

CHAPTER 1

THE PHYSICS OF MAGNETISM

BACKGROUND: Read chapters on magnetism from your favorite college physics book for review.

Paleomagnetism is the study of the magnetic properties of rocks. It is one of the most broadly applicable disciplines in geophysics, having uses in diverse fields such as geomagnetism, tectonics, paleoceanography, volcanology, paleontology, and sedimentology. Although the potential applications are varied, the fundamental techniques are remarkably uniform. Thus, a grounding in the basic tools of paleomagnetic data analysis can open doors to many of these applications. One of the underpinnings of paleomagnetic endeavors is the relationship between the magnetic properties of rocks and the Earth's magnetic field.

In this chapter, we will review the basic physical principles behind magnetism: what are magnetic fields, how are they produced, and how are they measured? Although many find a discussion of scientific units boring, much confusion arose when paleomagnetists switched from “cgs” to the Système International (SI) units, and mistakes abound in the literature. Therefore, we will explain both unit systems and look at how to convert successfully between them. There is a review of essential mathematical tricks in Appendix A, to which the reader is referred for help.

1.1 WHAT IS A MAGNETIC FIELD?

Magnetic fields, like gravitational fields, cannot be seen or touched. We can feel the pull of the Earth's gravitational field on ourselves and the objects around us, but we do not experience magnetic fields in such a direct way. We know of the existence of magnetic fields by their effect on objects such as magnetized pieces of metal, naturally magnetic rocks such as lodestone, or temporary magnets such as copper coils that carry an electrical current. If we place a magnetized needle on a cork in a bucket of water, it will slowly align itself with the local magnetic field. Turning on the current in a copper wire can make a nearby compass needle jump. Observations like these led to the development of the concept of magnetic fields.

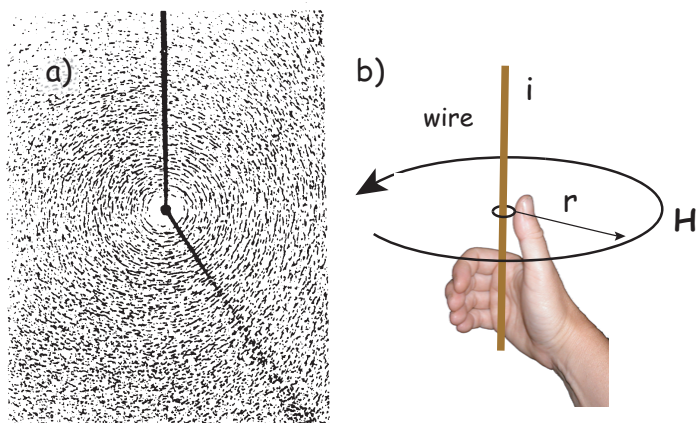


FIGURE 1.1. a) Distribution of iron filings on a flat sheet pierced by a wire carrying a current i . [From Jiles, 1991.] b) Relationship of magnetic field to current for straight wire. [Photo by author.]

Electric currents make magnetic fields, so we can define what is meant by a “magnetic field” in terms of the electric current that generates it. Figure 1.1 a is a picture of what happens when we pierce a flat sheet with a wire carrying a current i . When iron filings are sprinkled on the sheet, the filings line up with the magnetic field produced by the current in the wire. A loop tangential to the field is shown in Figure 1.1b, which illustrates the *right-hand rule*. If your right thumb points in the direction of (positive) current flow (the direction opposite to the flow of the electrons), your fingers will curl in the direction of the magnetic field.

The magnetic field \mathbf{H} points at right angles to both the direction of current flow and to the radial vector \mathbf{r} in Figure 1.1b. The magnitude of \mathbf{H} (denoted H) is proportional to the strength of the current i . In the simple case illustrated in Figure 1.1b, the magnitude of \mathbf{H} is given by Ampère’s law:

$$H = \frac{i}{2\pi r},$$

where r is the length of the vector \mathbf{r} . So, now we know the units of H : Am^{-1} .

Ampère’s Law, in its most general form, is one of Maxwell’s equations of electromagnetism: in a steady electrical field, $\nabla \times \mathbf{H} = \mathbf{J}_f$, where \mathbf{J}_f is the electric current density (see Section A.3.6 in the appendix for review of the ∇ operator). In words, the curl (or circulation) of the magnetic field is equal to the current density. The origin of the term “curl” for the cross product of the gradient operator with a vector field is suggested in Figure 1.1a, in which the iron filings seem to curl around the wire.

