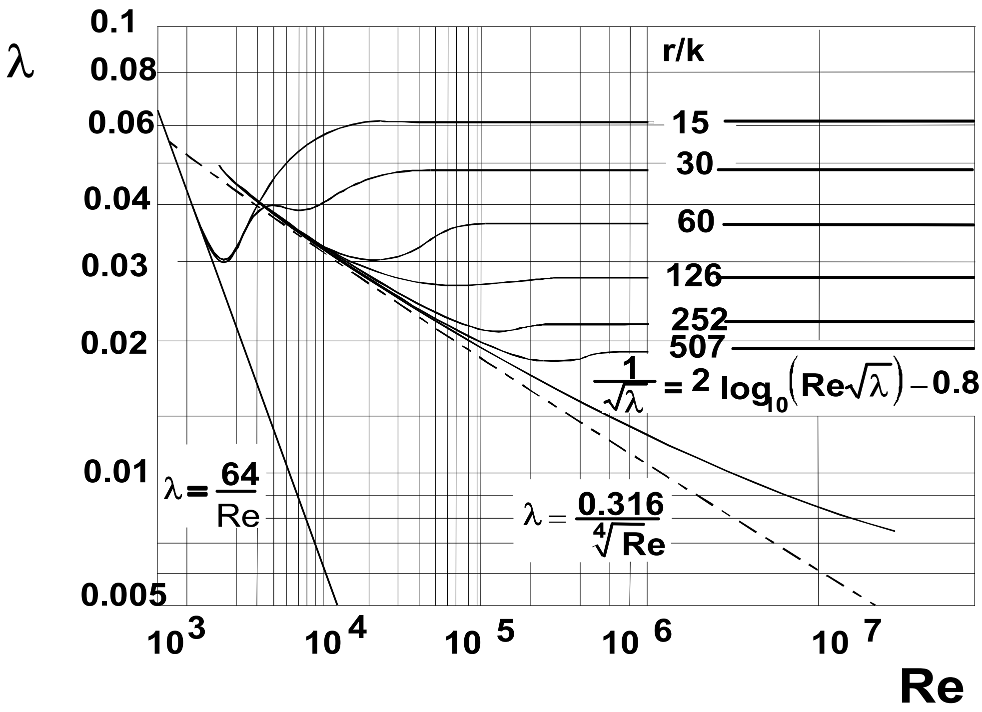
$$\frac{y_v}{d} = \frac{14.1}{Re \cdot \sqrt{\lambda}}, \qquad 16.11$$

which can be deduced from the boundary layer theory. It can be seen that in increasing numbers of Reynolds the thickness of the viscous substrate becomes smaller.

16.2.2 Nikuradse chart

The change of the friction coefficient was carried out by ***Nikuradse***, Prandtl's pupil (***Ludwig Prandtl 1875-1953 German physicist***). The roughness of the wall was made by spreading homogeneous grain sand on the inner tube wall with the glue. Thus he achieved an almost constant roughness inside surface. The results of each of the different Reynolds numbers at a constant relative roughness are summarized in a diagram, which is shown in **Figure 16.6**.

In the case of laminar flow, the roughness has no effect on the friction coefficient. In the turbulent flow (Re> 2300), the effect of roughness is significant: the curves on the Reynolds number rising up to a limit Reynolds number run on the same curve, with a larger Reynolds number they are separated from the curve and are moved horizontally. In this Reynolds number range, "λ" is only a function of $\frac{r}{k}$, it is called a full range of roughness. The curve from which the curves belonging to the different roughness tubes run off are described in the context.



Nikuradse chart
Figure 16.6

$$\frac{1}{\sqrt{\lambda_{turb}}} = 2 \cdot \lg\left(Re \cdot \sqrt{\lambda_{turb}}\right) - 0.8 \qquad 16.12$$

While the "λ" curve of the rough tube runs along the curve of the smooth tube curve *16.12*, the roughness has no effect, and then it is referred to as a hydraulically smooth tube. The tubular friction coefficient of **hydraulically smooth pipes** can be determined by the Reynolds number from the *relation 16.12* or by the **Blasius formula** approximating it in the range $4000 \leq Re \leq 10^5$:

$$\lambda_{turb} = \frac{0.316}{\sqrt[4]{Re}} \qquad 16.13$$

The effect of the roughness on the friction coefficient is explained by the following: If the thickness of the viscous substrate calculated from the *16.11 relationship* covers the roughness of the pipe wall, the roughness has no effect on the pipe friction coefficient, but if the viscous substrate is overturned, it can affect the internal flow. With increasing Reynolds number, an increasing part of roughness emerges from a viscous wall layer, and above a given Re number the smoothing role of the viscous wall layer completely disappears, thus reaching the entire roughness range where the curves are already horizontal lines.

16.2.3 The Moody Diagram

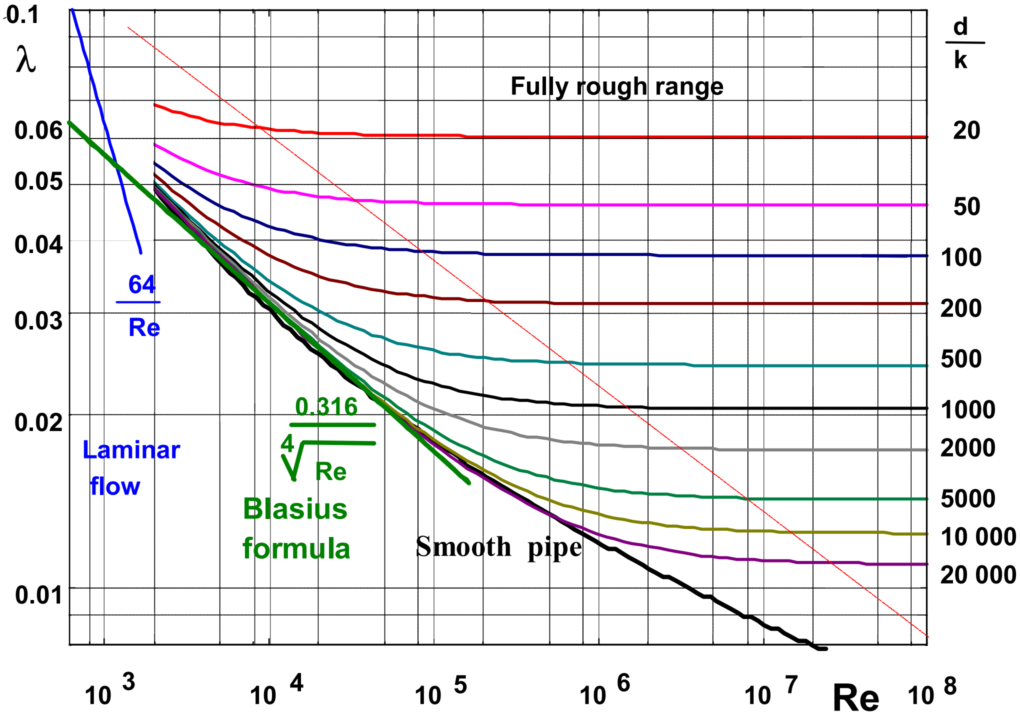
The Nikuradse diagram was made using uniformly sand-blasted tubes. The roughness of the pipes used in the industry is caused by protrusions with non-homogeneous distribution. The roughness at the production is usually dependent on the material and the technology.

The measurements showed that, in the case of general roughness, each tube behaves in the same way as the smooth tube, up to approx $Re = 4000$. Above this, however, its clamping factor suddenly rises, then gradually decreases its total roughness value.

In ***1939, C.F. Coolebrook*** recommended the following formula, which combines the expressions of smooth and rough tubes.

$$\frac{1}{\sqrt{\lambda}} = -2.0 \cdot \log_{10}\left(\frac{1}{3.7 \cdot \frac{d}{k}} + \frac{2.51}{Re \cdot \sqrt{\lambda}}\right) \qquad 16.14$$

In order to avoid the implicit term, ***L. Moody*** made a diagram in 1944, which has since been



Moody-diagram
16.7 ábra

referred to as the Moody diagram and is shown in **Figure 16.7**.
Moody also compiled a table on the basis of different measurements, listing the roughness of ordinary pipes, as shown in **Table 16.1**.
Instead of using the Moody diagram, several approximate terms are also recommended for describing the turbulent region from which the "λ" friction coefficient can be explicitly calculated for a given Re-number and relative roughness. For example, ***Haaland*** recommended the following:

$$\frac{1}{\sqrt{\lambda}} = -1.8 \cdot \log_{10}\left[\frac{6.9}{Re} + \left(\frac{\frac{k}{d}}{3.7}\right)^{1.11}\right] \qquad 16.15$$

According to the author, the difference between the diagram and the formula in the turbulent range is less than 2%.

Table 16.1 Average roughness of the materials

Material	k [mm]
Riveted steel tube	0.9 -16.0
Concrete	0.3 - 3.0
Wood channel	0.18 - 0.9
Cast iron	0.26-0.6
Galvanized Steel	0.1-0.15
Asphalt cast iron	0.1-0.15
Steel, slightly rusty	0.1-0.3
Conventional steel	0.02-0.046
Pulled steel	0.0015-0.03
Glass	Smooth

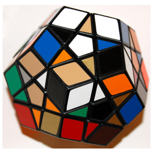
16.3 Three types of pipe flow problem

The Moody Diagram with the approximate formulas in it, almost all pipe flow problems can be solved with only straight pipe.
For many tasks, iteration should be used when using the diagram or related formulas, because the calculation of "λ" requires the relative roughness and the Re number.
(Hereinafter, the average velocity in the pipeline will only be marked with "v".)

There are three basic tasks for calculating the pipe loss:

I. The diameter of the tube is "d", the length "l" and the mean velocity "v" or the volume flow rate "q_v" and the medium density "ρ" and viscosity "ν", and calculate the **pressure drop "Δp" .**
II. The diameter of the pipe "d", the length "l" and the pressure loss Δp ', and the density of the medium "ρ" and the viscosity "ν" are given and calculate the **volume flow rate "q_v".**
III. The pipe length "l", the pressure loss Δp ', the flow rate "q_v" and the density of the medium "ρ" and its viscosity "ν" are given and calculate the **diameter of the pipe "d".**

16.3.1 Task I: Calculate the pressure drop!



Calculate the pressure loss in an asphalted cast iron pipe in which water flows.

Data: $\ell = 60\text{m}$; $d = 150\,\text{mm}$; $\bar{v} = 0.15\frac{\text{m}}{\text{s}}$

Qestions: Δp' pressure drop

Solution: First, the density and dynamic viscosity of the water should be determined from the table. Its density $\rho = 10^3\frac{\text{kg}}{\text{m}^3}$. Viscosity can be derived from Table 8.1:

$$\nu = 1.3 \cdot 10^{-6}\frac{\text{m}^2}{\text{s}}.$$

In the next step, the Re number is calculated according to *equation 16.1*:

$$\text{Re} = \frac{v \cdot d \cdot \rho}{\mu} = \frac{v \cdot d}{\nu} = \frac{0.15 \cdot 0.15}{1.3 \cdot 10^{-6}} = 1.73 \cdot 10^4$$

The value of roughness taken from **Table 16.1**, eg. value and the relative roughness of the data:

$$\frac{d}{k} = \frac{0.15}{0.00012} = 1250$$

Look for the line in the Moody Diagram $\frac{d}{k} = 1250$ and follow it until we close the vertical $Re = 1.73 \cdot 10^4$. We can read that, or instead of the diagram, we can use the *term 16.15*, which states that:

$$\frac{1}{\sqrt{\lambda}} = -1.8 \cdot \log_{10}\left[\frac{6.9}{Re} + \left(\frac{\frac{k}{d}}{3.7}\right)^{1.11}\right] = -1.8 \cdot \log_{10}\left[\frac{6.9}{17300} + \left(\frac{\frac{1}{1250}}{3.7}\right)^{1.11}\right] = 5.966,$$

from which

$$\lambda = 0.028 .$$

But even then, we can not make a big mistake when calculating the Blasius formula for the smooth pipe formula, calculating the friction loss coefficience.

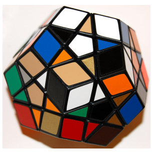
$$\lambda = \frac{0.316}{\sqrt[4]{Re}} = \frac{0.316}{\sqrt[4]{17300}} = 0.0275$$

Finally, the pressure drop is calculated from the *relation 16.7*:

$$\Delta p© = \frac{\rho}{2} v^2 \cdot \frac{\ell}{d} \cdot \lambda = \frac{1000}{2} \cdot 0.15^2 \cdot \frac{60}{0.15} \cdot 0.028 = 126\,Pa$$

Moody pointed out that the calculation does not contain a bigger mistake than about ± 10 percent.

16.3.2 II. task: Find the average speed!



Since the velocity (or flow) will appear in both the "λ" in both the number is issued, iteration can only solve the problem. Fortunately iteration is very fast, because "λ" slowly changes with the Re number. Calculate the average speed in the asphalt cast iron line that supplies water!

Data: $\ell = 60m$; $d = 150mm$; $\nu = 1.3 \cdot 10^{-6}\,\frac{m^2}{s}$; $\Delta p' = 5000\,Pa$

Question: Volume flow rate q_v

From **Table 16.1**, look for the roughness of the particular pipe material and determine the relative roughness that corresponds to the data of the previous problem, so, but we do not know the value of the Re-number because the average velocity is unknown. So we have to take a starting value for λ. Let's start with $\lambda_o = 0.02 - 0.03$, or the full friction coefficient corresponding to the roughness, which is in this case. Writing *equation 16.7* from which we can arrange the average velocity. This is the following:

16.16

$$v = \sqrt{\frac{2 \cdot \Delta p' \cdot d}{\rho \cdot \ell}} \cdot \sqrt{\frac{1}{\lambda}}$$

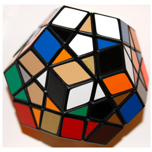
Of course, "λ" remains in the expression.

