SWCC measurements on fractal gradings, evaluation methods, identified parameters, parameter error, uses

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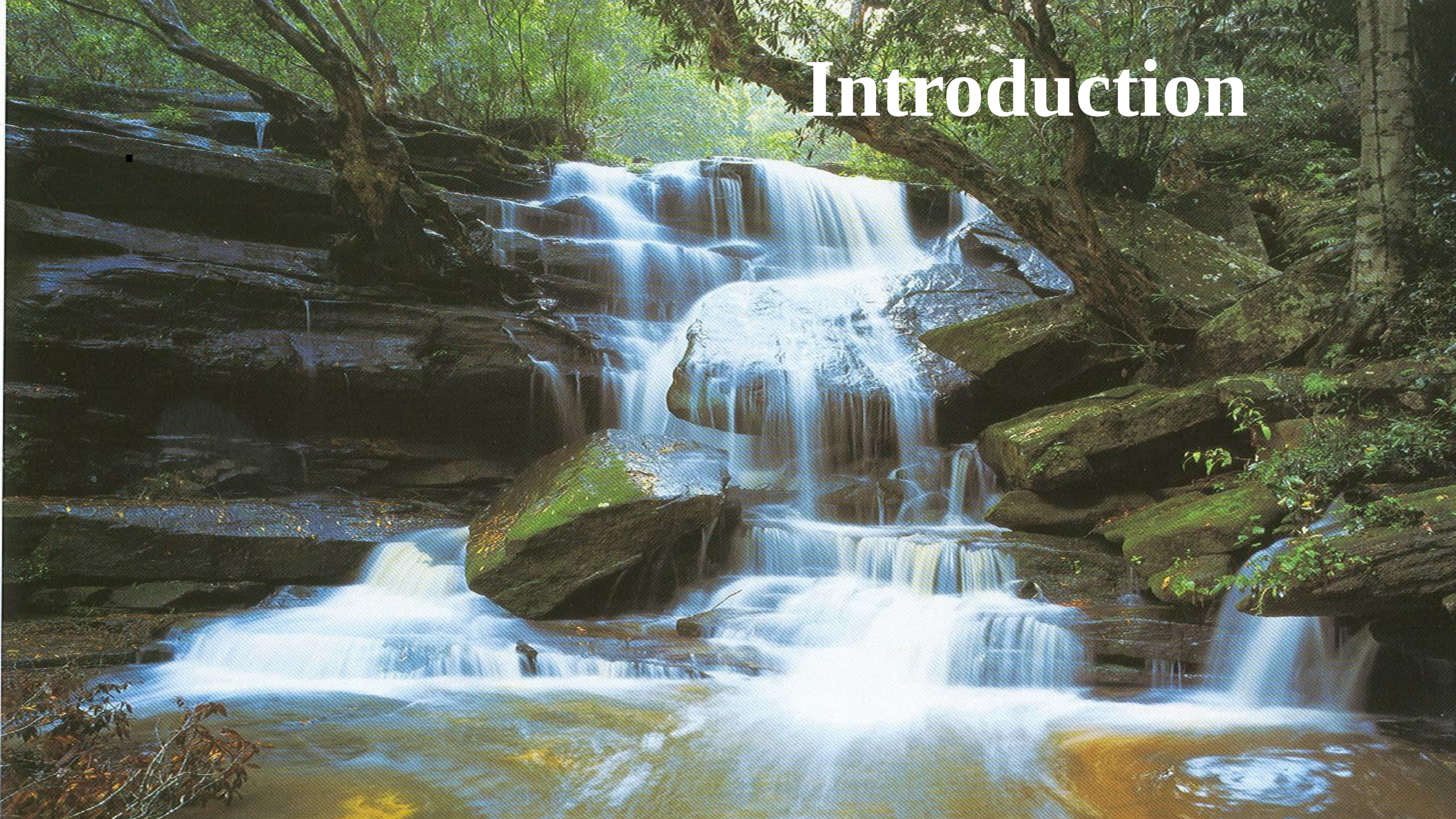
Introduction



In a large testing program of SWCC measurements on sand, the data were evaluated with the Van Genuchten model. Double samples were made for every mixture and a very first check of the reliability of the measurement could be made by comparing the inverse problem solutions of these.