1.2 MAGNETIC MOMENT

An electrical current in a wire produces a magnetic field that “curls” around the wire. If we bend the wire into a loop with an area πr^2 that carries a current i (Figure 1.2a),

the current loop would create the magnetic field shown by the pattern of the iron filings. This magnetic field is the same as the field that would be produced by a permanent magnet. We can quantify the strength of that hypothetical magnet in terms of a *magnetic moment* \mathbf{m} (Figure 1.2b). The magnetic moment is created by a current i and also depends on the area of the current loop (the bigger the loop, the bigger the moment). Therefore, the magnitude of the moment can be quantified by $m = i\pi r^2$. The moment created by a set of loops (as shown in Figure 1.2c) would be the sum of the n individual loops:

$$m = ni\pi r^2. \quad (1.1)$$

So, now we know the units of \mathbf{m} : Am^2 . In nature, magnetic moments are carried by magnetic minerals, the most common of which are magnetite and hematite (see Chapter 6 for details).

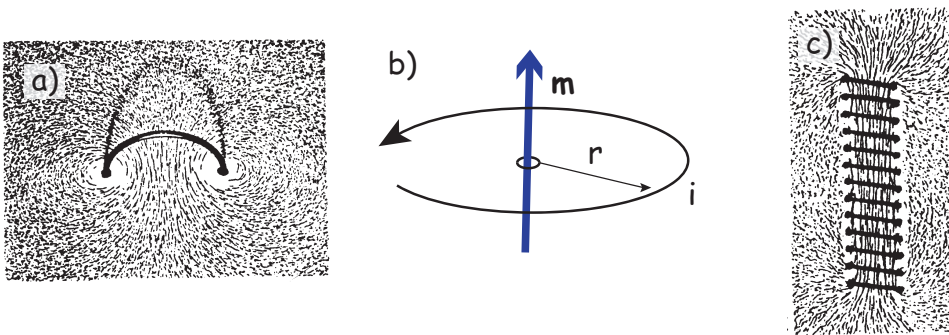


FIGURE 1.2. a) Iron filings show the magnetic field generated by current flowing in a loop. b) A current loop with current i and area πr^2 produces a magnetic moment \mathbf{m} . c) The magnetic field of loops arranged as a solenoid is the sum of the contribution of the individual loops. [Iron filings pictures from Jiles, 1991.]

1.3 MAGNETIC FLUX

The magnetic field is a vector field because, at any point, it has both direction and magnitude. Consider the field of the bar magnet in Figure 1.3a. The direction of the field at any point is given by the arrows, while the strength depends on how close the field lines are to one another. The magnetic field lines represent *magnetic flux*. The density of flux lines is one measure of the strength of the magnetic field: the magnetic induction \mathbf{B} .

Just as the motion of electrically charged particles in a wire (a current) creates a magnetic field (Ampère's Law), the motion of a magnetic field creates electric currents in nearby wires. The stronger the magnetic field, the stronger the current in the wire.

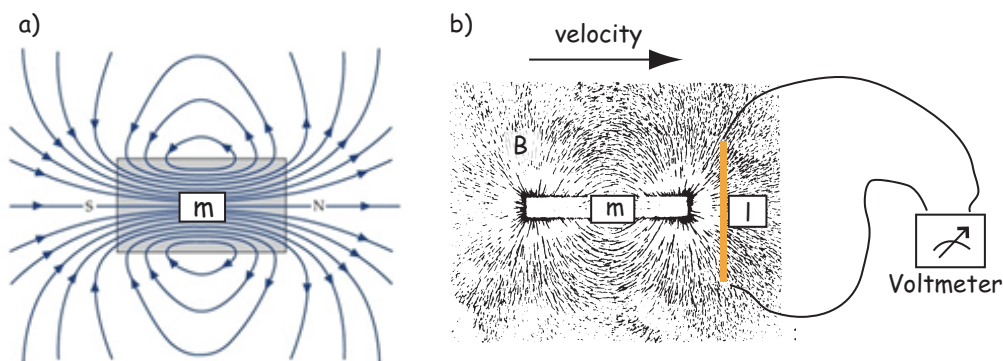


FIGURE 1.3. a) A magnetic moment \mathbf{m} makes a vector field \mathbf{B} . The lines of flux are represented by the arrows. [Adapted from Tipler, 1999.] b) A magnetic moment \mathbf{m} makes a vector field \mathbf{B} , made visible by the iron filings. If this field moves with velocity \mathbf{v} , it generates a voltage V in an electrical conductor of length l . [Iron filings picture from Jiles, 1991.]

We can therefore measure the strength of the magnetic induction (the density of magnetic flux lines) by moving a conductive wire through the magnetic field (Figure 1.3b).

Magnetic induction can be thought of as something that creates a potential difference with voltage V in a conductor of length l when the conductor moves relative to the magnetic induction B with velocity \mathbf{v} (see Figure 1.3b): $V = v l B$. From this, we can derive the unit of magnetic induction: the tesla (T). One tesla is the magnetic induction that generates a potential of 1 volt in a conductor of length 1 meter when moving at a rate of 1 meter per second. So now we know the units of \mathbf{B} : $V \cdot s \cdot m^{-2} = T$.