Table 16.2

λ	$v = \sqrt{\frac{2 \cdot \Delta p' \cdot d}{\rho \cdot \ell}} \cdot \sqrt{\frac{1}{\lambda}}$	Re_d
0.02	0.353	40730
0.026	**0.31**	35770

The steps of the iteration are given in **Table 16.2**, using the Moody Diagram. The calculation converges very quickly. Of course, if we add a different starting value "λ", convergence will be quick and the end result will not change, only the number of iteration steps will increase or decrease.

16.3.3 III. Task: Calculate the pipe diameter!



Now the pipe diameter is the question. Since both the "λ", the Re number and the relative roughness are dependent on the diameter, we can only solve the problem with iteration again. Fortunately, iteration is also very fast in this case. Express the velocity $v = \frac{4 \cdot q_V}{d^2 \pi}$ with the volume flow rate and the diameter from the continuity, and then put this into $\Delta p' = \frac{\rho}{2} v^2 \cdot \frac{\ell}{d} \cdot \lambda$ equation. The following is obtained:

$$\Delta p' = \frac{\rho}{2} \left(\frac{4 \cdot q_V}{d^2 \pi} \right)^2 \cdot \frac{\ell}{d} \cdot \lambda = \frac{C}{d^5} . \qquad 16.17$$

Then the diameter is expressed, which, of course, depends on the pipe friction coefficient:

$$d = \sqrt[5]{\frac{\rho}{2} \left(\frac{4 \cdot q_V}{\pi} \right)^2 \cdot \frac{\ell}{\Delta p'}} \cdot \sqrt[5]{\lambda} \qquad 16.18$$

If we could have the value "λ" then we could calculate the diameter. The calculation is given in the following task. Calculate the diameter of the asphalted steel pipe that supplies water!

Data: $\ell = 60\,m$; $\nu = 1.3 \cdot 10^{-6} \frac{m^2}{s}$; $\Delta p' = 12000\,Pa$, $q_V = 5.37 \cdot 10^{-3} \frac{m^3}{s}$

Question: *diameter „d"*

We do not know the relative roughness or the Re-number. Because of the diameter, both the λ "and the" Re "number, as well as the relative roughness, we can solve the problem again with iteration. To solve a problem, a standard commercially available pipe diameter, eg from **Table16.3.** When selecting a pipe diameter, for example, water system is designed, the water velocity should not exceed 1-2 m /s. The diameter of the selected tube diameter is 4 in (colos) d=102.3mm.

Then the problem resembles the 1st pipe problem. First we calculate the flow rate.

$$v = \frac{q_v}{d^2 \cdot \pi / 4} = \frac{5{,}37 \cdot 10^{-3}}{0{,}1023^2 \cdot \pi / 4} = 3{,}99 \frac{m}{s}$$

Apparently, the first pipe diameter chosen was small because velocity was too high. We choose a larger one, 6 in (colos) d = 154.1 mm.

$$v = \frac{q_v}{d^2 \cdot \pi / 4} = \frac{5{,}37 \cdot 10^{-3}}{0{,}1541^2 \cdot \pi / 4} = 1{,}76 \frac{m}{s}$$

Here's the velocity in a good range, so we can count further, followed by the Re-number.

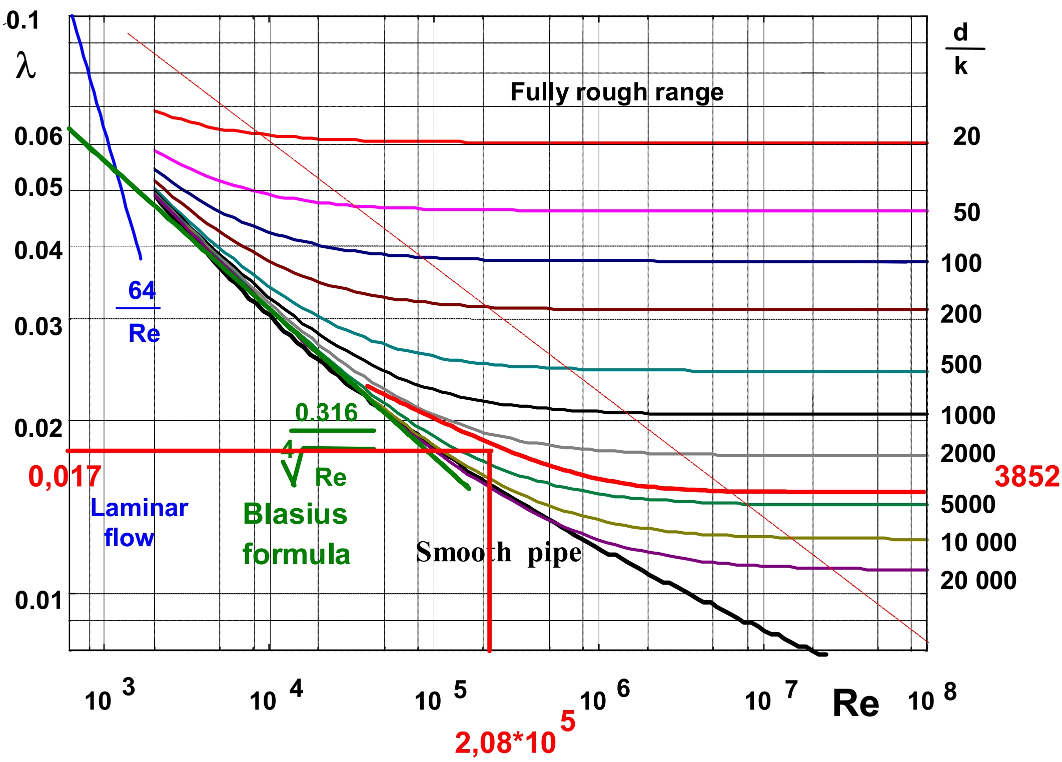
$$Re = \frac{v \cdot d \cdot \rho}{\mu} = \frac{1{,}76 \cdot 0{,}1541 \cdot 1000}{1{,}3 \cdot 10^{-3}} = 208764 = 2{,}08 \cdot 10^5$$

Table 16.3 Manufactured pipes

Nominal diameter [in]	Actual internal diameter [mm]
2	52.5
$2\frac{1}{2}$	62.7
3	77.9
$3\frac{1}{2}$	90.1
4	102.3
5	128.2
6	**154.1**
8	202.7
10	254.5

The roughness should be in the case of steel tubes $k = 0{,}04\,mm$. The roughness ratio

$$\frac{d}{k} = \frac{0{,}1541}{0{,}00004} = 3852$$



Select the friction coefficient from the Moody diagram

Figure 16.8

Calculate the pressure loss with the specified values.

$$\Delta p'_{sz} = \frac{\rho}{2} v^2 \cdot \frac{\ell}{d} \cdot \lambda = \frac{1000}{2} \cdot 1{,}76^2 \cdot \frac{60}{0{,}1541} \cdot 0{,}017 = 10251\,\text{Pa}$$

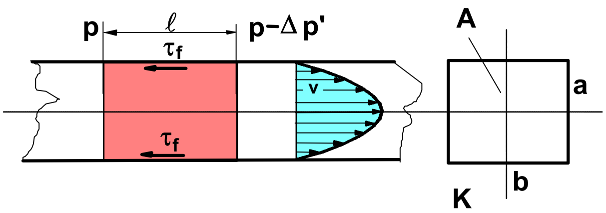
If the calculated loss of pressure is greater than the specified pressure loss, then a larger pipe diameter should be selected and the task recalculated. In this solution, this is the case, so we have to choose another bigger diameter for the good solution. I leave this solution to the reader.

If the calculated loss of pressure is smaller then our calculation is correct. But if it is much smaller than the specified one, we have to choose a smaller pipe diameter because the tube will be unnecessarily big and its price!

Table 16.3 shows the available tube diameter series. From the *16.17*, we know that the pressure loss is inversely proportional to the fifth power of the diameter. Thus, if the pipe diameter decreases by 10%, the pressure loss will increase by about 50%. Therefore, in the case of the diameter obtained from the calculation, a larger pipe diameter must be chosen. From the flow point of view, the larger the diameter choice is appropriate, but the larger the pipe diameter, the more expensive the wire. So just choose a diameter larger than one step. **A good choice is the 8 in nominal, 202.7 mm actual internal diameter.** In spite of the worldwide trend, despite the SI (International Quality System), the nominal diameter of pipelines is given in inch (in coll $1\,\text{col} = 2.54\text{cm}$).

16.4 Pressure loss in non-circular pipes

Non-circular pipelines are often used, eg. in building engineering structures. In the formed pipe flow, the calculation of loss is a much more complicated task for non-circular pipes than in laminar circular pipes. Laminar flow can be used in the Navier-Stokes equation but only in a three-dimensional version. In turbulent cases, the boundary layer theory can also be based here. However, for practice, a much simpler and more useful concept, the introduction of an **equivalent diameter** (or hydraulic radius), allows for simple loss calculation.



Non-circular cross-section
Figure 16.9

Let us examine a "λ" piece of rectangular cross-section of wire as shown in **Figure 16.9**. The force generated from the sliding force and the force from the pressure drop on the wall are in balance, as the desired liquid part does not accelerate. Write the equilibrium of the two forces.

$$\Delta p' \cdot A = \tau_f \cdot K \cdot \ell \,, \qquad 16.19$$

where "K" is the so-called wetted circumference (there may also be a flow so that the liquid does not fully fill the cross-section, e.g. in open-surface channels) "A" is the liquid filled cross section.
Of this, the pressure loss is:

$$\Delta p' = \tau_f \cdot \frac{\ell}{A/K} \qquad 16.20$$

Consider a circular cross-section pipe with the same pressure drop at the same length and the same wall shear stress. Find the diameter of this circle, which is called the equivalent diameter.

$$\Delta p' \cdot \frac{d_e^2 \cdot \pi}{4} = \tau_f \cdot d_e \cdot \pi \cdot \ell \qquad 16.21$$

Express the pressure drop here:

$$\Delta p' = \tau_f \cdot \frac{\ell}{\frac{d_e}{4}} \qquad 16.22$$

Comparison of *equations 16.20* and *16.22* results from:

$$\underline{d_e = \frac{4 \cdot A}{K}} \qquad 16.23$$

The pipe friction losses are also calculated with respect to the circular cross-section pipes *16.7*, with the exception that the diameter "d" is replaced by "d_e" the equivalent diameter:

$$\Delta p' = \frac{\rho}{2} v^2 \frac{\ell}{d_e} \lambda \quad \textbf{, ahol} \quad \lambda = \lambda\left(\text{Re}, \frac{k}{d_e}\right) \textbf{és} \quad \text{Re} = \frac{v d_e}{\nu} \qquad 16.24$$

In the above contexts, "v" the average velocity calculated with the actual pipe cross section. In the knowledge of the Reynolds number, the quench factor "" can be determined from the Moody diagram depending on the "Re" value and the roughness. Equivalent diameter yields acceptable precision results only in the case $\frac{a}{b} > \frac{1}{3}$ ahol $a < b$ of a rectangular rectangle and turbulent flow. A more accurate result is obtained if the pipe is rectangular and has a side-to-side relationship $\frac{a}{b} < 0.5$: then, for the calculation of "λ", the Reynolds number is determined by the Reynolds number, as follows:

$$\Phi \cong \frac{2}{3} + \frac{11}{24}\frac{a}{b}\left(2 - \frac{a}{b}\right)$$

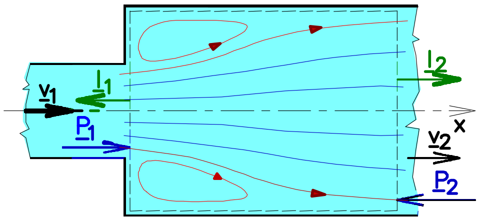
16.5 Calculation of loss of pipe fittings

Of course, wire systems include not only straight pipes, but also arches, valves, reducers, and joints. Consider the most important loss sources. The pressure loss is usually given by a loss factor which is the ratio between the pressure loss and the dynamic pressure, in most cases the ratio of the dynamic pressure before the assembly, i.e.

$$\zeta = \frac{\Delta p'}{\frac{\rho}{2} \cdot v^2} \qquad 16.25$$

16.5.1 Borda-Carnot transition loss

Consider the sudden cross-sectional growth shown in **Figure 16.10**, which has been introduced into the flow literature by **Borda-Carnot enlargement**. (***Jean Charles Borda 1733-1799*** French physicist, member of the French Academy, participated in the formation of a metric system of measurements, ***Nikolas Leonard Sadi Carnot, French physicist 1796-1833***, the creator of the second term of thermodynamics, independently of the problem.) The streamlines show a current picture close to the actual flow. Let the flow be steady and the density of the flow medium is constant.