An automatic and precise inverse problem solver was used, the reliability of the solution was tested.

Out of the models

- Gardner (1958),
- Fredlund – Xing (1994)
- the simplified
- and non-simplified Van Genuchten (1980)

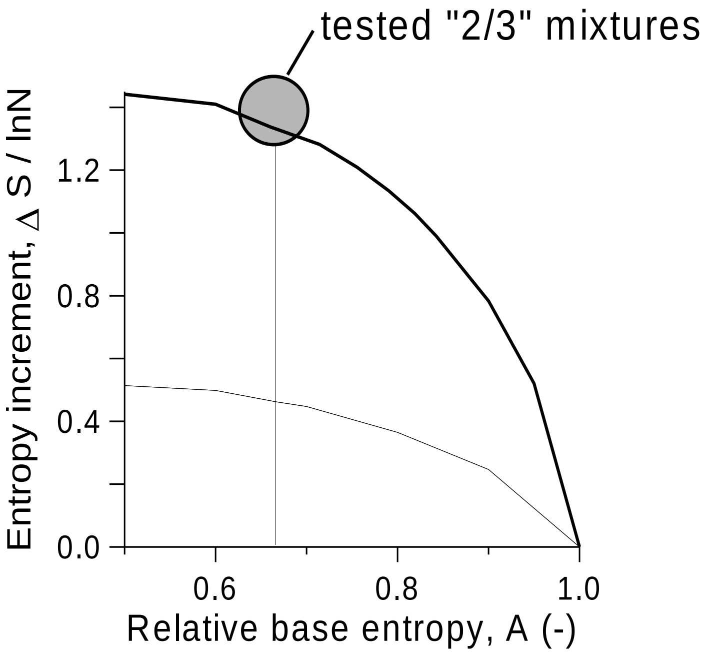
the non-simplified Van Genuchten model was with the smallest error.

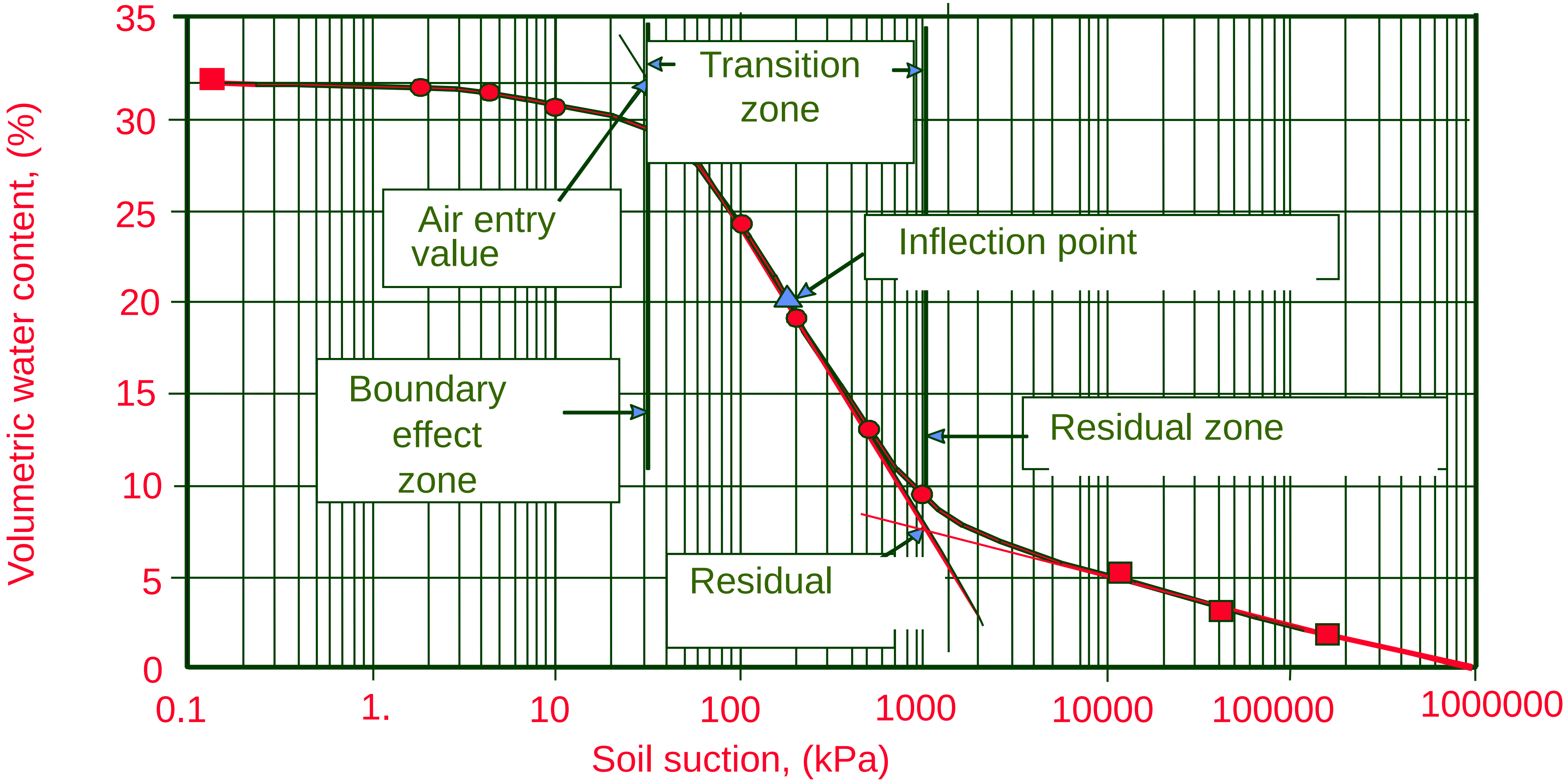
This work is related to the Van Genuchten model