Another way of looking at \mathbf{B} is that if magnetic induction is the density of magnetic flux lines, it must be the flux Φ per unit area. So an increment of flux $d\Phi$ is the field magnitude B times the increment of area dA . The area here is the length of the wire l times its displacement ds in time dt . The instantaneous velocity is $dv = ds/dt$, so $d\Phi = B dA$, and the rate of change of flux is

$$\frac{d\Phi}{dt} = \left(\frac{ds}{dt}\right) B l = v B l = V. \quad (1.2)$$

Equation 1.2 is known as Faraday's Law and, in its most general form, is the fourth of Maxwell's equations. We see from Equation 1.2 that the units of magnetic flux must be a volt-second, which is a unit in its own right: the weber (Wb). The weber is defined as the amount of magnetic flux which, when passed through a one-turn coil of a conductor carrying a current of 1 ampere, produces an electric potential of 1 volt. This definition suggests a means to measure the strength of magnetic induction and is the basis of the "fluxgate" magnetometer.

1.4 MAGNETIC ENERGY

A magnetic moment \mathbf{m} in the presence of a magnetic field \mathbf{B} has a *magnetostatic energy* (E_m) associated with it. This energy tends to align compass needles with the magnetic field (see Figure 1.4). E_m is given by $-\mathbf{m} \cdot \mathbf{B}$ or $-mB \cos \theta$, where m and B are the magnitudes of \mathbf{m} and \mathbf{B} , respectively (see Section A.3.4 in the appendix for a review of vector multiplication). Magnetic energy has units of joules and is at a minimum when \mathbf{m} is aligned with \mathbf{B} .

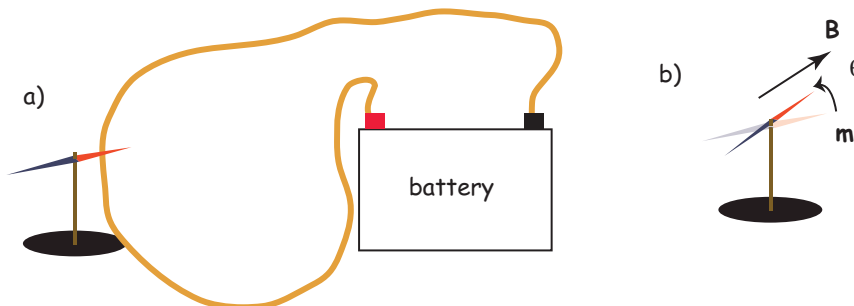


FIGURE 1.4. The magnetic moment \mathbf{m} of, for example, a compass needle will tend to align itself with a magnetic field \mathbf{B} . a) Example of when the field is produced by a current in a wire. b) The aligning energy is the magnetostatic energy, which is greatest when the angle θ between the two vectors of the magnetic moment \mathbf{m} and the magnetic field \mathbf{B} is at a maximum.

1.5 MAGNETIZATION AND MAGNETIC SUSCEPTIBILITY

Magnetization \mathbf{M} is a normalized moment (Am^2). We will use the symbol \mathbf{M} for volume normalization (units of Am^{-1}) and Ω for mass normalization (units of $\text{Am}^2\text{kg}^{-1}$). Volume-normalized magnetization therefore has the same units as \mathbf{H} , implying that there is a current somewhere, even in permanent magnets. In the classical view (pre-quantum mechanics), sub-atomic charges such as protons and electrons can be thought of as tracing out tiny circuits and behaving as tiny magnetic moments. They respond to external magnetic fields and give rise to an induced magnetization. The relationship between the magnetization induced in a material \mathbf{M}_I and the external field \mathbf{H} is defined as

$$\mathbf{M}_I = \chi_b \mathbf{H}. \quad (1.3)$$

The parameter χ_b is known as the *bulk magnetic susceptibility* of the material; it can be a complicated function of orientation, temperature, state of stress, time scale of observation, and applied field, but it is often treated as a scalar. Because \mathbf{M} and \mathbf{H} have the same units, χ_b is dimensionless. In practice, the magnetic response of a substance to an applied field can be normalized by volume (as in Equation 1.3), or by mass, or

not normalized at all. We will use the symbol κ for mass-normalized susceptibility and K for the raw measurements (see Table 1.1) when necessary.

Certain materials can produce magnetic fields in the absence of external magnetic fields (i.e., they are permanent magnets). As we shall see in later chapters, these so-called “spontaneous” magnetic moments are also the result of spins of electrons that, in some crystals, act in a coordinated fashion, thereby producing a net magnetic field. The resulting spontaneous magnetization can be fixed by various mechanisms and can preserve records of ancient magnetic fields. This *remanent magnetization* forms the basis of the field of paleomagnetism and will be discussed at length in subsequent chapters.