Borda-Carnot enlargement

Figure 16.10

Between cross sections "1" and "2" there is a loss of flow, so the Bernoulli equation can not be used without a loss member. The momentum equation, on the other hand, gives a very good result to reality. Using the assumption that pressure **on the left side of the control surface is everywhere** p_1**.**

$$-\rho \cdot v_1^2 \cdot A_1 + \rho \cdot v_2^2 \cdot A_2 = (p_1 - p_2) \cdot A_2$$

Using continuity: $v_1 \cdot A_1 = v_2 \cdot A_2$;

$$(p_2 - p_1)_{val.} = \rho \cdot v_2 \cdot (v_1 - v_2)$$

So there is more pressure at "2". However, in the direction of flow there is a loss of pressure, because according to the Bernoulli equation:

$$(p_2 - p_1)_{id} = \frac{\rho}{2} \cdot \left(v_1^2 - v_2^2\right)$$

The difference between the two is the Borda-Carnot loss:

$$\Delta p'_{BC} = (p_2 - p_1)_{id.} - (p_2 - p_1)_{val} = \frac{\rho}{2} \cdot (v_1 - v_2)^2 \qquad 16.26$$

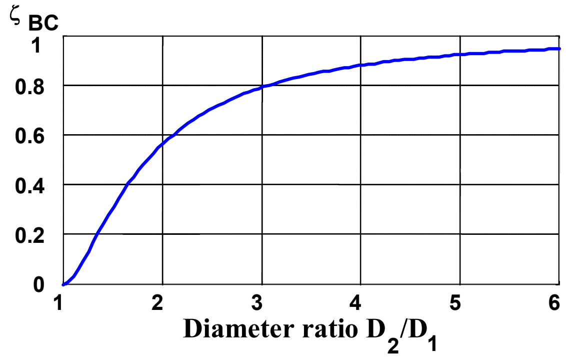
The loss factor is obtained after dividing by dynamic pressure entering, so:

$$\zeta_{BC} = \frac{\Delta p'_{BC}}{\frac{\rho}{2} \cdot v_1^2} = \frac{\frac{\rho}{2} \cdot (v_1 - v_2)^2}{\frac{\rho}{2} \cdot v_1^2} = \left(1 - \frac{v_2}{v_1}\right)^2$$

If we use continuity $A_1 \cdot v_1 = A_2 \cdot v_2$ and assume the connection of circular pipes then

$$\varsigma_{BC} = \left(1 - \frac{A_1}{A_2}\right)^2 = \left[1 - \left(\frac{D_1}{D_2}\right)^2\right]^2 \qquad 16.27$$

expression is obtained. **Figure 16.11** depicts the loss factor as a function of the diameter relationship.



Borda-Carnot transition loss factor
Figure 16.11

One of the special cases of Borda-Carnot loss is when the cross-section "2" is infinitely large, that is, a pipe connects to an infinite large space. This loss is called a loss loss. It is derived from *the expression 16.26* that:

$$\Delta p'_{ki} = \frac{\rho}{2} \cdot v_1^2$$

The loss factor is apparently "1".

16.5.2 Entry loss

From a relatively large container into a small pipeline with a small cross-section, a disintegration occurs at the entrance. Immediately after the entry, the flow passes through a small cross section of the pipe cross section (see **Figure 16.11**) this cross section is called a contralateral cross section or "vena contracta" (A '). The medium can not follow the sudden change of direction of the solid wall. After joining, it narrows to the cross section A ', which is smaller than the pipe cross section A_{cs}. The flow of liquid flowing through the narrowest cross section is arranged after a certain length and fills the entire cross section of the tube, but with the loss of Borda-Carnot loss.

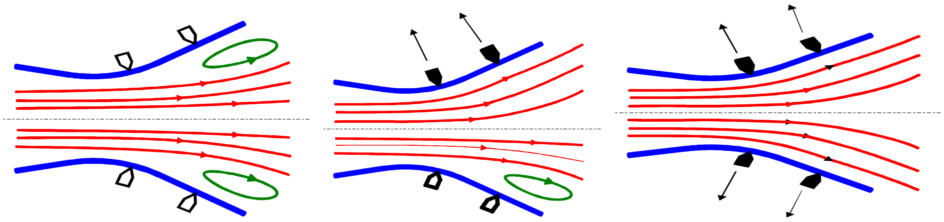
The more the tube enters the container (the larger "b") and the thinner the pipe wall (s), the greater the contraction rate and thus the loss. Obviously, the maximum loss factor can be "1". This is called Borda Spillage. Entry loss factor **Figure 16.12**. The variation of the loss factor can be seen in **Figure 16.12**, depending on the different parameters.

The loss factor, unlike the previous one, does not apply to the speed at the pre-piece speed, but rather to the velocity from the entrance to the pipe, so:

$$\zeta_{be} = \frac{\Delta p'}{\frac{\rho}{2} \cdot v_{cs}^2}$$

16.5.3 Diffuser

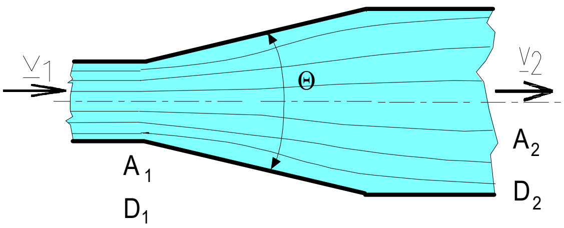
In the direction of flow, expansion pipes are called diffusers. With the loss of the Borda-Carnot transition, the flow rate can be reduced by means of a diffuser. In the diffuser, liquid particles must flow in the direction of pressure increase. The work required for this is covered by the reduction of their movement energy. According to the Bernoulli equation, this decrease is exactly the same as the increase in pressure. In reality, some of the motion energy is devoted to work against friction. Friction is particularly felt along the pipe wall. The moving energy of the particles going above, especially in the case of suddenly expanding diffusers, is insufficient to cover the work that the pressure increase in the medium flowing in the middle of the tube would require. Therefore, particles flowing through the wall stop and even turn back. At this time, a heavily bulging layer is created next to the wall. The sloping layers do not follow the expanding direction of the pipe wall and are separated from it. This phenomenon is called **separated bubble**, the loss of pressure caused by it by the loss of separation.



Different flow pattern in diffuser
16.13 ábra

On the left-hand side of **Figure 16.13**, a completely ineffective diffuser, which is offset on one side of the center and a well-functioning state on the right-hand edge.
There are many ways of eliminating separation, the simplest is to reduce the angle of the diffuser, in a circular symmetric case the optimum angle is approximately 8°. The lower angle diffuser is very long and therefore the friction loss exacerbates the recoverable pressure.
Other ways of eliminating separation include: high-speed fluid flow to the wall, or the "tired" wall layer is absorbed, the latter being used as shown.

Due to the separation and friction, the pressure does not increase to such an extent in the diffuser as it would do according to the Bernoulli equation. So the pressure is increasing in the diffuser, but not as much as it would ideally grow.



Diffuser
Figure 16.14

How can we determine the goodness of a diffuser?
We define a diffuser efficiency, which has achieved the pressure increase in real case $(p_2 - p_1)_{real}$, relative to the ideal increase in pressure $(p_2 - p_1)_{id.}$ from the Bernoulli equation. This fraction is called diffuser efficiency.
By describing the Bernoulli equation between cross sections "1" and "2", we get:

$$(p_2 - p_1)_{id.} = \frac{\rho}{2} \cdot \left(v_1^2 - v_2^2\right)$$

So the diffuser efficiency is:

$$\eta_{diff.} = \frac{(p_2 - p_1)_{real}}{\frac{\rho}{2} \cdot \left(v_1^2 - v_2^2\right)} \qquad 16.28$$

Another characteristic that differs from the diffuser is the loss factor of the diffuser, which can be defined by the following expression:

$$\zeta_{diff.} = \frac{\Delta p'_{diff}}{\frac{\rho}{2} \cdot v_1^2} = \frac{(p_2 - p_1)_{id.} + (p_2 - p_1)_{real}}{\frac{\rho}{2} \cdot v_1^2} \qquad 16.29$$

The pressure loss in the diffuser is related to the inlet dynamic pressure.
Of course there is a close relationship between efficiency and loss factor, which is as follows:

16.30

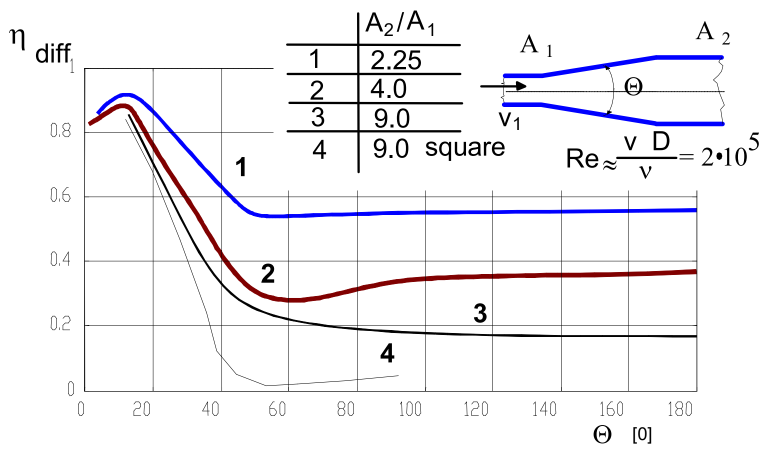
$$\zeta_{diff.}=\frac{\frac{\rho}{2}\cdot\left(v_1^2-v_2^2\right)-\eta_{diff}\cdot\frac{\rho}{2}\cdot\left(v_1^2-v_2^2\right)}{\frac{\rho}{2}\cdot v_1^2}=\left(1-\eta_{diff}\right)\cdot\left[1-\left(\frac{v_2}{v_1}\right)^2\right]$$

$$\zeta_{diff.}=\left(1-\eta_{diff.}\right)\cdot\left[1-\left(\frac{A_1}{A_2}\right)^2\right]$$

In the second part of the term, we applied the continuity according to which

$$v_1\cdot A_1=v_2\cdot A_2$$

Apparently the best efficiency is 5-10°, then above a certain angle the efficiency is independent of the angle of aperture, only depends on the cross-sectional relationship. The constant efficiency obtained here equals the diffuser efficiency of the Borda-Carnot transition. Obviously, in cross-section 2.25, this efficiency is approx. 55%, which does not even matter to a diffuser. In the case of small cross-sectional growth, it is not worth making a diffuser, because the sudden cross-sectional growth works as a suitable diffuser.



Efficiency of diffuser

Forrás: Varga J.: Hidraulikus és pneumatikus gépek

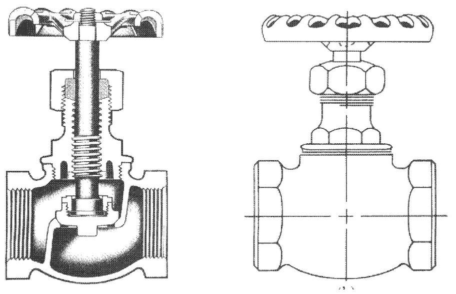
Figure 16.15

The diffuser flow is dealt with in many researches because most of the flow machines are of great importance to the flow in the direction of flow. For example, in the radial fan described in **section 13**, a diffuser flow is also generated in the impeller and the spiral haus. **Figure 16.15** shows the change in the efficiency of the diffuser as a function of the aperture angle and the cross section.

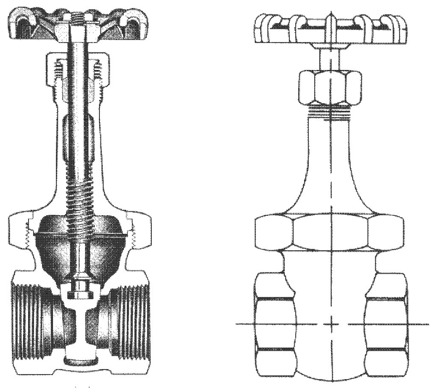
16.5.4 Valves, pins, gate valve

The loss of valves, latches and gate valves is also largely attributable to the loss of Borda-Carnot. Their moving elements narrow the flow cross section, resulting in sudden cross-sectional growth after the narrowed cross section. The flow losses of the valves and gate valves are also characterized by a loss factor ζ_{sz}, which is the ratio between the loss $\Delta p'_{sz}$ and the dynamic pressure at a typical velocity, usually at pre-assembly speeds.

$$\zeta_{sz}=\frac{\Delta p'_{sz}}{\frac{\rho}{2}\cdot v^2} \qquad 16.31$$

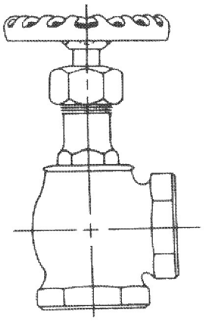


Directional break valve
Figure 16.16

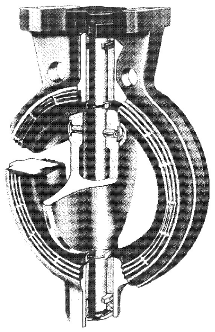


Gate valve
Figure 16.17

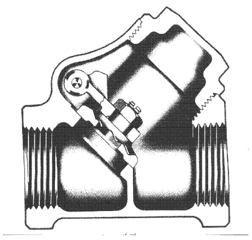
Figures 16.15 and **16.21** show different types of fittings. The loss factor depends to a large extent on the make and wear of the assembly. The mean values of the loss factor are given in **Table 16.4**, but there may be large variations depending on the product.



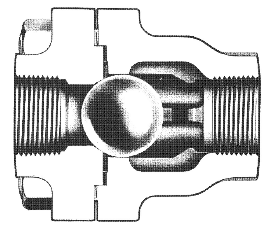
Angle valve
Figure 16.18



Butterfly valve
Figure 16.19



Non-return valve
Figure 16.20



Ball check valve
Figure 16.21

The table shows the equivalent pipe length. For straight pipes, the term can also be understood as a loss factor. The argument is also true: for a loss factor, "ζ" can also be given to what length of straight pipe it would correspond.
So

$$\zeta = \frac{\ell_e}{d} \cdot \lambda \qquad 16.32$$

from which the equivalent pipe length or the relative pipe length relative to the pipe diameter can be specified. The only problem is that if the loss factor is to be determined from the relative pipe length, it seems that the valve's loss factor depends on the pipe friction coefficient of the pipe into which the valve is installed. This obviously is not true.
The table also assumes an average friction coefficient $\lambda = 0.02$ and is equivalent pipe length is related to this.

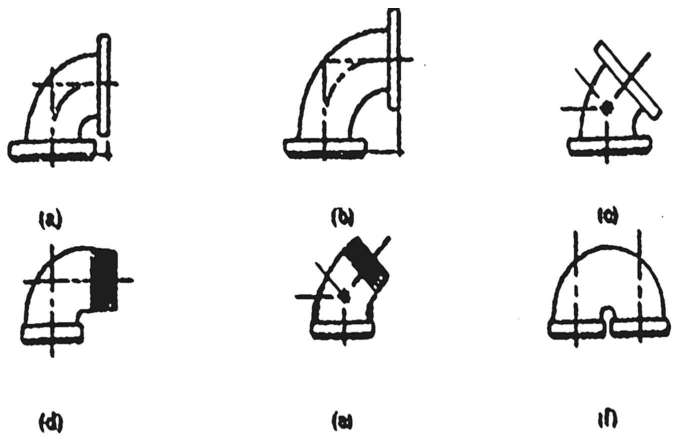
According to the experience, a given type of pipeline is connected to a given assembly, so that the value of "λ" can vary quite narrowly, so the introduction of the equivalent pipe length is a good approximation under these conditions.