Materials and Methods
Measurements

Soils

- 7 sand fractions and
- 21 internally stable, artificial sand mixtures with finite fractal distribution.

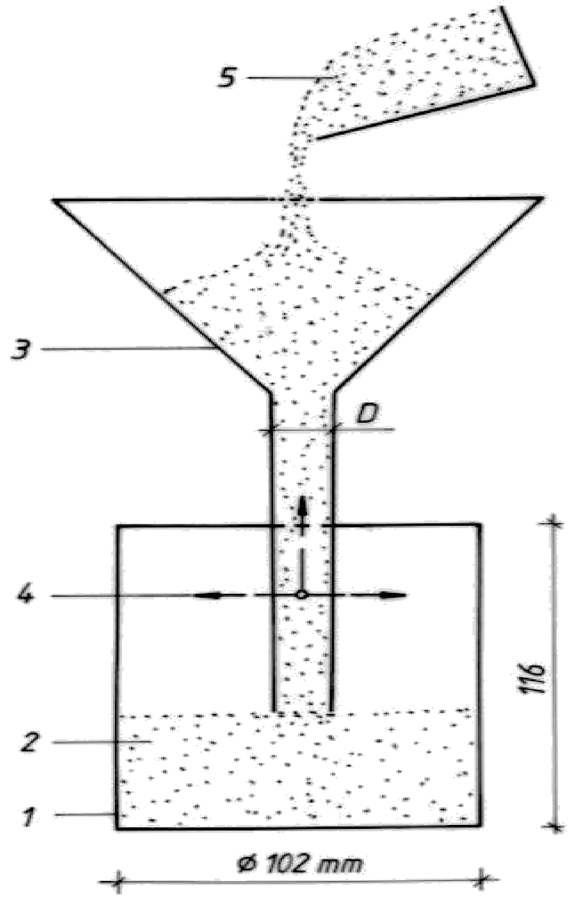



Transition zone
Air entry value
Inflection point
Boundary effect zone
Residual zone
Residual
Volumetric water content, (%)
35
30
25
20
15
10
5
0
0.1
1.
10
100
1000
10000
100000
1000000
Soil suction, (kPa)

Stage 1 Load suction step (kPa)	**Semi-permeable membrane**	**Suction load application**
0.1	Sand	gravitational
0.25	sand	gravitational
1	fine sand	gravitational
3.15	fine sand	gravitational
10.0	Kaolinit	gravitational
20.0	Kaolinit	gravitational
50.1	Kaolinit	vacuum