1.6 RELATIONSHIP OF \mathbf{B} AND \mathbf{H}

\mathbf{B} and \mathbf{H} are closely related, and in paleomagnetic practice, both \mathbf{B} and \mathbf{H} are referred to as the “magnetic field.” Strictly speaking, \mathbf{B} is the induction, and \mathbf{H} is the field, but the distinction is often blurred. The relationship between \mathbf{B} and \mathbf{H} is given by

$$\mathbf{B} = \mu(\mathbf{H} + \mathbf{M}), \quad (1.4)$$

where μ is a physical constant known as *the permeability*. In a vacuum, this is the permeability of free space, μ_o . In the SI system, μ has dimensions of henries per meter, and μ_o is $4\pi \times 10^{-7} \text{H} \cdot \text{m}^{-1}$. In most cases of paleomagnetic interest, we are outside the magnetized body, so $\mathbf{M} = 0$, and $\mathbf{B} = \mu_o \mathbf{H}$.

1.7 A BRIEF TOUR OF MAGNETIC UNITS IN THE CGS SYSTEM

So far, we have derived magnetic units in terms of the Syst eme International (SI). In practice, you will notice that people frequently use what are known as cgs units, based on centimeters, grams, and seconds. You may wonder why any fuss would be made over using meters as opposed to centimeters because the conversion is trivial. With magnetic units, however, the conversion is far from trivial and has been the source of confusion and many errors. So, in the interest of clearing things up, we will briefly outline the cgs approach to magnetic units.

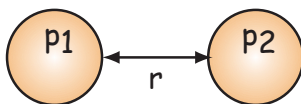


FIGURE 1.5. The force between two magnetic monopoles p_1, p_2 is $\frac{p_1 p_2}{r^2}$.

The derivation of magnetic units in cgs is entirely different from SI. The approach we will take here follows that of Cullity (1972). We start with the concept of a magnetic pole with strength p , instead of with current loops as we did for SI units. We will consider the force between two poles p_1, p_2 (see Figure 1.5) Coulomb's Law. This states that the force between two charges (q_1, q_2) is

$$F_{12} = k \frac{q_1 q_2}{r^2}, \quad (1.5)$$

where r is the distance between the two charges. In cgs units, the proportionality constant k is simply unity, whereas in SI units it is $\frac{1}{4\pi\epsilon_0}$, where $\epsilon_0 = \frac{10^7}{4\pi C^2}$, and c is the speed of light in a vacuum (hence $\epsilon_0 = 8.859 \cdot 10^{-12}$ As $V^{-1}m^{-1}$). (You can see why many people really prefer cgs, but we are not allowed to publish in cgs in most geophysical journals, so we must grin and bear it!)

For magnetic units, we use pole strength p_1, p_2 in units of *electrostatic units*, or esu, so Equation 1.5 becomes

$$F = \frac{p_1 p_2}{r^2}.$$

Force in cgs is in units of dynes (dyn), so

$$F = 1 \text{ dyn} = \frac{1 \text{ g cm}}{\text{s}^2} = \frac{1 \text{ esu}^2}{\text{cm}^2},$$

so 1 unit of pole strength is rather awkwardly $1 \text{ gm}^{1/2} \text{ cm}^{3/2} \text{ s}^{-1}$. Of course, there are no isolated magnetic poles in nature, only dipoles, but the concept of a unit of pole strength lies at the heart of cgs magnetic units.

A magnetic pole, as an isolated electric charge, would create a magnetic induction $\mu_o H$ in the space around it. One unit of field strength (defined as one *oersted*, or Oe) is the unit of field strength that exerts a force of 1 dyne on a unit of pole strength. The related induction ($\mu_o H$) has units of gauss or G.

The relationship between force, pole, and magnetic field is written as

$$F = p \mu_o H.$$

So, a pole with 1 pole strength, placed in a 1 Oe field, is acted on by a force of 1 dyne. This is the same force that it would experience if we placed the pole 1 centimeter away from another pole with 1 pole strength. Hence, the strength of the field produced by this monopole must be 1 oersted at 1 centimeter distance, and must decrease proportional to $1/r^2$.

Returning to the lines of force idea developed for magnetic fields earlier, let us define the oersted to be the magnetic field which would produce an induction with 1 unit of induction per square centimeter. Imagine a sphere with a radius r surrounding the magnetic monopole. The surface area of such a sphere is $4\pi r^2$. When the sphere is a unit sphere ($r = 1$) and the field strength at the surface is 1 Oe, then there must be a magnetic flux of 4π units of induction passing through it.

You will have noticed the use of the permeability of free space μ_o in the above treatment—a parameter missing in many books and articles using the cgs units. The

reason for this is that μ_o is unity in cgs units and simply converts from oersteds (\mathbf{H}) and gauss ($\mathbf{B} = \mu_o \mathbf{H}$). Therefore, in cgs units, B and H are used interchangeably. We inserted it in this derivation to remind us that there is a difference and that the difference becomes very important when we convert to SI because μ_o is not unity, but $4\pi \times 10^{-7}$! For conversion between commonly used cgs and SI parameters, please refer to Table 1.7.