Valves loss factors and equivalent length
Table 16.4

Type	**loss factor ζ**	**equivalent pipe length** ℓ_e/d
Directional break valve open	6.8	340
Angle valve fully open	2.9	145
Gate valve open	0.26	13
1/4 open	18	900
1/2 open	3.2	160
3/4 open a	0.7	35
Check valve	2.7	135
Ball check valve	3	150
Butterfly valve open	0.3	15

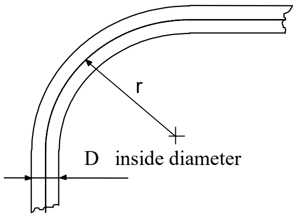
16.5.5 Elbows, arces

Fluctuations in the flow of fluids in the tubes, tubes and elbows can result in significant flow



Pipe elbows
Figure 16.22

losses, which are also characterized by a loss factor. Among the causes of the losses, the wall shear stress usually plays a subordinate role. Losses due to secondary flows and disconnections are of greater importance.



Elbow
Figure 16.23

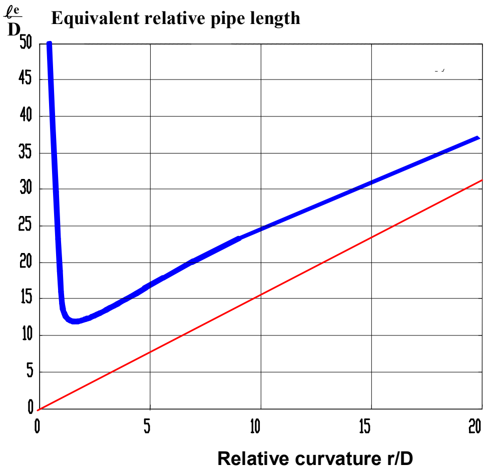
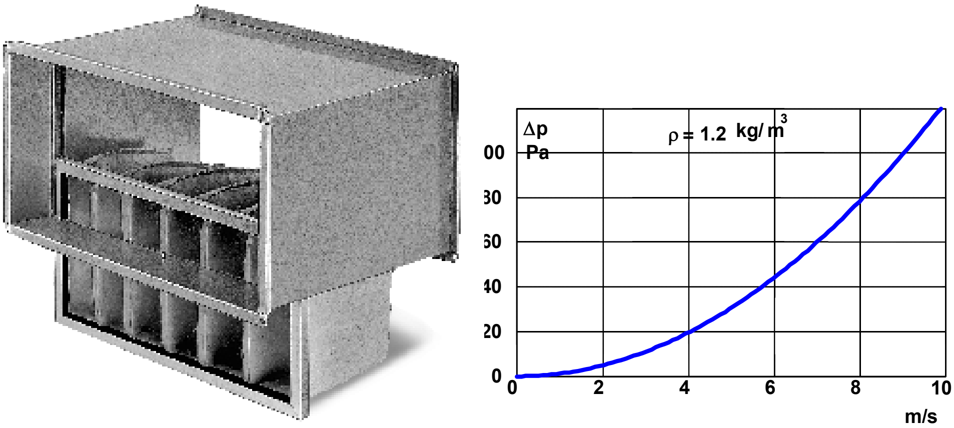


Figure 16.24

The loss of the arches and elbows can be reduced by the rounding of the transitions and by increasing the relative curvature "r/d", or by applying baffles with which curves of greater relative curvature or you can create part elbows.
Figure 16.22 shows preformed arches and elbows. These are manufactured in threaded and flanged versions for standardized straight pipes.
But not just prefabricated arches can be used, but pipe assemblies can also be used to make pipes on site. **Figure 16.24** shows the equivalent pipe length of a 90° elbow relative to the relative curvature (r/D). The equivalent pipe length includes the loss of direction change plus the friction loss along the length of the pipe. It can be seen that the loss has a minimum of approx. 1.5-2 r/D.
The figure also shows the length of the straight pipe of the same length as the arc. For very long arches, the two functions are approaching each other.

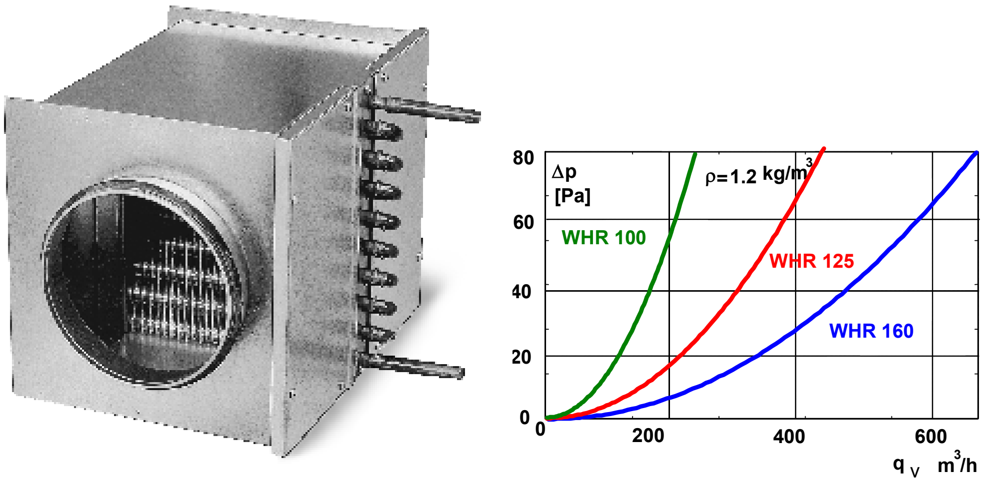
16.5.6 Air-conditioning equipment

In the previous subsection we saw mainly fittings used in plumbing systems. Without the necessity of completeness, we also outline some air-technology equipment.
The air vents are relatively well-known, since these are the elements of the ventilation systems that can be seen in the ceiling and ceiling of the ventilated room. In the ventilation technique mainly rectangular wires are used for space utilization and simpler manufacturing and assembly. **Figure 16.25** shows an air filter (in semi-disassembled condition).
The air is fed through filter cartridges in the "V" shape. The main advantage of such an arrangement is that the air velocity through the filter cloth is small due to the relatively large surface, so that the resulting air resistance of the filter is much smaller than if the cross section has been provided with a single flat filter. The air resistance of the filter could be given with a "ζ" factor (it can be calculated from the curve $\zeta = 2$, the control is given to the reader as a useful exercise at $\rho = 1.2\frac{kg}{m^3}$ medium density), but to facilitate the user's handling, the air resistance is determined by the average velocity of the pipe cross section. The diagram also designates the velocity range in which it is recommended to use the equipment. In ventilation systems, they do not usually go over $10\ \frac{m}{s}$ air velocity. Another frequently used element is the air heater. It is shown in **Figure 16.26**.
The heat exchanger is heated with $80°C/60°C$ water. (The first number is the incoming and the second the output of the water.) The air and water resistance are also given in the catalog, only air resistance is shown on the WHR 100, WHR 125, WHR 160 models. The number after the letter is the number of connecting pipe diameters in mm. The catalog gives zhe pressure loss depending on the airflow volume. (Of course, there is a loss factor here, although here the density of air at the flow through the heat exchanger can be significantly different due to the heating, so it can be misleading in the calculation of the loss factor.)



Air filter Figure 16.25

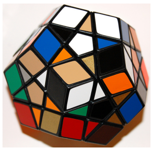
We have reviewed some of the most important sources of loss above. The loss factors of other elements (eg pipe joints, shutter curlers, etc.) or direct loss of pressure can be extracted from technical books, catalogs, product information sheets.
The " ζ " loss factors and the " λ " friction coefficient are calculated or are out of range, and the values taken out in the actual applications are only approximate. measured values do not take into account the effect of friction losses on each other. For direct flow into a straight pipe, for example. the formed pipe flow occurs only at 5-20d from the entrance, the loss due to the rearrangement of the flow is slightly larger than the flow of pipe formed. It is also not indifferent to the value of the loss factor of an elbow, that a straight pipe or another elbow is placed in front of it. In the latter case, the value of the loss factor of the second tube " ζ " can be significantly affected by the fact that the two arches are "U", "S", or twisted in space. In one system, the loss calculation must be performed by a security factor, apply.



Air heater unit and air resistance curve
Figure 16.26

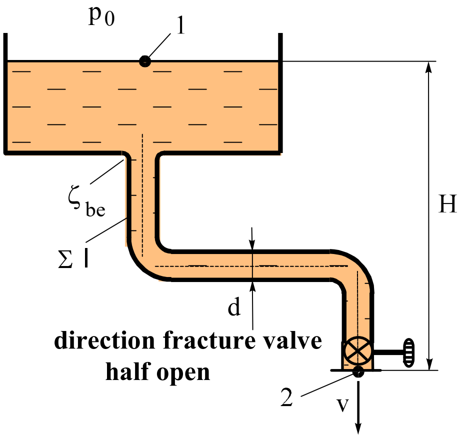
17. Calculation of piping systems

In **Chapter 16.3**, three typical cases of pipe flow were discussed on a straight pipe. The three basic tasks can be extended to pipeline systems where straight pipes also include fittings and pipe joints in the system.



17.1 Oiling line

The lubricating device outlined in the figure flows with oil viscosity "ν" with stationary flow.



Oiling line
Figure 10.1

Data: $\Sigma\ell = 2\,\text{m}$ ***;*** *nominal pipe diameter;*

$NA = \frac{1}{4}"$; $H = 1.5\,\text{m}$; $\xi_{be} = 1.2$ **;** $\nu = 2\cdot 10^{-4}\frac{\text{m}^2}{\text{s}}$

Questions:

a./ Calculate the oil flow velocity!
During the calculation, the tube can be considered straight.
b./ What is the error in calculating if we only consider the loss of straight pipe?

Solution:

The **II. type** problem group when the differential pressure is given and the flow rate or average velocity is determined. Instead of the iteration applied there (**chapter 16.3.2**), we can give a direct calculation of the velocity, as the flow will be laminar.
If friction has a decisive role in flow formation, then the *16.6 loss-making Bernoulli equation* should be used when solving the problem.
a./ Write between "1" and "2":

$$p_1 + \frac{\rho}{2}v_1^2 + \rho g z_1 = p_2 + \frac{\rho}{2}v_2^2 + \rho g z_2 + \Sigma\Delta p' \qquad 17.1$$

The pressure loss member, members must be added to the side of the flow direction, in this case this is the "2" point. In the given task we know that:

$$p_1 = p_2 = p_0,\ v_1 = 0, \qquad v_2 = v, \qquad z_2 = 0, \qquad z_1 = H$$

Using these, the Bernoulli equation with losses is:

$$\rho\cdot g\cdot H = \frac{\rho}{2}v^2 + \Sigma\Delta p' \qquad 17.2$$

The pressure loss consists, in this case, of the loss of input during the formation of the velocity profile and the loss of pipe friction caused by the wall shear stress and the pressure drop on the valve:

$$\Sigma\Delta p' = \left(\zeta_{be} + \zeta_{sz} + \frac{\Sigma\ell}{d}\lambda\right)\frac{\rho}{2}v^2$$

Given the high viscosity of the oil and the small diameter of the tube, it can be assumed that the flow will be laminar. At the end of the calculation we have to verify the correctness of this assumption by determining the Reynolds number. In the case of laminar flow, the friction coefficient can be calculated by the relation.

$$\lambda = \frac{64}{Re},$$

As the pipe has a nominal diameter, it is necessary to look for the actual internal diameter from **table 17.1** (at the end of this chapter):

$$d = 9.2\,\text{mm}$$

Taking all these into account, *equation 17.2* can be written as follows:

$$\rho \cdot g \cdot H = \frac{\rho}{2} \cdot v^2 \cdot \left(1 + \zeta_{be} + \zeta_{sz} + \frac{\Sigma\ell}{d}\frac{64 \cdot \nu}{v \cdot d}\right)$$

We get a quadratic equation for velocity, which is:

$$\left(1 + \zeta_{be} + \zeta_{sz}\right) \cdot v^2 + \frac{64 \cdot \nu \cdot \Sigma\ell}{d^2} v - 2 \cdot g \cdot H = 0 \qquad 17.3$$

There is no data for the valve's loss factor, so we can get approximate loss factors from **Table 9.5**

$$\zeta_{sz} = 6.8.$$

Second degree equation:

$$\left(1 + 0.2 + 6.8\right) \cdot v^2 + \frac{64 \cdot 2 \cdot 10^{-4} \cdot 2}{0.0092^2} \cdot v - 2 \cdot 9.81 \cdot 1.5 = 0$$

$$9 \cdot v^2 + 302.45 \cdot v - 29.43 = 0$$

$$v_{12} = \frac{-302.45 \pm \sqrt{302.45^2 + 1059.48}}{18}$$

by solving the velocity (of course, only a positive solution can be obtained):

$$\underline{v = 0.0970 \frac{m}{s}}$$

Check the Reynolds number:

$$Re = \frac{v \cdot d}{\nu} = \frac{0.0970 \cdot 0.0092}{2 \cdot 10^{-4}} = 4.46$$

so really the laminar flow, because it is much smaller than 2300.

b./ If only the friction loss in the straight pipe is taken into account, then in *equation 17.3* the square member can be omitted so that the equation to be solved

$$\frac{64 \cdot \nu \cdot \Sigma\ell}{d^2} v - 2 \cdot g \cdot H = 0$$

Replacing the data and expressing the velocity of the

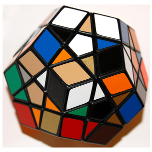
$$v = \frac{2 \cdot 9.81 \cdot 1.5 \cdot 0.0092^2}{64 \cdot 2 \cdot 10^{-4} \cdot 2} = 0.0973 \frac{m}{s}$$

we get results. Obviously, the difference between the two results is negligible. This means that the straight pipe loss is much greater than the loss of input and the valve loss. The friction coefficient and the pipe loss factor are quantified as follows:

$$\lambda = \frac{64}{Re} = \frac{64}{4.46} = 14.34 \qquad \text{and} \qquad \frac{\sum \ell}{d} \cdot \lambda = \frac{2}{0.0092} \cdot 14.34 = 3119.51$$

In addition to the loss of a straight pipe of magnitude of 1000, the loss factors in in one order of magnitude are negligible.