Sample preparation

- Duplicate samples in the e_{max} state
- Saturation



SWCC measurement additional load steps at low suctions for coarse sand

Completed Load suction step (kPa)	Material for the semi-permeable membrane	The method of the suction load application
0.1	Sand	gravitational
0.25	sand	gravitational
0.4*	fine sand	gravitational
0.7*	fine sand	gravitational
1	fine sand	gravitational
1.5*	fine sand	gravitational
2*	fine sand	gravitational
3.15	fine sand	gravitational
10.0	Kaolinit	gravitational
20.0	Kaolinit	gravitational
50.1	Kaolinit	vacuum

Approximate threshold suction values for fractions 1-4

Fraction	Air entry suction [kPa]	Residual suction [kPa]
1	0.25	0.4
2	0.4	0.7
3	0.7	2
4	2	3


Inverse problem solution
METHOD

$$w = w_r + \frac{w_s - w_r}{\left(1+\left[a\left(u_a - u_w\right)\right]^n\right)^m}$$

was the model and the following merit function was minimised:

$$F(\underline{p}) = \frac{\sqrt{\sum_{i=1}^{L} w_{me}(u_i) - w(u_i, \underline{p})}}{L \quad \max_i (w_{me}(u_i))}$$

where subscript *me* is measured, $\underline{p}$ is parameter vector consisting of the model parameters [w_r,, w_s-w_r ,1/a^n, *n*, *m]* and u_i, (i=1..*L*) are the applied suction load steps

Parameter identification

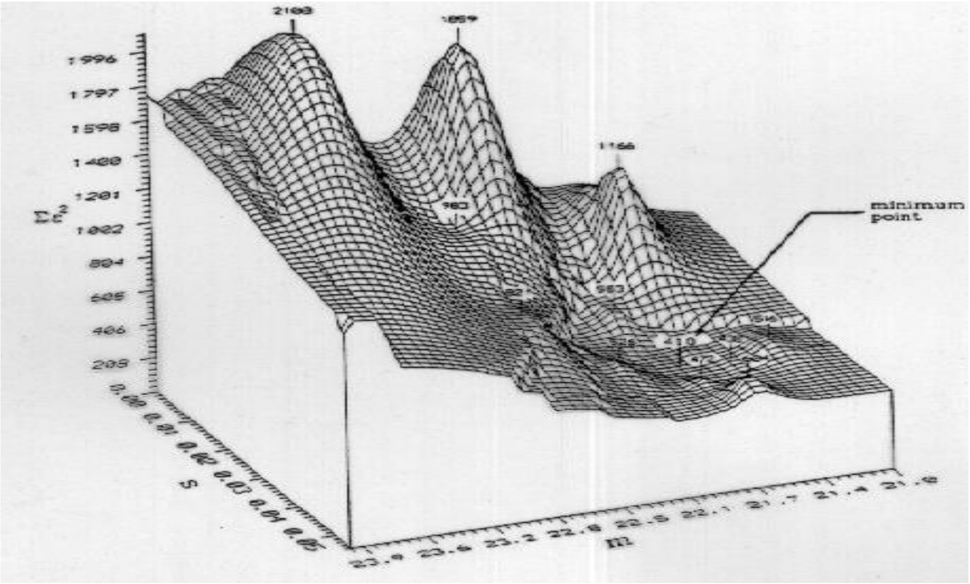
By using measured data, the real-life merit function $F(\boldsymbol{p})$ has an unimaginable complexity of the shape to hinder the minimization.

However, there is a closest, noise-free LS merit function $F_{\text{min}}(\boldsymbol{p})$, such that the difference of this “follower” and the real-life merit function is extremely small in case of a good model and small noise.

Especially, for convex noise-free LS merit functions, the minimum can be bracketed automatically, as an exception since there is no such possibility in the general multidimensional case.

Two types of objective functions

- Real-life objective or merit function (measured data) – HARD GEOMETRY
- Follower noise-free objective or merit function (simulated data using $\boldsymbol{p} = \boldsymbol{p}_{min}$) – NICE GEOMETRY, CAN BE CONVEX



The follower noise-free merit function (i) has critical points only due to the model and (ii) is similar to the real-life merit function outside the error domain.

