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Trajectory Planning for a Four-Wheel-Steering Vehicle

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Abstract

This paper develops a trajectory planning algorithm for a four-wheel-steering (4WS) vehicle based on vehicle kinematics. The flexibility offered by the steering is utilized fully in the trajectory planning. A two-part trajectory planning algorithm consists of the steering planning and velocity planning. The limits of vehicle mechanism and drive torque are taken into account. Simulation results are presented to illustrate the application of the proposed algorithm.

Introduction

Research works on Autonomous Guided Vehicles (AGV) have been extensively carried out in the last four decades, and still in the process of rapid development at present. One application of AGV has been explored in harbor automation operation. Figure 1 shows an AGV for transporting cargo containers in harbor area. In order to move in cluster space within the harbor with flexibility, the vehicle structure is designed so that all the four wheels can be driven and steered individually. This kind of vehicle is referred to as a *Four-Wheel-Steering (4WS)* vehicle.

Figure 1 A 4WS vehicle for cargo transport

The problem of *trajectory planning and generation* is to design the configuration of the vehicle motion as a function of time, as well as generate corresponding reference inputs to the trajectory tracking system so that the vehicle will move along a specified path. The vehicle trajectory planning is not as well researched as that of vehicle path planning, and to our knowledge, only several works [2] [4]-[12] addressed this problem.

This paper develops a methodology that consists of so-called rotation planning and translation planning for the trajectory planning for 4WS vehicles. The methodology utilizes the flexibility of the 4WS vehicle to plan the vehicle orientation.

Vehicle Kinematic Analysis

Under the basic assumptions of planar motion, rigid body and non-slippage of tire, the large size vehicle with four steering wheels as shown in Figure 1 can be approximated using a *bicycle model.* To describe the vehicle motion, a global coordinate *X-Y* is fixed on the horizontal plane on which the vehicle moves. The motion status of the vehicle at an arbitrary moment can be described using the *Bicycle Model* as illustrated in Figure 2.

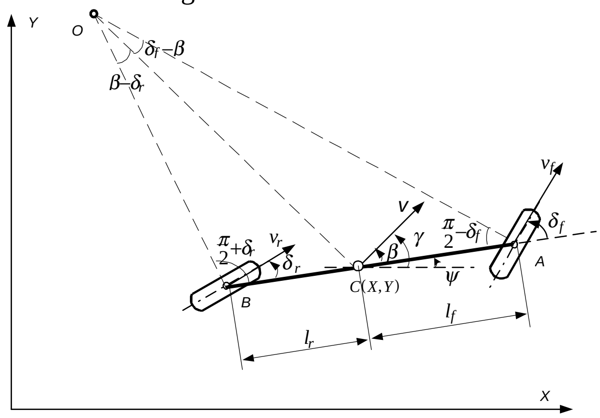

Figure 2 Kinematic model

Reference point *C* is chosen at the center of gravity of the vehicle body. Its coordinates (X,Y) represents the position of the vehicle; **Vehicle Velocity *v*** is defined at the reference point *C*; **Heading Angle ψ** is the angle from the *X*-axis to the longitudinal axis of the vehicle body *AB*; **Course Angle γ** is the angle from the X-axis to the direction of the vehicle velocity, v; **Side-slip Angle β** is the angle from the longitudinal axis of the vehicle body AB to the direction of vehicle velocity, v; **Turning radius r** is the distance between the reference point C and *the Instant Rotating Center* (*IRC*) O; **Front Wheel Velocity v_f** is the velocity defined at the intersection of the mid-plane of the virtual front wheel and the front wheel axle, A; **Rear Wheel Velocity v_r** is the velocity defined at the intersection of the mid-plane of the virtual rear wheel and the rear wheel axle, B; **Front wheel steering angle δ_f** is the angle from the longitudinal axis of the vehicle body AB to the direction of v_f; **Rear wheel steering angle δ_r** is the angle from the longitudinal axis of the vehicle body AB to the direction of v_r.