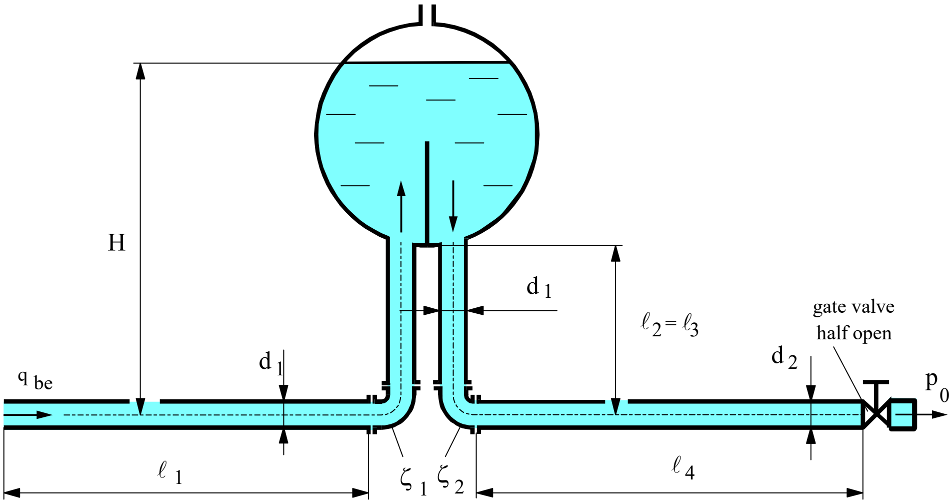
17.2 Water tower



In a reservoir of a water tower, the water level is constant "H". The consumption is compensated by supplying the volume flow. Consumption is through a semi-open valve to irrigate outdoors.
The geometry of the flow rate and the pipeline are known, and the task is to determine the differential pressure. Thus, the **type I task** discussed in **section 16.3.1** is to be solved, widened with fittings losses.

Data: $\ell_1 = 50\text{m}$; $\ell_2 = \ell_3 = 20\text{m}$; $\ell_4 = 20\text{m}$; $NA_1 = 2\text{col}$; $NA_2 = 1\text{col}$

$\zeta_1 = \zeta_2 = 1.2$; $q_{be} = 4 \cdot 10^{-3} \frac{m^3}{s}$; $\nu = 1.3 \cdot 10^{-6} \frac{m^2}{s}$; $\rho = 10^3 \frac{kg}{m^3}$



Water tower
Figure 10.3

Questions:

a./ Calculate the overpressure required at the point of entry if the loss factors of the flow-through and the network structure are calculated!
b./ What is the water level "H"?
Calculate the friction coefficient using the Moody diagram. Missing data can be taken from the diagrams and tables in the note.

Solution:

a./ First we determine the actual diameter of the pipes from **table 17.1**:

$$NA_1\ d_1 = 52.5mm\ A_1 = 2.168\cdot 10^{-3}\,m^2$$

$$NA_2\ d_2 = 26.6mm\ A_2 = 5.574\cdot 10^{-4}\,m^2$$

The roughness of the straight pipes from **Table 16.1**:

$$k = 0.02mm$$

assuming that the pipes in the system are no new and therefore slightly rusty.
The half-open gate valve loss factor is taken from **Table 16.3**

$$\zeta_t = 3.2$$

it is given. Calculate flow rates for pipes "d_1" in diameter:

$$v_1 = \frac{q_{be}}{A_1} = \frac{4\cdot 10^{-3}}{2.168\cdot 10^{-3}} = 1.84\frac{m}{s}$$

and the diameter of the "d_2":

$$v_2 = \frac{q_{be}}{A_2} = \frac{4\cdot 10^{-3}}{5.574\cdot 10^{-4}} = 7.17\frac{m}{s}$$

Among the points of supply and consumption we apply the Bernoulli equation with losses:

$$p_1 + \frac{\rho}{2}v_1^2 = p_2 + \frac{\rho}{2}v_2^2 + \Delta p'_{12}$$

The majority of practitioners working on the "water" profession prefer to use the Bernoulli equation with its dimensional shape, so we use it in this case:

$$\frac{p_1}{\rho\cdot g} + \frac{v_1^2}{2\cdot g} = \frac{p_2}{\rho\cdot g} + \frac{v_2^2}{2\cdot g} + h'_{12}$$

The loss term in the equation $\frac{\Delta p'_{12}}{\rho\cdot g} = h'_{12}$ is called a loss height or head loss.

The head loss:

$$\frac{\Delta p'_{12}}{\rho\cdot g} = \frac{v_1^2}{2\cdot g}\cdot\left(\frac{\ell_1 + 2\ell_2}{d_1}\lambda_1 + \zeta_1 + \zeta_2 + 1\right) + \frac{v_2^2}{2\cdot g}\cdot\left(\frac{\ell_4}{d_2}\lambda_2 + \zeta_t\right) \qquad 17.4$$

The loss factors as a function of the Reynolds number and the relative roughness taken from the Moody diagram (**Figure 16.7**):

$$\frac{d_1}{k} = \frac{52.5}{0.02} = 2625 \quad \text{and} \quad \frac{d_2}{k} = \frac{26.6}{0.02} = 1330,$$

$$Re_1 = \frac{v_1 d_1}{\nu} = \frac{1.84\cdot 0.0525}{1.3\cdot 10^{-6}} = 7.43\cdot 10^4, \quad \text{from the diagram} \quad \lambda_1 = 0.02 \text{ and}$$

$$Re_2 = \frac{v_2 d_2}{\nu} = \frac{7.14\cdot 0.0266}{1.3\cdot 10^{-6}} = 1.46\cdot 10^5, \quad \text{from the diagram} \quad \lambda_2 = 0.021$$

Using these, the pressure loss is calculated as follows:

$$h'_{12} = \frac{\Delta p'_{12}}{\rho\cdot g} = \frac{1.84^2}{2\cdot 9.81}\cdot\left(\frac{50 + 2\cdot 20}{0.0525}0.02 + 1.2 + 1.2 + 1\right) + \frac{7.17^2}{2\cdot 9.81}\cdot\left(\frac{20}{0.0266}\cdot 0.021 + 3.2\right)$$

$$h'_{12} = 56.26\,m, \qquad (\Delta p'_{12} = h'_{12}\cdot\rho\cdot g = 5.52\cdot 10^5\ Pa)$$

from which the overpressure at the feed point:

$$p_1 - p_0 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) + h'_{12} \cdot \rho \cdot g = \frac{10^3}{2}\left(7.17^2 - 1.84^2\right) + 5.52 \cdot 10^5 = 5.76 \cdot 10^5 \, \text{Pa}.$$

b./ To determine the height "H", use the Bernoulli equation with losses between the point of entry and the point on the surface of the water in the water tower (be "0"):

$$\frac{p_1}{\rho \cdot g} + \frac{v_1^2}{2 \cdot g} = \frac{p_0}{\rho \cdot g} + H + h'_{10}$$

Express the desired "H" value of the equation:

$$H = \frac{p_1 - p_0}{\rho \cdot g} + \frac{v_1^2}{2 \cdot g} - h'_{10}$$

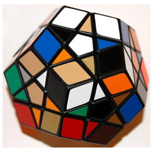
On the right side of the formula, we know all things, since the loss height "h'_{10}" has been calculated in *equation 17.4* with the loss factors belonging to the velocity v_1, so:

$$h'_{10} = \frac{v_1^2}{2 \cdot g} \cdot \left(\frac{\ell_1 + 2\ell_2}{d_1}\lambda_1 + \zeta_1 + \zeta_2 + 1\right) = \frac{1.84^2}{2 \cdot 9.81} \cdot \left(\frac{50 + 2 \cdot 20}{0.0525} 0.02 + 1.2 + 1.2 + 1\right) = 6.5 \text{ m}$$

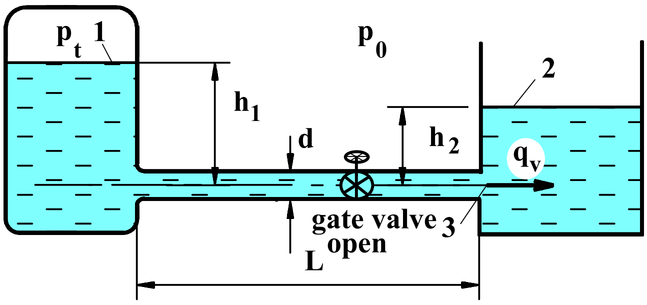
The pressure difference was obtained in response to the previous question, so the water height required:

$$H = \frac{p_1 - p_0}{\rho \cdot g} + \frac{v_1^2}{2 \cdot g} - h'_{10} = \frac{5.52 \cdot 10^5}{1000 \cdot 9.81} + \frac{1.84^2}{2 \cdot 9.81} - 6.5 = 49.95 \cong 50 \text{ m}$$

17.3 From tank to tank flow



In the figure water from a pressurized container into an open container flows. The phenomenon can be considered steady flow.
The connecting pipe is a standard, slightly rusty 2-inch steel pipe.



Flow from tank to tank
Figure 10.3

Data: $\nu = 1.3 \cdot 10^{-6} \frac{m^2}{s}$;

$p_0 = 10^5 \text{ Pa}$; $p_t = 2 \cdot 10^5 \text{ Pa}$; (absolute pressure) $L = 15 \text{m}$; $h_1 = 1.4 \text{m}$; $h_2 = 0.8 \text{m}$; $d = 52.5 \text{mm}$

Question:

What is the "q_v" volume flow rate on the "L" length tube?

The nature of the task is given in Annex II. (**see section 16.3.2**), in which the volume flow is to be found with a given geometry and pressure difference.

I. Solution:

Apply the Bernoulli equation with losses between points 1 and 2:

$$\frac{p_t}{\rho \cdot g} + h_1 = \frac{p_0}{\rho \cdot g} + h_2 + h'_{12}.$$

Write down the losses:

$$h'_{12} = \frac{v^2}{2 \cdot g} \cdot \zeta_{be} + \frac{v^2}{2 \cdot g} \cdot \frac{L}{d} \cdot \lambda + \frac{v^2}{2 \cdot g} \cdot \zeta_t + \frac{v^2}{2 \cdot g}$$

The first loss is the entry loss, the loss factor of which is based on **figure 16.11** $\zeta_{be} = 0.5$, the second is the loss of the straight pipe. The third member is the open gate valve loss, which is $\zeta_t = 0.26$ selected from **table 16.5**. The fourth member is the exit loss at the entry into the right-hand tank.

In the above terms, we do not know the velocity, so we can not specify a Re-number to calculate "λ". It is advisable to use iteration for the solution. As a first step, from the above two equations we create a term in which the velocity is given as a function of the "λ" and other known quantities. Express from the Bernoulli equation also h'_{12}.

$$h'_{12} = \frac{p_t}{\rho \cdot g} - \frac{p_0}{\rho \cdot g} + (h_1 - h_2)$$

Then replace it with the expression of loss and express the "v" velocity and enter the numeric values.

$$v = \sqrt{2 \cdot g \cdot \frac{\frac{p_t}{\rho \cdot g} - \frac{p_0}{\rho \cdot g} + (h_1 - h_2)}{\frac{L}{d} \cdot \lambda + \zeta_{be} + \zeta_t + 1}} = \sqrt{2 \cdot 9.81 \cdot \frac{20.38 - 10.19 + (1.4 - 0.8)}{\frac{15}{0.0525} \cdot \lambda + 0.5 + 0.26 + 1}}$$

$$v = \sqrt{\frac{211.7}{285.7 \cdot \lambda + 1.76}} \qquad 17.5$$

The *term 17.5* serves as the basis for iteration.

The iteration process for velocity:

A./ The first step is to choose the "λ" friction coefficient.