The domain defined by $F'(\boldsymbol{p}) < F(\boldsymbol{p}_{min})$ is such a vicinity of the $\boldsymbol{p}_{min}$ global minimiser of the $F(\boldsymbol{p})$ real-life merit function where - due to the presence of the many critical points -, the minimisation algorithm are no more effective due to the flat and irregular surface. This geometrical feature may allow to define an "error domain" in the vicinity of the global minimum irrespective of the type of the noise.

Due to (i), it can be used for uniqueness testing and for the determination of a parameter error.

Due to (ii), it can be used outside the error domain for minimisation as an approximation of the real-life merit function, if convex, the minum can be bracketed. We assumed convexity.

Hierarchical solution of the inverse problem

(ii) The hierarchical technique was used to eliminate the „linear part" of the parameter vector in the inverse problem solution, to reduce the parameter number in the non-linear bracketing algorithm,
the to collect some pieces of information on parameter sensitivity / error.

Hierarchical solution of the inverse problem

- The original problem:

$$F(\mathrm{p}) = \min!$$

- The parameter space is split into the direct sum of two subspaces and the parameter vector into two parts:

$$\mathrm{p} = [\mathrm{p}_1, \mathrm{p}_2]$$

Hierarchical solution of the inverse problem

Hierarchical inverse problem:

$$F(\mathrm{p}_1, \mathrm{p}_2) = \min! \quad \mathrm{p}_2 = \text{fixed}$$

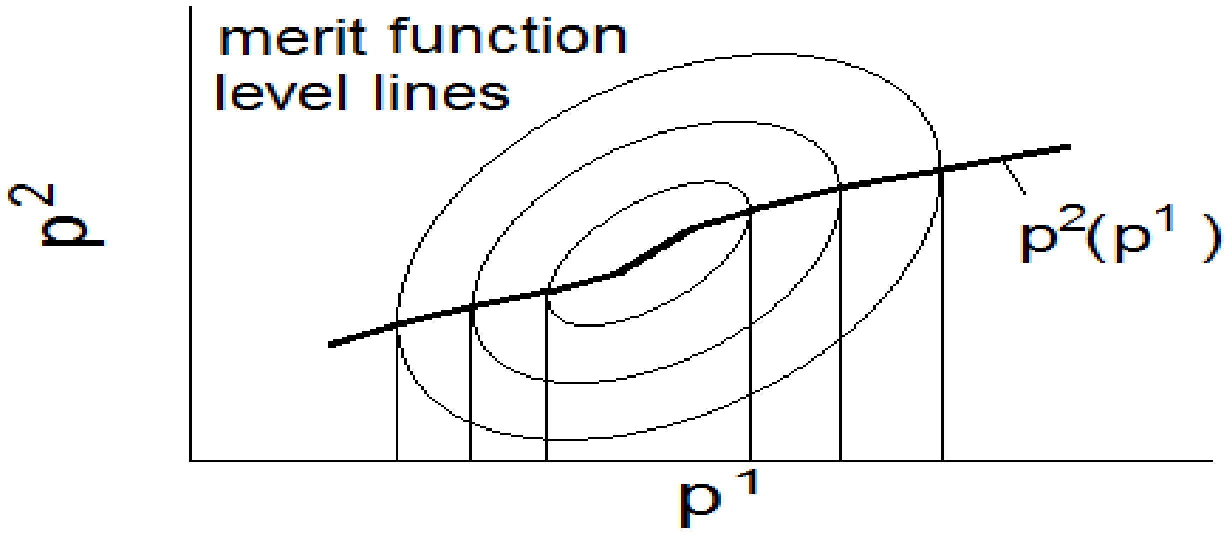
- Conditional minimization performed for each, fixed value of ***p***2. Solution is the relation $\boldsymbol{p}_1 = \boldsymbol{a}(\boldsymbol{p}_2)$, containing those $\boldsymbol{p}_1$ at each fixed $\boldsymbol{p}_2$ which minimises $F(\boldsymbol{p}_1,\boldsymbol{p}_2)$.