Proceeding to the notion of magnetic moment, from a cgs point of view, we start with a magnet of length l with two poles of strength p at each end. Placing the magnet in a field $\mu_o \mathbf{H}$, we find that it experiences a torque Γ proportional to p, l , and \mathbf{H} such that

$$\Gamma = pl \times \mu_o \mathbf{H}. \quad (1.6)$$

Recalling our earlier discussion of magnetic moment, you will realize that pl is simply the magnetic moment m . This line of reasoning also makes clear why it is called a “moment.” The units of torque are energy, which are ergs in cgs, so the units of magnetic moment are technically erg per gauss. But because of the “silent” μ_o in cgs, magnetic moment is most often defined as erg per oersted. We therefore follow convention and define the “electromagnetic unit” (emu) as being $1 \text{ erg} \cdot \text{oe}^{-1}$. (Some use emu to refer to the magnetization [volume normalized moment, see above], but this is incorrect and a source of a lot of confusion.)

TABLE 1.1: CONVERSION BETWEEN SI AND CGS UNITS.

Parameter	SI unit	cgs unit	Conversion
Magnetic moment (\mathbf{m})	Am^2	emu	$1 \text{ A m}^2 = 10^3 \text{ emu}$
Magnetization			
by volume (\mathbf{M})	Am^{-1}	emu cm^{-3}	$1 \text{ Am}^{-1} = 10^{-3} \text{ emu cm}^{-3}$
by mass (Ω)	$\text{Am}^2 \text{kg}^{-1}$	emu gm^{-1}	$1 \text{ Am}^2 \text{kg}^{-1} = 1 \text{ emu gm}^{-1}$
Magnetic field (\mathbf{H})	Am^{-1}	Oersted (oe)	$1 \text{ Am}^{-1} = 4\pi \times 10^{-3} \text{ oe}$
Magnetic induction (\mathbf{B})	T	Gauss (G)	$1 \text{ T} = 10^4 \text{ G}$
Permeability			
of free space (μ_o)	Hm^{-1}	1	$4\pi \times 10^{-7} \text{ Hm}^{-1} = 1$
Susceptibility			
total ($\chi: \frac{\mathbf{m}}{\mathbf{H}}$)	m^3	emu oe^{-1}	$1 \text{ m}^3 = \frac{10^6}{4\pi} \text{ emu oe}^{-1}$
by volume ($\chi: \frac{\mathbf{M}}{\mathbf{H}}$)	—	$\text{emu cm}^{-3} \text{ oe}^{-1}$	$1 \text{ S.I.} = \frac{1}{4\pi} \text{ emu cm}^{-3} \text{ oe}^{-1}$
by mass ($\kappa: \frac{\mathbf{m}}{m} \cdot \frac{1}{\mathbf{H}}$)	$\text{m}^3 \text{kg}^{-1}$	$\text{emu g}^{-1} \text{ oe}^{-1}$	$1 \text{ m}^3 \text{kg}^{-1} = \frac{10^3}{4\pi} \text{ emu g}^{-1} \text{ oe}^{-1}$

Note. $1 \text{ H} = \text{kg m}^2 \text{A}^{-2} \text{s}^{-2}$, $1 \text{ emu} = 1 \text{ G cm}^3$, $B = \mu_o H$ (in vacuum), $1 \text{ T} = \text{kg A}^{-1} \text{s}^{-2}$

1.8 THE MAGNETIC POTENTIAL

An isolated electrical charge produces an electrical field that begins at the source (the charge) and spreads (diverges) outward (see Figure 1.6a). Because there is no return

flux to an oppositely charged “sink,” there is a net flux out of the dashed box shown in the figure. The *divergence* of the electrical field is defined as $\nabla \cdot \mathbf{E}$, which quantifies the net flux (see Section A.3.6 for more). In the case of the field around an electric charge, the divergence is non-zero.

Magnetic fields are different from electrical fields, in that there is no equivalent to an isolated electrical charge; there are only pairs of “opposite charges”—magnetic *dipoles*. Therefore, any line of flux starting at one magnetic pole returns to its sister pole, and there is no net flux out of the box shown in Figure 1.6b; the magnetic field has no divergence (Figure 1.6b). This property of magnetic fields is another of Maxwell’s equations: $\nabla \cdot \mathbf{B} = 0$.