The roughness of the usual steel pipe is now selected from **Table 16.1**, now value $k = 0.03$, so the relative roughness value is: $\frac{d}{k} = \frac{52.5}{0.03} = 1750$

The starting "λ" is free to choose. Recommended start up value:

$\lambda_0 = 0.02 \div 0.03$, in this case $\lambda_0 = 0.02$

B./ In the next step, find the first approximate velocity "v_0" for the given "λ" using the *term 17.5*

$$v_0 = \sqrt{\frac{211.7}{285.7 \cdot \lambda + 1.76}} = \sqrt{\frac{211.7}{285.7 \cdot 0.02 + 1.76}} = 5.32 \frac{m}{s}$$

C./ Then, with the value "v_0" you can now count Reynolds number:

$$Re_0 = \frac{v_0 \cdot d}{\nu} = \frac{5.32 \cdot 0.0525}{1.3 \cdot 10^{-6}} = 2.149 \cdot 10^5$$

D./ Using the Re number, we use the expression suggested by *Haaland 9.15*, which is approximate to the Moody diagram:

$$\frac{1}{\sqrt{\lambda_1}} = -1.8 \cdot \log_{10}\left[\frac{6.9}{Re} + \left(\frac{\frac{k}{d}}{3.7}\right)^{1.11}\right] = -1.8 \cdot \log_{10}\left[\frac{6.9}{2.149 \cdot 10^5} + \left(\frac{\frac{1}{1750}}{3.7}\right)^{1.11}\right] = 7.229,$$

of which the second approximate value of the friction coefficient:

$$\lambda_1 = 0.0191.$$

E./ In this step, comparing the "λ_1" received with the "λ_0" now, if the deviation is at a desired value, eg. less than 3%, the calculation is considered complete and the last-obtained diameter is accepted as the final result.
In this case the deviation

$$H[\%] = \frac{|0.02 - 0.0191|}{0.0191} \cdot 100 = 4.71\%,$$

which is even greater than the target error limit, so we go back to step **B./** and we do iterate again.

$$v_1 = 5.41\frac{m}{s}; Re_1 = 2.187 \cdot 10^5; \lambda_2 = 0.01887$$

Examining the difference, we get the following:

$$H[\%] = \frac{|\lambda_2 - \lambda_1|}{\lambda_2} \cdot 100 = \frac{|0.01887 - 0.0191|}{0.01887} \cdot 100 = 1.2\%.$$

Thus, the results obtained in the last step will be accepted as final. The flow rate after these:

$$q = v \cdot \frac{d^2 \cdot \pi}{4} = 5.41 \cdot \frac{0.0525^2 \cdot \pi}{4} = 0.0117\frac{m^3}{s}.$$

II. <u>solution:</u>

Apply the Bernoulli equation with losses between "1" and "3".

$$\frac{p_t}{\rho \cdot g} + h_1 = \frac{p_t + \rho \cdot g \cdot h_2}{\rho \cdot g} + \frac{v^2}{2 \cdot g} + h'_{13}$$

The first member of the right-hand side expresses that the pressure at the height of the fluid entering the open container is greater than the atmospheric pressure. The velocity member is now part of the Bernoulli equation, not a loss of exit. The loss is due to losses in the entrance, the straight pipe and the gate valve:

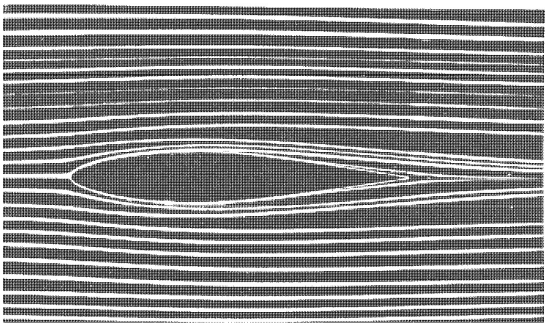
$$h'_{13} = \frac{v^2}{2 \cdot g} \cdot \zeta_{be} + \frac{v^2}{2 \cdot g} \cdot \frac{L}{d} \cdot \lambda + \frac{v^2}{2 \cdot g} \cdot \zeta_t$$

The next steps of the solution are the same as the previous one.
Section **III. type task**, where pipe diameter is unknown and not just straight pipes, but also other sources of loss, the direct solution can become very complicated. It is preferable to recur to one of **Type I or Type II** problems.
For a given pressure drop and flow rate, you need to find the pipeline that can meet your needs.
To solve this problem, select the pipe line, the directional breaks, etc., and then select the pipe diameters and other features of the line.
The task can be traced back
a./ with a given flow rate and pipe diameters for **type I** work, or

b./ with a given pressure drop and with the pipe diameters for **type II.**
In case **a./,** the calculated pressure drop should be smaller than the required value,
and in **b./,** the calculated flow rate should be greater than the value expressed as the claim.
The better the design task, the smaller the difference between the demand and the calculated results. If there is a big difference, we have unreasonably large pipe diameters, so the cost of purchasing and installing the pipeline will be greater than necessary.
If the above mentioned conditions are not met then new pipe diameters should be selected.

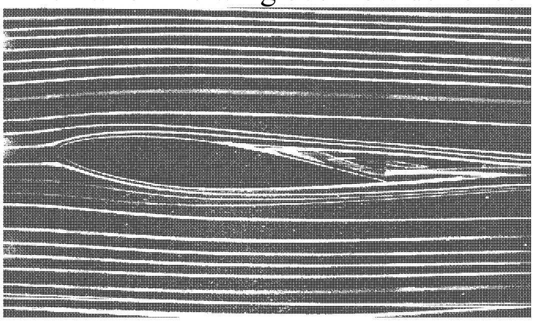
18. Flow forces acting on the bodies

For flowing bodies, flow in most cases exerts some force. In **Chapter 14**, we have already dealt with the power of infinite wing grids, and the magnitude and direction of the force on the single wing was also determined. According to Zhukovsky's law, the lift force acting on the wing is perpendicular to the direction of the infinite velocity. The component parallel to the velocity is not in the ideal flow. When flowing with real fluid, the force acting against the direction of flow is always the resistance force or drag force.



Airfoil
Figure 18.1

(The ideal fluid can also be a resistance force, for example, when the body is accelerated by fluid.)
The resistance in a real fluid can be divided into two parts: friction, pressure or shape resistance.
The friction resistance is the result of shear stress between the surface of the body and the fluid. The pressure or shape resistance results from the fact that, due to friction, the pressure behind the body is less than in a frictionless situation.
The streamlined body resistance is small. For example, **Figure 18.1** shows a NACA 64A015 symmetric wing profile that is streamlined. The current flow around it is very similar to the ideal flow; the wing resistance is very small. The resistance increases rapidly when the flow of the body is removed. **Figure 18.2** shows the previous wing section at a higher angle of attack when the wing overflow detaches from the upper wing surface.



Flow separation on the airfoil
Figure 18.2

On non-streamlined bodies, almost always appears the flow separation, and most of the resistance is caused by this force.
There are bodies with the shape of the detachment regardless of the Reynolds number. Such are bodies with angles with sharp edges (eg cubes, iron structures, angular buildings). In such cases, the friction resistance is negligible in the form of resistance to form, these bodies are referred to as Reynolds number-resisting bodies. In contrast, bodies with curved surfaces where the location of detachment depends on the Reynolds number are referred to as Reynolds-number susceptible bodies.
From the point of view of resistance, we have divided the bodies into two groups. Specifically, the resistance factor depends on or does not depend on the Reynolds number.
The drag coefficient is equal to the drag force (F_e) divided by a typical surface (A) of the body and by a dynamic pressure that can be measured far beyond the body, ie:

$$c_e = \frac{F_e}{\frac{\rho}{2} \cdot v_\infty^2 \cdot A} \qquad 18.1$$

The Reynolds number is the undisturbed flow velocity (v_∞) measured far in front of the body, multiplied by a typical size of the body (eg cylinder, diameter of the cylinder, d) and divided by the kinematic viscosity (ν) of the medium, i.e.:

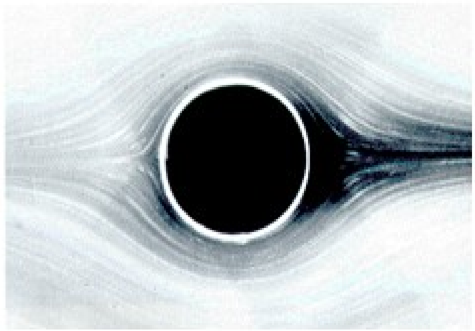
$$Re = \frac{v_\infty d}{\nu} \qquad 18.2$$

Both the drag coefficient and the Reynolds number are dimensionless quantities.

18.1 The flow around the cylinder and the sphere

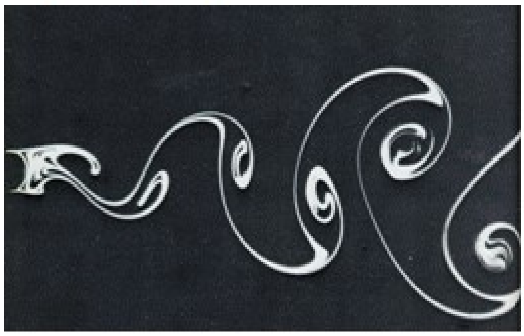
Much of the resistance of the cylinder and the ball is made by the form resistance, and the friction resistance is negligible. **Figure 18.3** shows a flow around an infinite long circular cylinder at different Reynolds numbers. The resistance factor is shown in **Figure 18.4**, depending on Reynolds number. It can be noticed that an increasing number of Reynolds numbers will be getting the separation zone behind the roller. For Re numbers less than 1, the drag coefficient decreases linearly with velocity, so the drag force increases linearly with the velocity. This is a current picture according to **Figure 18.3/a**. For Re numbers greater than 1, the curve gradually passes horizontally and the resulting current images are shown in **Figures 18.3/b** and **/c.** Behind the cylinder the flow separation is getting larger and larger.
From about Re=100 onwards, vortex on both sides periodically follow each other. The resulting vortex wind is called the Kármán vortex street, its discovery (see **Figure 18.3/d**). (***Tódor Kármán 1881-1957 Hungarian-American engineer inventor***). The resistance factors for each Reynolds number can be seen in **Figure 18.4** by dots.
In $Re \approx 100 \div 10^5$, the drag coefficient is nearly constant, i.e., the drag force (F_e) is proportional to the square of the velocity. Before the release, the boundary layer around the cylinder will remain laminar in this range.

a./ Re=0.3



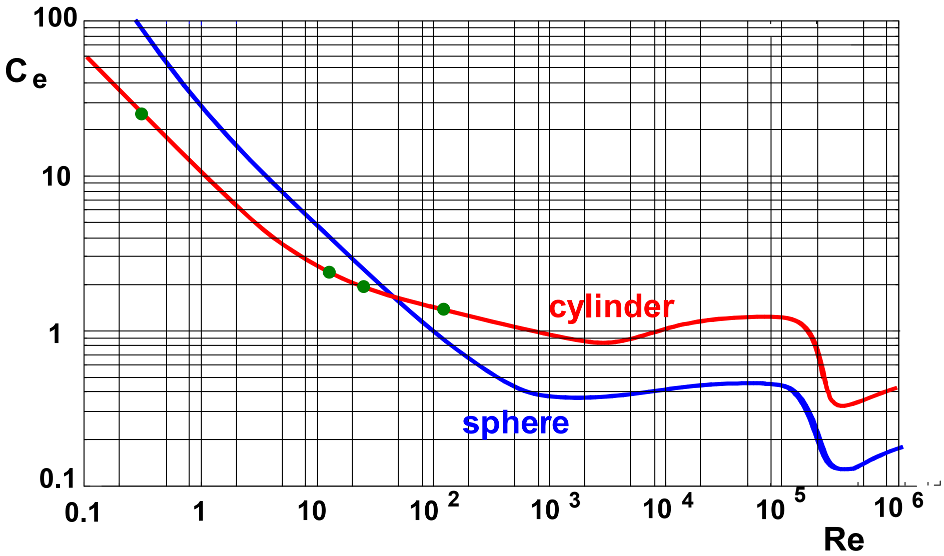
b./ Re= 13

c./ Re= 26



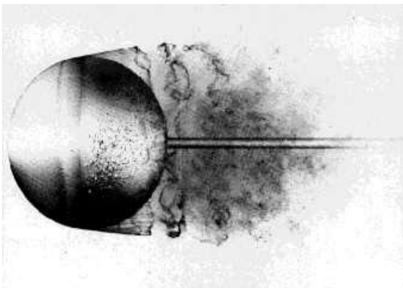
d./ Re=140

Flow around a cylinder at different Reynolds numbers
Figure 18.3

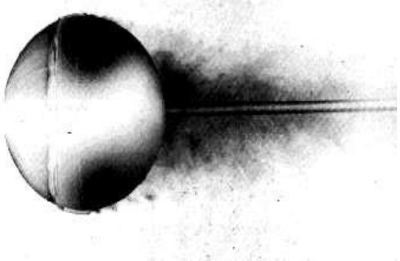


Cylindrical and spherical drag coefficience as a function of the Reynolds number
Figure 18.4

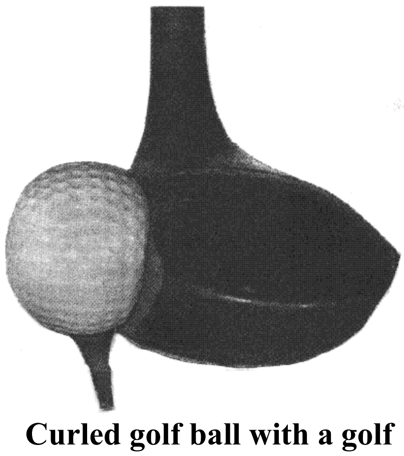
Periodically swirling vortexes can revert to the cylinder and, in some cases, also vibrate. This phenomenon can be feel when to stick is pulled at a certain velocity in the water, but this causes the vibration and swirling of the highway led wires in windy weather.
For Reynolds number above 10^5, the boundary layer becomes turbulent even before separation point. In the turbulent boundary layer, the particles have a moving energy greater than the laminar, so they can continue to follow the shape of the cylinder against the higher pressure behind the cylinder, that is, the release occurs later. In this case, the separation bobble behind the cylinder will be smaller than at smaller Re numbers. This is explained by the fact that approx. $2 \div 5 \cdot 10^5$. At the Re-number, the drag coefficient suddenly decreases. The Re-number belonging to the curve is called a critical Reynolds number.
The shape of the resistance of the sphere is quite similar to its cylinder, which is also shown in **Figure 18.4**. In the reader, the question is involuntarily that if the boundary layer can be turbulent in a flow around the cylinder or the sphere, then its resistance can be reduced even under the critical Reynolds number. One of these methods, the reduction of the pressure on the sphere, is shown in **Figure 18.5. Fig. 18.5/a**. On the smooth sphere approx. Re = 15000 values show the separation zone behind the sphere. The start of separation is approx. 80-90 ° from the blowing direction. In **Figure 18.5/b**, a thin wire ring was placed on the side being pushed, causing the boundary layer to turbulence, 120-130° relative to the direction of the blowing and the detachment zone shrunk much less. Of course, the drag coefficient has also decreased.
The hundreds of small crater-like recesses on the surface of the golf ball serve to make the boundary around the sphere turbulent. The smooth surface of the golf ball is 30-40% bigger away from roughened balls due to reduced air resistance. In **Figure 18.6** you can see a golf ball that has just hit. Interestingly, besides the design of the surface, it is also possible to observe the deformation of the spherical shape caused by the impact, when it is made of hard plastic.



a./ Separation zone behind the sphere



b./ Separation zone behind the sphere with turbulence wires
Figure 18.5

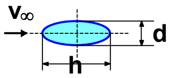
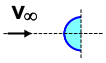
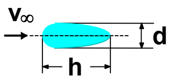
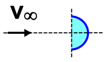


Curled golf ball with a golf club at the time of the hit
Figure 18.6

18.2 Drag force of different bodies

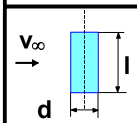
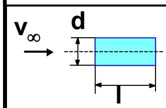
The drag coefficient for some bodies is shown in **Tables 18.1 and 18.2**.
The drag coefficient is the area of the largest cross-section perpendicular to the bowing direction for each body. The half-sphere drag coefficient are different blowed from opposite directions, but it does not depend on the Reynolds number. The spoon anemometer uses this fact to function.