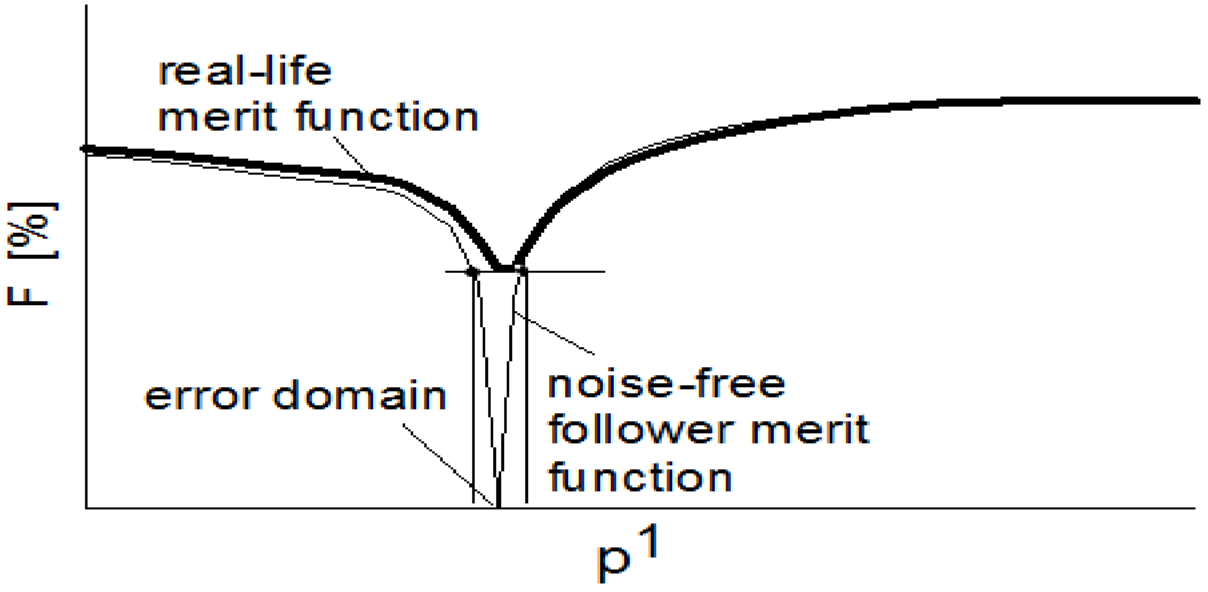
- $F(\boldsymbol{a}(\boldsymbol{p}_2),\boldsymbol{p}_2)$ is an *M–J* dimensional 'minimal' section of the objective function depending only on $\boldsymbol{p}_2$

$$F(\mathrm{a}(\mathrm{p}_2), \mathrm{p}_2) = \min!$$

Hierarchical solution of the inverse problem – minimal section

The deepest (‘minimal’) section with respect tp parameter p^1

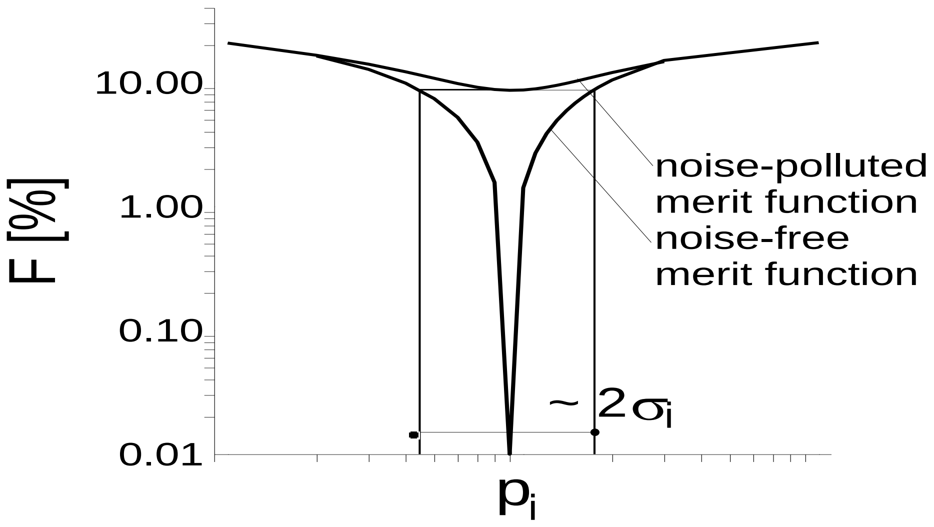




Hierarchical solution of the inverse problem – minimal section

We assume convexity and define bracketing coordinate hyperplanes with discrete sets of parameter values in the space of the parameters. We explore the topography and construct minimal sections of the „nonlinear" parameters by the hierarchical solution of the inverse problem.