Referring to Figure 2, the kinematic model of 4WS vehicles can be expressed as follows

$$\dot{X} = v\cos(\psi + \beta) \tag{1-1}$$

$$\dot{Y} = v\sin(\psi + \beta) \tag{1-2}$$

$$\dot{\psi} = \frac{v\cos\beta\left(\tan\delta_f + \tan\delta_r\right)}{l_f + l_r} \tag{1-3}$$

where

$$\beta = \arctan\frac{l_f \tan\delta_r + l_r \tan\delta_f}{l_f + l_r} \tag{1-4}$$

and

$$v = \frac{v_f \cos\delta_f + v_r \cos\delta_r}{2\cos\beta} \tag{1-5}$$

In this model, there are four inputs: two steering angles, δ_f and δ_r, and two wheel velocities, v_f and v_r. The state variables of kinematic motion are the vehicle configuration (X, Y, ψ).

We assume that both the front and rear wheels of this 4WS vehicle can only vary within the following vehicular mechanical range

$$-\delta_{\max} \le \delta_f \le \delta_{\max} \tag{1-6}$$

$$-\delta_{\max} \le \delta_r \le \delta_{\max} \tag{1-7}$$

where $\delta_{\max}$ is the maximum of the steering angles to both sides. The side-slip angle reaches its extreme value only when both front and rear steering angles reach their positive or negative maximum simultaneously.

$$-\delta_{\max} \le \beta \le \delta_{\max} \tag{1-8}$$

Problem Formulation

The trajectory of a 4WS vehicle can be expressed as

$$\mathbf{T}(t) = \begin{Bmatrix} X(t) \\ Y(t) \\ \psi(t) \end{Bmatrix}, \quad t: t_0 \to t_n \tag{1-9}$$

where t_0 is the initial moment of motion, t_n is the final moment of motion and '→' indicates that time t changes from t_0 to t_n.

The path is a geometric curve defined in the XY coordinates

$$Y = P(X), \quad X: X_0 \to X_n;\ Y: Y_0 \to Y_n \tag{1-10}$$

where $\{X_0, Y_0\}$ are the coordinates of the vehicle at the starting point, and $\{X_n, Y_n\}$ are the coordinates of the vehicle at the destination. It can also be expressed as the function of a single variable s,

$$\mathbf{P}(s) = \begin{Bmatrix} X(s) \\ Y(s) \end{Bmatrix}, \quad s: 0 \to s_n \tag{1-11}$$

where s represents the *length of path* traveled by the reference point of vehicle from the starting point, s_n indicates the distance from the starting point to the destination.

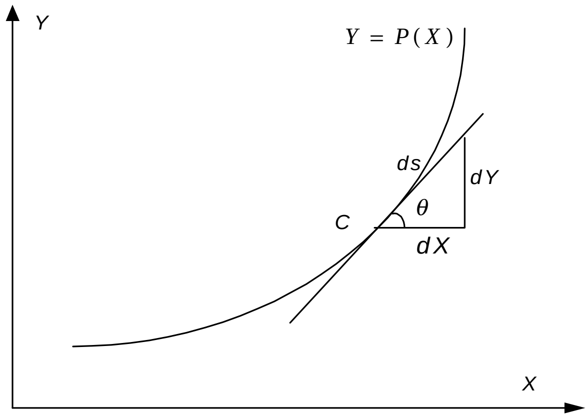

Figure 3 Path and tangential direction angle

Suppose that the path is smooth. At a point C on the path, the incremental change, ds, can be expressed as

$$ds = \sqrt{dX^2 + dY^2} \tag{1-12}$$

where dX and dY are the incremental changes in X and Y directions, respectively. The tangential angle θ at point C can be expressed as

$$\theta = \arctan \frac{dY}{dX} \tag{1-13}$$

If the function $\theta(s)$ is differentiable with respect to s, the *path curvature* can be obtained as

$$\kappa = \frac{d\theta}{ds} = \frac{d^2Y/dX^2}{\left(1+\left(dY/dX\right)^2\right)^{3/2}} \tag{1-14}$$

For a given path, both the *tangent direction angle* and the *path curvature*, if any, are functions of *s*.

Based on the definitions of trajectory and path, the objective of the trajectory planning along specified path can be described as follows: Given a path which the vehicle is expected to follow

$$\mathbf{P}_d(s) = \begin{Bmatrix} X_d(s) \\ Y_d(s) \end{Bmatrix}, \qquad s: 0 \rightarrow s_n \tag{1-15}$$

Design a trajectory of a 4WS vehicle configuration

$$\mathbf{T}(t) = \begin{Bmatrix} X(t) \\ Y(t) \\ \psi(t) \end{Bmatrix}, \qquad t: t_0 \rightarrow t_n \tag{1-16}$$

The designed trajectory should also meet the following criteria to satisfy the requirement of the transportation in port areas. i). *Safety criterion*: In port operation, a fleet of vehicles travels closely next to each other. To avoid potential collision, the trajectory of the vehicle motion should strictly follow the specified path. Furthermore, the orientation of vehicles should be planned so that the protrusion of the vehicle body to adjacent lanes be minimized; ii). *Efficiency criterion*: Under the hypothesis of the *Safety criterion*, the vehicle is expected to move at its highest feasible velocity to ensure the efficiency.