Table 18.1 Drag coefficient of bodies I.

Body	Scale	C_e $Re = 10^3 \div 10^5$	C_e $Re > 5 \cdot 10^5$	Body	C_e $Re > 5 \cdot 10^5$
 ellipsoid	$h:d = 1.8$ (sphere)1 0.75 circuit plate 0	0.4 0.5 0.6 1.1	0.08 0.15 0.2 1.1	 half globe shell	0.34
 Drutch profile endless long	$h:d = 2$ 3 5 10 20		0.2 0.1 0.06 0.083 0.094	 half globe shell	1.33

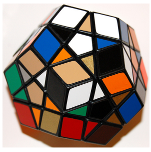
The drag coefficient of the cylinder that is paralyzed with its axis does not depend on the Reynolds number.

Table 18.2 Drag coefficient of bodies II.

Body	Scale	C_e $Re = 10^3 \div 10^5$	C_e $Re > 5 \cdot 10^5$
 cylinder	$\ell : d = 1$ 2 5 10 20 ∞	0.63 0.68 0.74 0.82 0.98 1.2	 0.35
 cylinder	$\ell : d = 0$ 1 2 4 7	1.11 0.91 0.85 0.87 0.99	

The ellipsoidal axis of rotation is the direction of blowing. Its largest diameter is "d", its length is "h". The drutch profile is infinitely long (plane flow) in one direction, the maximum thickness perpendicular to the blowing direction is "d". Reduced drag of rod-mounted structures by inserting a suitable profile. Nowadays, high-speed skiing is fashioned, using a leg-reinforced stud profile to achieve a lower air resistance.
The resistance profile of the rod profile and its cylinder parallel to its axis decreases with increasing flow rate initially and then increases. The reason for the decrease is that the body-forming behind the body is reduces the separated flow bobble. However, on the surface of a very long body, the friction loss again increases the resistance of the body. The drag coefficient increases with the increase in the length of the finite cylinder, which is perpendicular to its axis.

The drag coefficients of a body that are placed behind each other are altered by the interaction. Cyclists, swimmers, and flying birds in the form of "V" are used to reduce the resistance of the backside riders.
Measurement of drag coefficient in wind tunnels is most commonly determined by force measurement. In our department, we conducted countless measurements of this kind, and we measured the air resistance of various buildings, parabola and traditional antennas, vehicles, skiers etc. with wind tunnel measurements.
After a short period of time, the free bodies are falling down reach a state where the weight and the force from the air drag force are balanced. At this point, the body does not accelerate and falls at constant velocity. In our next example, we are examining such a case.



Determine the rate of falling or descending ice in the air that can be considered a sphere shape. Let's neglect the melting of ice during the fall.

Data: The diameter of the ice sphere $d = 15mm$, ice density, $\rho_{ice} = 900\frac{kg}{m^3}$. air density $\rho_{air} = 1.2\frac{kg}{m^3}$, kinematic viscosity $\nu = 13 \cdot 10^{-6}\frac{m^2}{s}$.

Solution:

The resistance of the sphere depends on the velocity through the Re number (see **Figure 18.4**) so we can only solve it by iteration. Assume that the Re number is between 10^3 and 10^5, then the resistance of the sphere is well approximated $c_e = 0.5$,

The weight of the sphere keeps balance with the drag force, so:

$$G = F_e$$

Weight of ice:

$$G = \frac{d^3 \cdot \pi}{6} \cdot \rho_{ice} \cdot g = \frac{0.015^3 \cdot \pi}{6} \cdot 900 \cdot 9.81 = 0.0156N$$

This force balances the resistance force, which according to *Equation 18.1*:

$$F_e = \frac{\rho_{air}}{2} \cdot v^2 \cdot A \cdot c_e.$$

Area „A” is the largest cross-section perpendicular to the flow, which is the area of its main circuit in the sphere, i.e. the resistance force:

$$F_e = \frac{\rho_{lair}}{2} \cdot v^2 \cdot \frac{d^2 \cdot \pi}{4} \cdot c_e$$

Equalizing the weight and using it in the first approximation we get the following:

$$G = \frac{\rho_{air}}{2} \cdot v^2 \cdot \frac{d^2 \cdot \pi}{4} \cdot c_e$$

Expressing the velocity and replacing the data we get the velocity:

$$v = \sqrt{\frac{2 \cdot G}{\rho_{lair} \cdot \frac{d^2 \cdot \pi}{4} \cdot c_e}} = \sqrt{\frac{2 \cdot 0.0156}{1.2 \cdot \frac{0.015^2 \cdot \pi}{4} \cdot 0.5}} = 17.1\frac{m}{s}.$$

Check the Re number at the received speed with the data:

$$Re = \frac{v \cdot d}{\nu} = \frac{17.1 \cdot 0.015}{13 \cdot 10^{-6}} = 19794 .$$

From **figure 18.4**, for the given Re number, we get a value that re-calculates the velocity, and slightly changes our result:

$$v = \sqrt{\frac{2 \cdot 0.0156}{1.2 \cdot \frac{0.015^2 \cdot \pi}{4} \cdot 0.45}} = 18.1 \frac{m}{s}$$

18.3 Wing forces

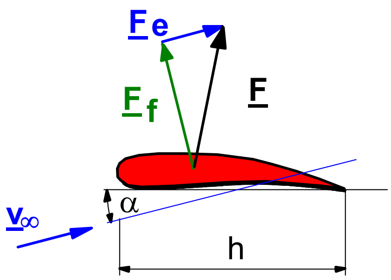


Figure 18.7

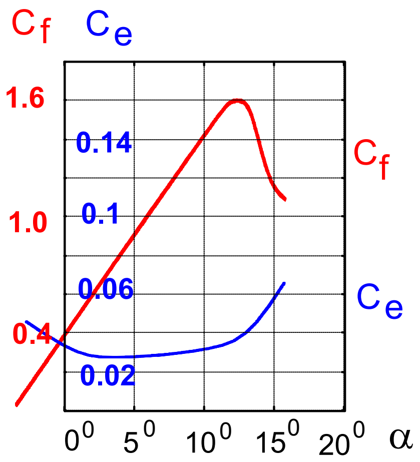
Figures 18.1 and 18.2 shows a flow around a symmetrical wing profile at two different angles of attack. Generally, the wings are not symmetrical. The wings shown in **Figure 18.7** are a general wing section, which is of great importance both in aviation and in flow technology. In the discussion of the momentum equation, we have already introduced the Zhukovsky law (see **section 14.5, 14.34**), according to which, in the case of a frictionless medium, the force acting on a unit length section of the wing can be calculated $|\underline{R}| = \rho \Gamma \cdot v_\infty$ by reference to the relationship between "v_∞" the distant uninterrupted flow velocity, "Γ" the circulation around the wing. The "$\underline{R}$" force applied to the wing is also perpendicular to the "v_∞" velocity. In a real medium, the force acting on the wing can be divided into two components: the "F_f" lift force perpendicular to the "v_∞" (blowing) velocity and the drag force F_e parallel to the blowing velocity. Similar to the *18.1* connection, the wing drag coefficient and lift coefficient can be introduced:

$$c_e = \frac{F_e}{\frac{\rho}{2} \cdot v_\infty^2 \cdot A} \qquad 18.3$$

$$c_f = \frac{F_f}{\frac{\rho}{2} \cdot v_\infty^2 \cdot A} \qquad 18.4$$

In the bodies discussed so far, the characteristic surface in the denominator of the expression of the drag coefficient was the largest cross-sectional area of the body's perpendicular to blowing direction. For the wings, this typical surface is the basic surface area the product of the "h" cord length of the wing and the width of the wing "ℓ":

For a given wing, the "c_f" and "c_e" coefficients depend on the Re-number so far, but in addition, it is important to see the angle between the direction of blowing and the wing, so the force factors depend on the angle of attact "α". The characteristic of the lift and drag coefficient of a particular wing is shown in **Figure 18.8**. The drag coefficient and the lift coefficient are represented on a different scale. The lift coefficient can also be 10 to 40 times the drag coefficient. It can be seen from the figure that the lift coefficient approximates linearly as a function of the angle of attect and reaches its maximum value $c_f \cong 1.2 \sim 1.6$ suddenly decreasing. The drag coefficient for increasing with α is less "sensitive": the value of a wide angle between the boundaries is nearly constant, relatively small ($c_e \cong 0.01 - 0.04$). The drag coefficient only grows rapidly when the "c_f" sudden drops. This is referred to as the fall of the wings, which can be explained by a large increase separation region (see **Figure 18.2**)



Wing drag and lift coefficient
Figure 18.8

The wings are characterized by the number of gliders, which is the ratio between the lift and the drag coefficient: $\frac{c_f}{c_e}$. (A glider number of a glider will give you how many meters the plane will glide horizontally, while sinking 1 meter.) The gliding number is generally between 10 and 50. The wings are highly valued by this property.

19. Speed and volume flow rate measurement

19.1 Velocity measurement and tools

Spoon cap anemometer
Figure 19.1

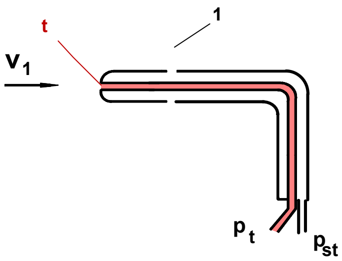
The velocity at a point in the flow can be measured by a variety of methods. Only some of the measuring instruments and measuring principles are described.
It measures the mechanical principle eg. the spoon anemometer (see **Figure 19.1**). Its operation is based on the fact that the resistance factor of an open hemispherical bulb from the open side is much bigger ($c_e = 1.33$) than scattered on the convex side ($c_e = 0.34$). Experience has shown that the anemometer's speed is proportional to the speed of the wind, so speed can be determined by simple speed measurement. I is often used the wing wheel anemometer, which is uses the mechanical principle .

19.1.1 The Prandtl tube or Pitot static tube

The Prandtl tube returns the velocity measurement to measure the pressure difference (see **Figure 19.2**). Its essence is a double-walled pipe facing the flow, at the beginning of which the space between the outer and inner tubes is closed by a rounded nose. In the inner tube, the nose groove on the symmetry axis extends away from the nose hole in the space between the outer and the inner tubes, and the dowel holes on the outer tube flange open. The nose bore is connected to the pressure gauge separately through the inner tube and the bore holes through the space between the two tubes. (**Fig. 19.3** shows the geometrical design of the Prandtl tube with some standard dimensions.) At the point of the tube, the velocity of the flow is zero and there is a stagnation point. On the cylindrical part, however, it is far from the stagnation point, the velocity is nearly the same as before the measuring device was placed. Apply the Bernoulli equation in a plane perpendicular to the axis of the nose and the Prandtl tube at its "1" point at the height of the side holes. Here, the velocity is very close to the same as before the tube was placed.

$$p_t = \frac{\rho}{2} v_1^2 + p_1$$

In the nose hole, therefore, the "v_1" vrlocity and the "p_1" pressure are generated. Pressure "p_1" is **static pressure**, "p_{st}" and "$p_t - p_1$" is **dynamic pressure** corresponding to "v_1" velocity:



Prandtl tube section
Figure 19.2

$$p_{din} = \frac{\rho}{2} v_1^2 = p_t - p_1 .$$

The " v_1 " velocity equal to the positioning of the Prandtl tube at the rate at which it is located:

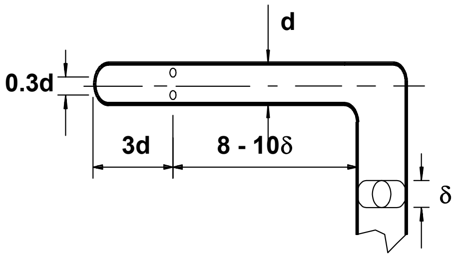
$$v_1 = \sqrt{\left(\frac{2}{\rho} \cdot (p_t - p_1)\right)} = \sqrt{\left(\frac{2}{\rho} \cdot p_{din}\right)}, \qquad 19.1$$

where " ρ " density for air and gases can be calculated from

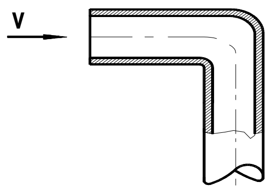
$$\frac{p}{\rho} = RT$$

relationship. R = 287 [J/kgK] for dry air.

The Prandtl tube can only measure the exact dynamic pressure when the cylindrical head symmetry axis is parallel to velocity and can deviate by up to 15°, because the Prandtl tube is insensitive to the smaller angular deviation.