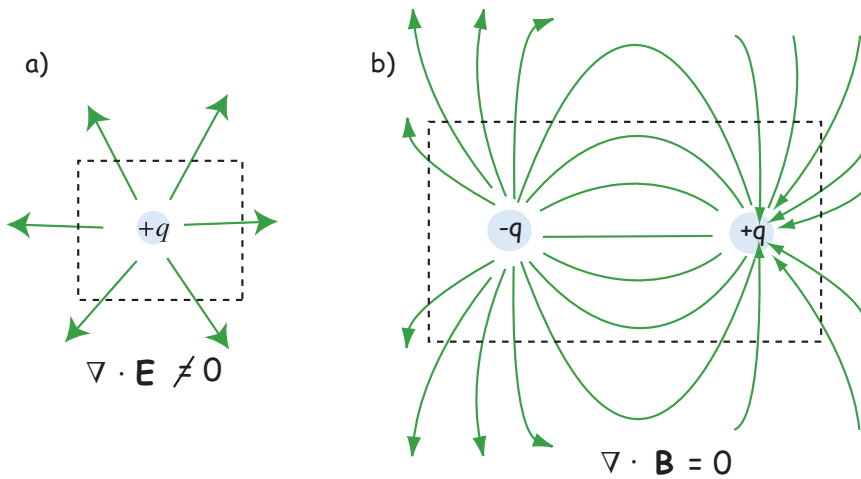


FIGURE 1.6. a) An electric charge produces a field that diverges out from the source. There is a net flux out of the dashed box, quantified by the divergence ($\nabla \cdot \mathbf{E}$), which is proportional to the magnitude of the sources inside the box. b) There are no isolated magnetic charges, only dipoles. Within any space (e.g., the dashed box), any flux line that goes out must return. The divergence of such a field is zero, i.e., $\nabla \cdot \mathbf{B} = 0$.

In the special case away from electric currents and magnetic sources ($B = \mu_o H$), the magnetic field can be written as the gradient of a scalar field that is known as the *magnetic potential*, ψ_m , i.e.,

$$\mathbf{H} = -\nabla\psi_m.$$

The presence of a magnetic moment \mathbf{m} creates a magnetic field, which is the gradient of some scalar field. To gain a better intuitive feeling about the relationship between scalar fields and their gradient vector fields, see Section A.3.6. Because the divergence of the magnetic field is zero, by definition, the divergence of the gradient of the scalar field is also zero, or $\nabla^2\psi_m = 0$. The operator ∇^2 is called the Laplacian, and $\nabla^2\psi_m = 0$ is *Laplace’s equation*. This will be the starting point for spherical harmonic analysis of the geomagnetic field discussed briefly in Chapter 2.

The curl of the magnetic field ($\nabla \times \mathbf{H}$) depends on the current density and is not always zero, and magnetic fields cannot generally be represented as the gradient of a scalar field. Laplace's equation is only valid outside the magnetic sources and away from currents.

So what is this magnetic potential and how does it relate to the magnetic moments that give rise to the magnetic field? Whatever it is, it has to satisfy Laplace's equation, so we turn to solutions of Laplace's equation for help. One solution is to define the magnetic potential ψ_m as a function of the vector \mathbf{r} , with radial distance r and the angle θ from the moment. Given a *dipole moment* \mathbf{m} , a solution to Laplace's equation is

$$\psi_m = \frac{\mathbf{m} \cdot \mathbf{r}}{4\pi r^3} = \frac{m \cos \theta}{4\pi r^2}. \quad (1.7)$$

You can verify this by making sure that $\nabla^2 \psi_m = 0$.

The radial and tangential components of \mathbf{H} at P (Figure 1.7) then would be

$$H_r = -\frac{\partial \psi_m}{\partial r} = \frac{1}{4\pi} \frac{2m \cos \theta}{r^3}$$

and

$$H_\theta = -\frac{1}{r} \frac{\partial \psi_m}{\partial \theta} = \frac{m \sin \theta}{4\pi r^3}, \quad (1.8)$$

respectively.

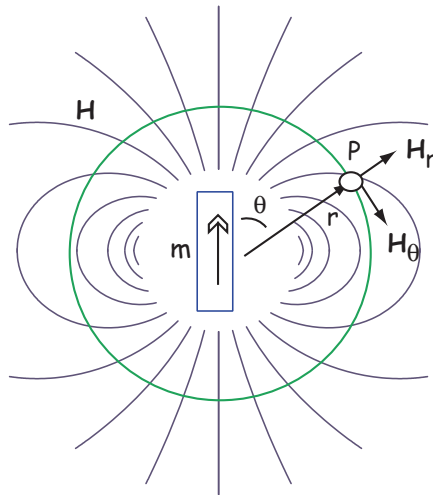


FIGURE 1.7. Field \mathbf{H} produced at point P by a magnetic moment \mathbf{m} . \mathbf{H}_r and \mathbf{H}_θ are the radial and tangential fields, respectively.

1.9 ORIGIN OF THE GEOMAGNETIC FIELD

Measurement and description of the geomagnetic field and its spatial and temporal variations constitute one of the oldest geophysical disciplines. However, our ability to

describe the field far exceeds our understanding of its origin. All plausible theories involve generation of the geomagnetic field within the fluid outer core of the Earth by some form of magnetohydrodynamic dynamo. Attempts to solve the full mathematical complexities of magnetohydrodynamics succeeded only in 1995 (Glatzmaier and Roberts, 1995).