Rotation Planning

The orientation of 4WS vehicle body is not uniquely determined when moving along a specified path. One constraint is needed to fully determine the orientation of the vehicle body along the path. When a path is specified, so is the course angle of the vehicle. The heading angle of the vehicle along the path is

$$\psi(s) = \theta(s) - \beta(s) \tag{1-17}$$

Due the limit to the steering angles, the heading angle is expected within the range as follows

$$\theta(s) - \beta_{\max} \le \psi(s) \le \theta(s) + \beta_{\max} \tag{1-18}$$

Furthermore, its changing rate with respect to s should be

$$\left|\frac{d\psi}{ds}\right| \ge \frac{1}{r_{\min}} \tag{1-19}$$

where $r_{\min}$ is the minimum turning radius.

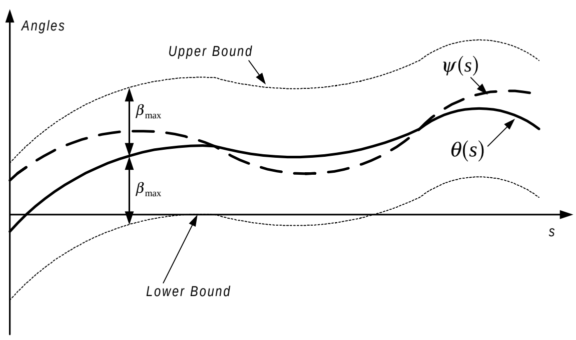

Figure 4 Range bounds on heading angle along path

As shown in Figure 4, the heading angle $\psi(s)$ should follow the course angle $\theta(s)$ within a range bound. The upper and lower bound of the heading angle are the *parallel shift* of the curve $\theta(s)$ with the amount of $\beta_{\max}$ in both up and down directions.

Two special maneuvers, the so-called *Zero-side-slip Maneuver* and *Parallel Steering Maneuver*, take advantage of the special kinematic characteristic of 4WS vehicles and are commonly used to meet the Safety criterion. In the following, we will show how these two maneuvers can be used in our rotation planning problem.

i). Zero-side-slip_Maneuver

In this maneuver, the side-slip angle is set to zero from the starting point s_0 to the ending point s_n when the vehicle moves along the path.

$$\beta(s) = 0, \quad s: s_0 \rightarrow s_n \tag{1-20}$$

The orientation of the vehicle $\psi(s)$ is set to match the tangential angle of the desired path $\theta_d(s)$

$$\psi(s) = \theta_d(s), \quad s{:}0 \to s_n \qquad (1\text{-}21)$$

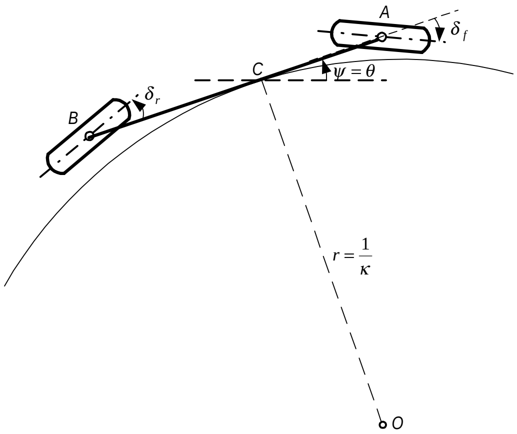

Figure 5 Zero-side-slip maneuver

This maneuver is desirable in vehicle motion since the vehicle body is always tangent to the path. As shown in Figure 5, it reduces the protrusion of the vehicle body to the neighboring lane to the minimum.

ii). Parallel Steering Maneuver

Parallel Steering is defined as that both two wheels are always steered at the same angle in the same direction. In this maneuver, two steering angles is set as follows

$$\delta_f = \delta_r \qquad (1\text{-}22)$$

and

$$\beta = \delta_f = \delta_r \qquad (1\text{-}23)$$

This leads to that the turning radius is always equal to infinity along the path.

$$r(s) = \infty, \quad s : s_0 \to s_n \qquad (1\text{-}24)$$

This implies that the vehicle translates without changing its orientation during the motion. Thus we have

$$\psi(s) = \psi_0, \quad s{:}0 \to s_n \qquad (1\text{-}25)$$

where ψ_0 is the initial heading angle of the vehicle.