Prandtl tube dimensions
Figure 19.3



Pitot tube
Figure 19.4

Static pressure can be measured by other means or measured in free jets, it is sufficient to measure the difference in the total pressure from the atmospheric pressure. In this case only the Pitot tube is used (see **Figure 19.4**)

19.1.2 Flow measurement in tube with Prandtl tube

The amount of fluid flowing through one time cross section of the pipe with the following terms and/today we can characterize standard units:

$$\text{Volume flow rate" } q_v \text{ " } \left[\frac{m^3}{s}; \frac{m^3}{h}; \frac{\ell}{s}; \frac{\ell}{min}\right].$$

$$\text{Mass flow rate: " } q_m \text{ " } \left[\frac{kg}{s}; \frac{t}{h}; \frac{kg}{min}\right] \; (q_m = \rho \cdot q_v).$$

The volume flow rate is usually expressed as

$$q = \iint_A \underline{v} \cdot d\underline{A} = v_{átl} \cdot A \qquad 19.2$$

can be defined. Using the integral form of a point-to-point velocity, the integration of scalar product can be performed on the surface bounded by pipe walls but on any surface. If a sufficiently long straight pipe section precedes the cross-section of the measurement, the velocity can be considered parallel to the walls defining the conductor, and the aspect of the arbitrary shape of the surface is only perpendicular to the velocity and thus to the longitudinal axis of the conductor. If the range of integration is the cross-section of the pipe - the plane perpendicular to the velocity - the vector and the velocity of the surface element can be

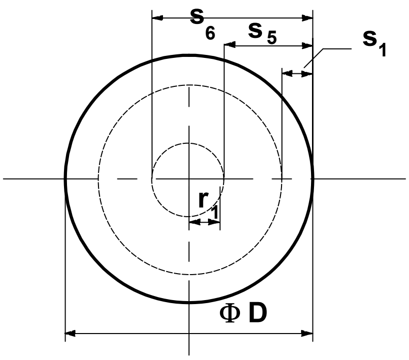
considered. (If the flow jet rotates about the longitudinal axis of the tube, determining the volume flow in this way results in inaccurate results.)
The function of velocity distribution is usually not known, so instead of integration, we define the flow rate by approximation:

$$q_v = \sum_{i=0}^{n} v_i \cdot \Delta A_i \qquad 19.3$$

If surface elements "ΔA_i" are assumed to have a size of 1/n times the full "A" cross section, i.e. constant, the "A/n" factor as constant, highlightable and thus:

$$q_v = \frac{A}{n} \cdot \sum_{i=0}^{n} v_i \qquad 19.4$$

relationship. So, "q_v" can only be calculated from the arithmetic mean of the measured velocity values if the measured values apply to **an equal surface element**. This means that the velocity should be measured at points in which **equally sized ranges** - surface elements.
In a circular cross-section tube, the velocity shall be measured along at least two diameters perpendicular to each other. Experience has shown that a circle of diameter "D" is sufficient to divide into five equal parts. Lines bounding elements are concentric circles. The measuring points should be on the circle that divides the surface elements into two equal rings. Thus, the total area is actually divided into 10 equidistant ring circles and the velocity of every odd number radius is measured. With this, a radius of 5 or more. a diameter of 10 measuring points is obtained. We call this method a ten point method, which can be found in MSZ EN 24006: 2002 in the Hungarian Standard. As an example, determine the location of one measuring point closest to the axis of the tube. The radius of a circle with these points "r_1", its area is just one tenth of the whole circle: $r_1^2 \cdot \pi = 0.10 \cdot R^2 \cdot \pi$ so $r_1 = R \cdot \sqrt{0.1} = 0.316 \cdot R$.



Pipe diameter splitting
Figure 19.5

Since the distance of the Prandtl tube from the axis of the tube can not be measured directly, it is preferable to mark the distance of the head from the pipe wall to the stem of the Prandtl tube. The figures in **Figure 19.**5 are therefore:

$$s_5 = R - r_1$$

The value relative to the diameter of the pipe is:

$$\frac{s_5}{D} = \frac{R - r_1}{2R} = \frac{1 - \frac{r_1}{R}}{2} = \frac{1 - 0.316}{2} = 0.342$$

Distance from pipe wall
Table19.1

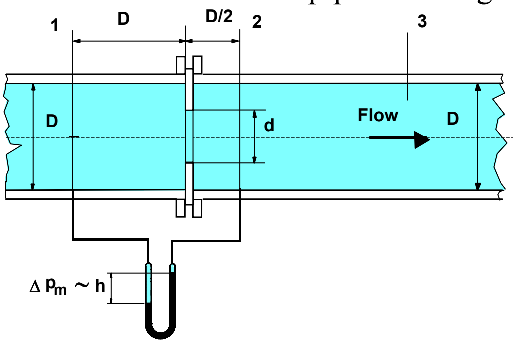
i	$\frac{s_i}{D}$
1	0.026
2	0.082
3	0.146
4	0.226
5	0.342
6	0.658
7	0.774
8	0.854
9	0.918
10	0.974

The ratios determining the location of the ten measuring points on the diameter are given in **Table 19.1**. The determination of the flow rate by point-by-step speeds requires a relatively large number of measurements. However, it has a great advantage that it can be measured in an air duct without breaking the pipeline, only two holes must be made, interrupted by operation and without a lengthy preparatory action.
Numerous other ways of measuring the flow rate based on speed

measurement are possible. Standardized methods are contained in MSZ EN 24006: 2002.

19.2 Flow measurement with reducers

If we are able to break down the pipeline and insert the restrictor into the system as required by the standard, the flow measurement itself is simpler. According to the standard, a straight pipe section should be provided in front of and behind the restrictor. The limiting elements (measuring flange, measuring tip, venturi meter) allow to measure the average velocity or volume flow rate in the pipe for a single differential pressure measurement.



Measuring flange in the pipeline
Figure 19.6

The plug-in limiter included in the line, measuring flange (see **Fig. 19.6)** and the pressure difference depending on the flow medium is measured. (The standard design of the corner-permeable measuring rim can be seen in **Figure 19.7** It is worth noting that the sharp edge of the through-hole is pointing towards the flow.) This volume flow can be determined by the following general context:

$$q_v = \alpha \cdot \varepsilon \cdot \frac{d^2 \pi}{4} \cdot \sqrt{\frac{2}{\rho}} \cdot \sqrt{\Delta p_m}\,, \qquad 19.5$$

where:

"$\Delta p_m = p_1 - p_2$";

"d" the diameter of the opening of the restrictor;

"ρ" density of medium;

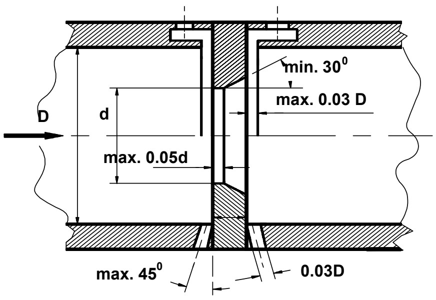
"ε" "expansion number" means the compressibility of the medium

"α" the "flow factor" is an experimentally determined factor whose value depends on the following quantities:

"β = d/D"("D" is the diameter of the pipe in front of the restrictor);

$Re_D = \frac{D \cdot v}{\nu}$ the Reynolds number for the front part tube.

Here "ν" is the kinematic viscosity of the medium.



Oriffice

The upper part of the figure is a ring box and the bottom shows a punctured puncture

Figure 19.7

The constraints: the measurement parameters of the measuring rim, measuring tip, Venturi meter, the way in which they are installed, and the values of the empirical factors (α and ε) needed to determine the flow rate flow are contained in the standard.

There are still many ways and means of measuring the volume flow rate, which are not detailed here. Without the fullness, we mention the simplest and perhaps the most accurate volume flow measurement of the dimensional measurement, the volume flow of liquid flowing in open surface channels is measured by gaps, using the principle of the constraint elements, etc.

The "Flow Measurements" [7], [14] manuals provide additional information for measurements.

From the Hungarian literature

[1] Dr. Bihari Páter: Műszaki termodinamika, BME 1999
Műszaki Könyvkiadó, Budapest, 1987

[2] Dr. Bobok Elemér: Áramlástan bányamérnököknek;
Műszaki Könyvkiadó, Budapest, 1987

[3] Fényes Imre: Termosztatika és termodinamika. Műszaki Könyvkiadó, Budapest, 1968.

[4] Gróf Gyula: Hőtechnika. Felsőfokú tankönyv. Phare Program HU-94.05, Budapest, 1999.

[5] Dr. Gruber József-Dr. Blahó Miklós: Folyadékok mechanikája
Tankönyvkiadó,1973.

[6] Harmatha András: Termodinamika műszakiaknak. Műszaki Könyvkiadó, Budapest, 1982.

[7] Hegel I.-Füredi G.-Szlivka F.: Áramlástani mérések
Műegyetemi Kiadó, Budapest, 1993.

[8] Horváth János: Termodinamika és statisztikai mechanika. Tankönyvkiadó, Budapest, 1960.

[9] Dr. Író Béla: Hő- és áramlástan HEFOP, Széchényi István Egyetem

[10] Jászay Tamás: Műszaki Hőtan (Termodinamika). Tankönyvkiadó, Budapest, 1990.

[11] Dr. Lajos Tamás: Áramlástan alapjai
Műegyetemi Kiadó, Budapest, 1994.

[12] L.D. Landau, E.M. Lifsic: Elméleti fizika VI. Hidrodinamika; Tankönyvkiadó, Budapest, 1980.

[13] Nyúli, Gy.-Vincze, Gy.: Áramlástan GATE, Gödöllő, 1987.

[14] Dr. Sassy László-Dr. Török Sándor: Áramlástani mérések Gödöllő, 1983.

[15] Dr. Sitkei György: Gyakorlati áramlástan; Mezőgazdasági Szaktudás Kiadó, Budapest, 1997

[16] Dr. Szlivka Ferenc: Áramlástan energetikai főiskolások számára
Műegyetemi Kiadó, Budapest, 1996.

[17] Dr. Szlivka Ferenc: Áramlástan Gödöllő 2002.

[18] Dr. Szlivka F.-Dr. Bencze F.-Kristóf G.: Áramlástan példatár
Műegyetemi Kiadó, Budapest, 1998.

[19] Willi Bohl: Műszaki áramlástan; Műszaki könyvkiadó, Budapest, 1983.

From the foreign language literature

[20]. Charles Hirsch: Numerical Computation of Internal and External Flows;
Volume 1, 2, John Wiley & Sons, 1991.

[21]. W. J. Duncan-A. S. Thom-A. D. Young: Mechanics of Fluids
Edward Arnold Publisher Ltd. London, 1970

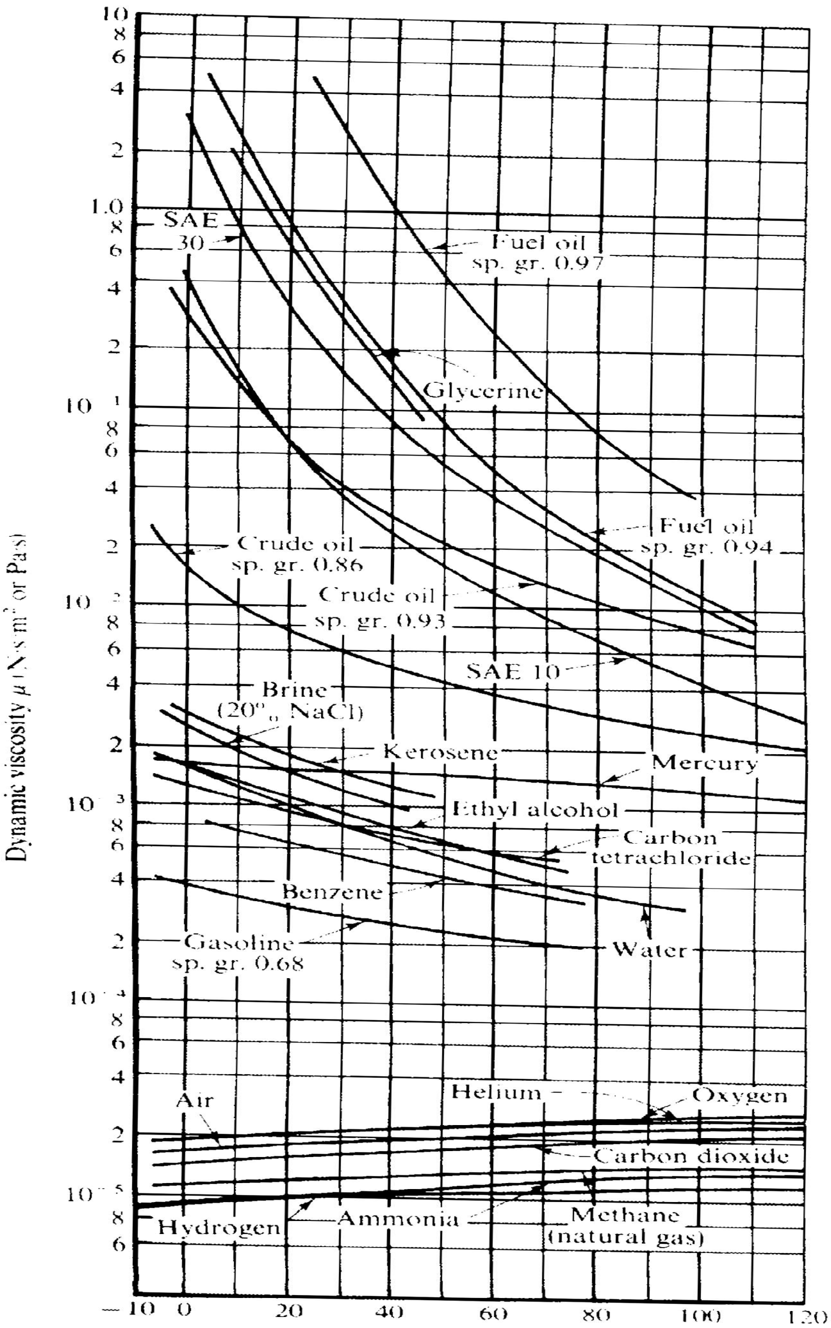
[22]. John D. Anderson, Jr.: Fundamentals of Aerodynamics; McGraw -Hill, Inc.1991.

[23] Klaus Oswatitch: Gasdynamik; Springer -Verlag, 1952.

[24] MIHEJEV, M. A.: A hőátadás gyakorlati számításának alapjai, Budapest, 1987.TK

[25] Robert L. Mott: Applied Fluid Mechanics Maxwell Macmillan International Editions, 1990

1. Annex



Dynamic viscosity μ [Pas]