Quantitative treatment of magnetohydrodynamics is (mercifully) beyond the scope of this book, but we can provide a qualitative explanation. The first step is to gain some appreciation for what is meant by a self-exciting dynamo. Maxwell's equations tell us that electric and changing magnetic fields are closely linked and can affect each other. Moving an electrical conductor through a magnetic field will cause electrons to flow, generating an electrical current. This is the principal of electric motors. A simple electromechanical disk-dynamo model such as that shown in Figure 1.8 contains the essential elements of a self-exciting dynamo. The model is constructed with a copper disk rotating attached to an electrically conducting (e.g., brass) axle. An initial magnetic induction field, B , is perpendicular to the copper disk in an upward direction. Electrons in the copper disk experience a push from the magnetic field known as the Lorentz force, F_L , when they pass through the field.

The Lorentz force is given by

$$F_L = q\mathbf{v} \times \mathbf{B}, \quad (1.9)$$

where q is the electrical charge of the electrons, and v is their velocity. The Lorentz force on the electrons is directed toward the axle of the disk, and the resulting electrical current flow is toward the outside of the disk (Figure 1.8).

Brush connectors are used to tap the electrical current from the disk, and the current passes through a coil under the disk. This coil is cleverly wound so that the electrical current produces a magnetic induction field in the same direction as the original field. The electrical circuit is a positive feedback system that reinforces the original magnetic induction field. The entire disk-dynamo model is a self-exciting dynamo. As long as the disk keeps rotating, the electrical current will flow, and the magnetic field will be sustained even if the original field disappears.

With this simple model, we encounter the essential elements of any self-exciting dynamo:

1. A moving electrical conductor is required and is represented by the rotating copper disk.
2. An initial magnetic field is required.
3. An interaction between the magnetic field and the conductor must take place to provide reinforcement of the original magnetic field. In the model, this interaction is the Lorentz force with the coil acting as a positive feedback (self-exciting) circuit.
4. Energy must be supplied to overcome electrical resistivity losses. In the model, energy must be supplied to keep the disk rotating.

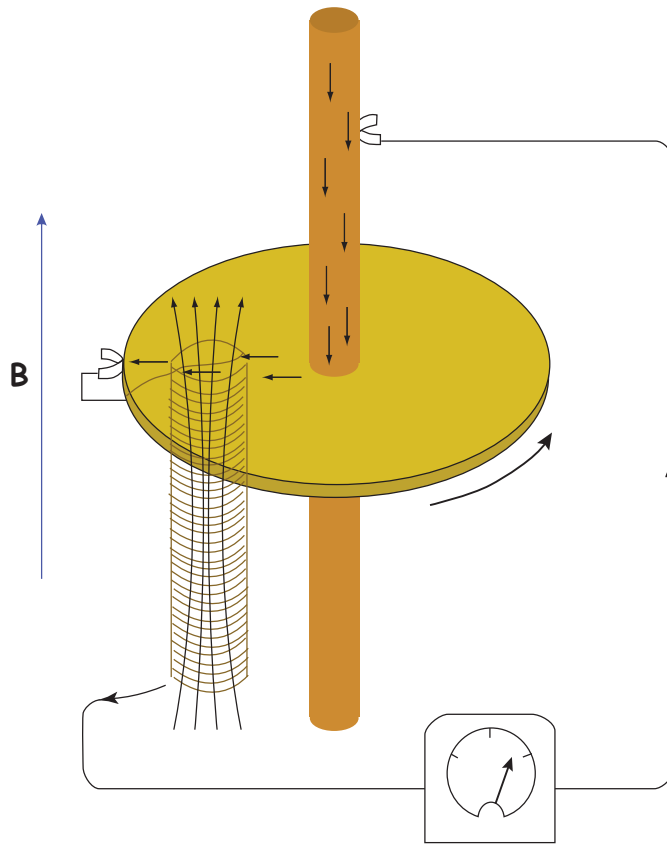


FIGURE 1.8. Self-exciting disk dynamo. An initial field B is reinforced by dynamo action. When the conducting plate is rotated, electric charge moves perpendicular to the magnetic field, setting up an electric potential between the inner conducting rod and the outer rim of the plate. If the conducting plate is connected to a coil, wound such that a current produces a magnetic field in the same direction as the initial field, then the magnetic field is enhanced. [After Elsasser, 1958; redrawn from R. F. Butler, 1992.]

More complicated setups using two disks whose fields interact with one another generate chaotic magnetic behavior that can switch polarities even if the mechanical motion remains steady. Certainly, no one proposes that systems of disks and feedback coils exist in the Earth's core. But interaction between the magnetic field and the electrically conducting iron-nickel alloy in the outer core can produce a positive feedback and allow the Earth's core to operate as a self-exciting magnetohydrodynamic dynamo. For reasonable electrical conductivities, fluid viscosity, and plausible convective fluid motions in the Earth's outer core, the fluid motions can regenerate the magnetic field that is lost through electrical resistivity. There is a balance between fluid motions regenerating the magnetic field and loss of magnetic field because of electrical resistivity. The dominant portion of the geomagnetic field detectable at the surface is essentially dipolar with the axis of the dipole nearly parallel to the rotational

axis of the Earth. Rotation of the Earth must therefore be a controlling factor on the time-averaged fluid motions in the outer core. It should also be pointed out that the magnetohydrodynamic dynamo can operate in either polarity of the dipole. Thus, there is no contradiction between the observation of reversals of the geomagnetic dipole and magnetohydrodynamic generation of the geomagnetic field. However, understanding the special interactions of fluid motions and magnetic field that produce geomagnetic reversals is a major challenge.