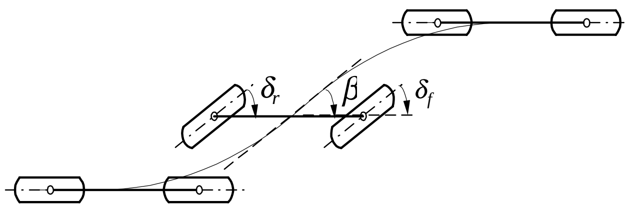

Figure 6 Parallel steering maneuver

This maneuver is very practical in vehicle lane-changing and obstacle-avoidance. As illustrated in Figure 6, the protrusion of vehicle body is eliminated compared to that of 2WS vehicles. In addition, the rotation of the vehicle is reduced as well, thus improves the vehicle stability at high speed.

Translation Planning

The aim of translation planning is to design the velocity profile $\dot{s}(s)$ of vehicle motion along the specified path. The feasible range of vehicle velocities is subjected to dynamic constraints. Some constraints, such as Limited driving and braking forces, Non-side-sliding constraints, are widely used in trajectory planning of 2WS vehicles [3], [10]. They can be used for the translation planning of 4WS vehicles.

In order to derive the dynamic constraints, a simplistic dynamic model is built as follows

$$f_\tau = m\ddot{s} \qquad (1\text{-}26)$$

$$f_n = m\dot{s}^2\kappa \qquad (1\text{-}27)$$

where f_τ and f_n are the net forces in the tangential and normal directions of the path, respectively, *m* is the mass of the vehicle, $\ddot{s}$ is the linear acceleration of the vehicle. Thus, $m\ddot{s}$ represents the inertial force due to the linear acceleration, and $m\dot{s}^2\kappa$ is the centrifugal force. Using this model, the dynamic constraints can be expressed in the following ways.

i). Bounds on Driving and Braking Forces

This is the main factor affecting the acceleration and deceleration of the vehicle motion. The driving force and the braking force are both bounded by the limitations to the engine power and the maximum braking torque. The vehicle acceleration is bounded as follows,

$$\frac{B_{\max}}{m} \le \ddot{s} \le \frac{D_{\max}}{m} \qquad (1\text{-}28)$$

where B_{max} and T_{max} are the maximum braking force and the maximum driving force, respectively.

ii). Non-side-sliding Constraint

In the case where the net friction force in the normal direction of the path exceeds the limit, the vehicle will skid and deviate from the desired path. To prevent the vehicle skidding away from the desired path, the following constraint should be observed

$$f_n \le \mu mg \qquad (1\text{-}29)$$

where μ is the friction coefficient between the tires and the ground. Substituting (1-27) into (1-29), we have the upper limit to the vehicle velocity when making turns

$$\dot{s} \le \sqrt{\frac{\mu g}{\kappa}} \qquad (1\text{-}30)$$

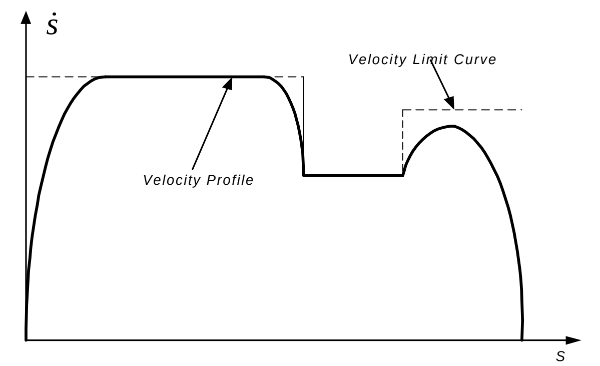

Figure 7 Vehicle velocity profile along a desired path

Plotting the velocity limits for all points along the path creates a velocity limit curve in the plane $s-\dot{s}$. The limit curve represents the upper bound of the vehicle velocity. In practice, the vehicle is often required to move at its maximum allowable velocity. The velocity profile is generated by integrating the maximum acceleration or deceleration along the path. Figure 7 shows one example of a velocity profile. In these segments of *s*, the maximum allowable speeds are limited by various conditions. Thus the vehicle starts and ends with zero speed but moves at the maximum allowable speeds subject to the constraints.