As wise economists have long observed, there is no free lunch. The geomagnetic field is no exception. Because of ohmic dissipation of energy, there is a requirement for energy input to drive the magnetohydrodynamic fluid motions, and thereby sustain the geomagnetic field. Estimates of the power (energy per unit time) required to generate the geomagnetic field are about 10^{13} W (roughly the output of 10^4 nuclear power plants). This is about one-fourth of the total geothermal flux, so the energy involved in generation of the geomagnetic field is a substantial part of the Earth's heat budget.

Many sources of this energy have been proposed, and ideas on this topic have changed over the years. The energy sources that are currently thought to be most reasonable are a combination of cooling of the Earth's core, with attendant freezing of the outer core and growth of the solid inner core. The inner core is pure iron, whereas the liquid outer core is some 15% nickel (and probably has trace amounts of other elements as well). The freezing of the inner core therefore generates a buoyancy force as the remaining liquid becomes more enriched in the lighter elements. These energy sources are sufficient to power the fluid motions of the outer core required to generate the geomagnetic field.

SUPPLEMENTAL READINGS: Jiles (1991), Chapter 1; Cullity (1972), Chapter 1.

1.10 PROBLEMS

PROBLEM 1

In axisymmetric spherical coordinate, ∇ (the gradient operator), is given by

$$\nabla = \frac{\partial}{\partial r} + \frac{\partial}{r\partial\theta}.$$

We also know that

$$\mathbf{H} = -\nabla\psi_m$$

and that ψ_m is a scalar function of position

$$\psi_m = \frac{\mathbf{m} \cdot \mathbf{r}}{4\pi r^3}.$$

Find the radial and tangential components of \mathbf{H} if \mathbf{m} is 80 ZAm² (remember that “Z” stands for Zeta, which stands for 10²¹), r is 6×10^6 m, and θ is 45°. What are these field values in terms of \mathbf{B} (teslas)?

PROBLEM 2

In this problem, we introduce beginning programming skills. Read Appendix F.1 for help installing and beginning to use Python. Then work the rest of this problem.

a) Write an interactive python script to convert induction, moment, and magnetic field quantities in cgs units to SI units. Use the conversion factors in Table 1.7. Use your program to convert the following:

- i) $B = 3.5 \times 10^5$ G
- ii) $m = 2.78 \times 10^{-20}$ G cm³
- iii) $H = 128$ oe

b) Modify your script to allow conversion from cgs => SI or SI => cgs. Rerun it to convert your answers from (a) back to cgs.

HINTS: Review Appendix F.1. Use the **raw_input** command in Section F.1.10 for getting information into a program. You will need to ask what the units of the input data will be and what output units are desired as well as what the number is. When you read in data using **raw_input**, it comes in as a string variable, and you will have to convert it to a floating point in order to change units.

PROBLEM 3

Figure 1.9 shows a meridional cross-section through the Earth in the plane of a magnetic dipole source m . At the location directly above the dipole, the field from the dipole is directed vertically downward and has intensity $10 \mu\text{T}$. The dipole source is placed at 3480 km from the center of the Earth. Assume a mean Earth radius of 6370 km. Adapt the geometry of Figure 1.7 and the equations describing the magnetic field of a dipole to the model dipole in Figure 1.9.

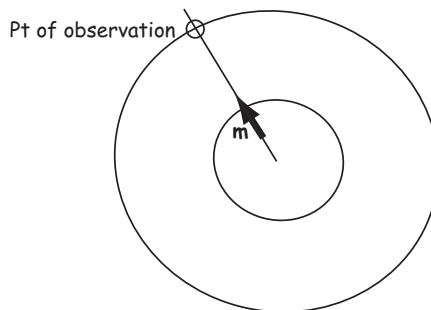


FIGURE 1.9. A magnetic dipole source (\mathbf{m}) located 3480 km below the point of observation at 45° latitude. The average radius of the Earth is 6370 km. The field at the point of observation is directed downward (toward the center) with a magnitude of $10 \mu\text{T}$.

a) Calculate the magnetic dipole moment of the model dipole. Remember to keep track of your units!

b) Compare this field to the total field produced by a centered axial magnetic dipole moment (i.e., one that is pointing straight up and is in the center of the circles) equivalent to that of the present geomagnetic field ($m \sim 80 \text{ ZAm}^2$; $Z = 10^{21}$). Assume a latitude for the point of observation of 60° . [HINT: the angle θ in Equation 1.7 is the co-latitude, not the latitude.]

PROBLEM 4

Knowing that $B = \mu_o H$, work out the fundamental units of μ_o in SI units.

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