Example and Simulation

The vehicle is expected to move along a combined path subject to dynamic constraints. The desired path is a combination of straight lines and circular arcs. Initial and final velocities: $v_0 = 0,\ v_n = 0$. Side-slip angle along the path: $\beta(s) = 0,\ \ s{:}0 \to s_n$. Maximum velocity along straight lines: $v_{\max} = 10$ m/s. Maximum driving force: $D_{\max} = 1500\ N$, Maximum braking force: $B_{\max} = 2500\ N$. Vehicle mass: $\mathrm{m} = 500\ kg$. Friction factor between wheels and the road: $\mu = 0.5$.

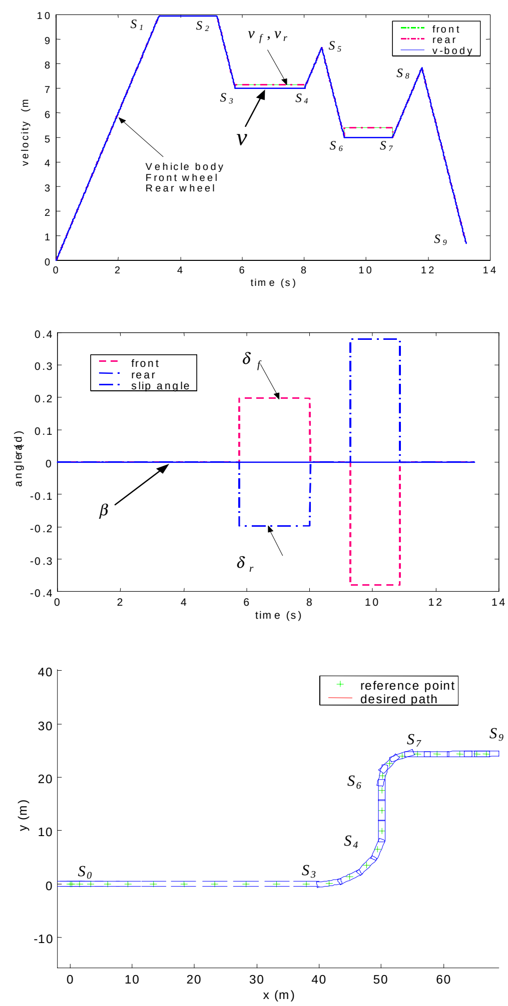

Figure 8 Moving along combined path subject to dynamic constraints

From s_0 to s_1, the vehicle speed increases with maximum acceleration that is limited by maximum engine torque (1-28) until the vehicle velocity reaches its maximum value (1-30) at point s_1. From s_1 to s_2, the vehicle moves at a constant velocity. From s_2 to s_3, the vehicle reduces speed with the maximum deceleration produced by maximum braking force (1-28). From s_3 to s_4, the vehicle moves along ¼ circle at the maximum velocity subject to side-slip constraint (1-28). From s_4 to s_6, the vehicle accelerates and then decelerates with the maximum acceleration and maximum decelerations, respectively. From s_6 to s_7, the vehicle turns along another arc with a smaller radius at its maximum velocity limited by the sliding constraint (1-28). From s_7 to s_9, the vehicle accelerates again and then decelerates until the vehicle velocity reaches zero. Note that the final velocity does not reach zero exactly because of errors caused by integration. Note that steering angles change discontinuously at connecting points between straight lines and arcs s_3, s_4, s_6 and s_7.

This result shows how to plan the vehicle velocity along a combined path when dynamic constraints are taken into account. The inputs generated make the vehicle move along the combined path at its highest feasible velocity.

Conclusion

Flexibility of vehicle orientation when moving along the specified path is exploited. Based on this feature, the trajectory planning of 4WS vehicle consists of two parts: rotation planning and translation planning. In rotation planning, the limitation of turning radius are taken into account. Two special maneuvers which are not available for 2WS vehicles, namely, *zero side-slip* and *parallel steering*, are introduced into the rotation planning. In translation planning, several dynamic constraints, such as based on the driving and braking forces and the non-side-sliding constraint are taken into account. The vehicle velocity profile is designed so that the vehicle will move at the maximum allowable speed without violating these constraints. Simulation example is used to evaluate the algorithm and to show the application of the algorithm in practice.

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