The bracketing is made on these minimal sections, and the diameter of the parameter error domain with respect to a specified parameter is also determined with these.

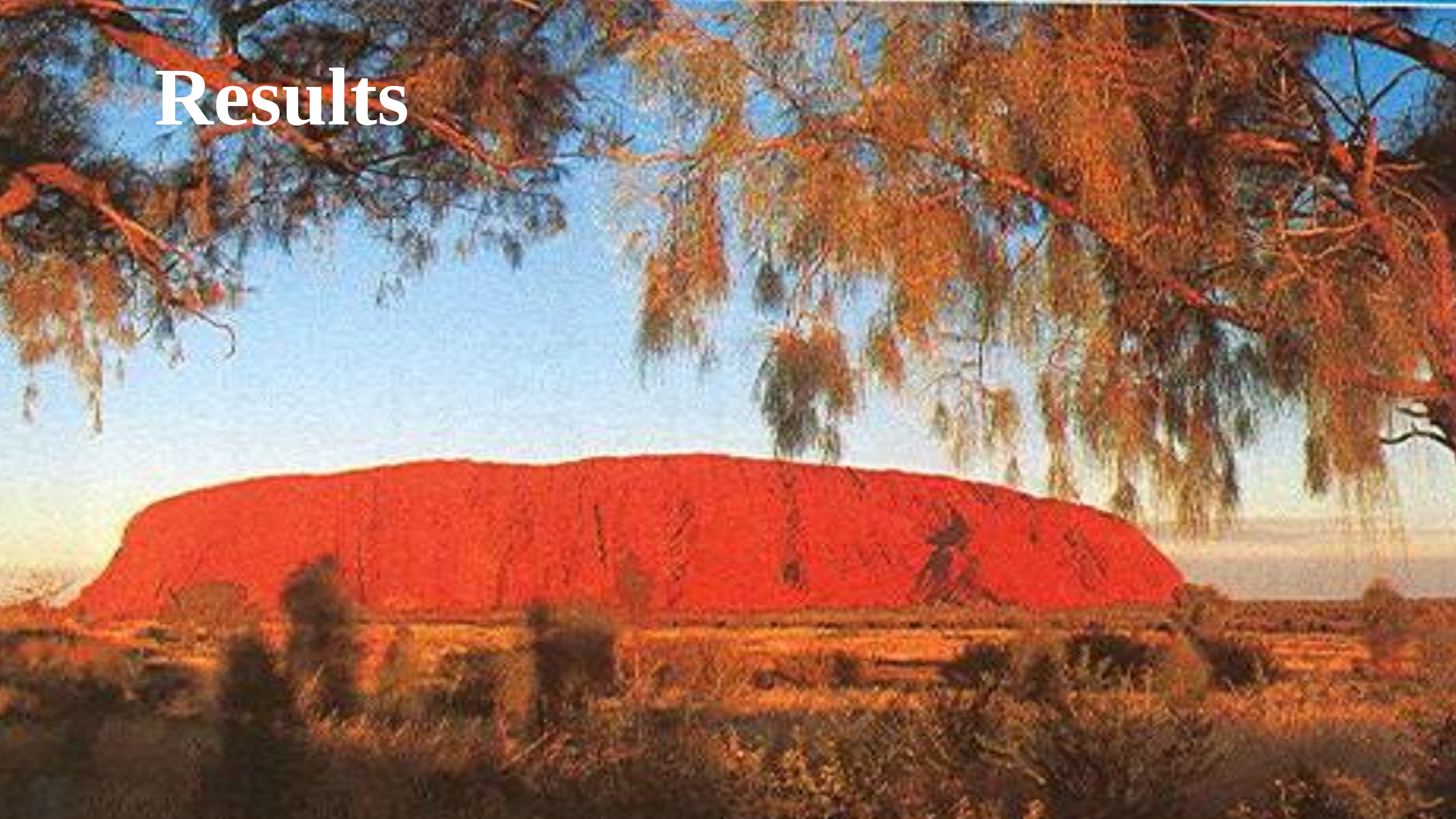


Summary

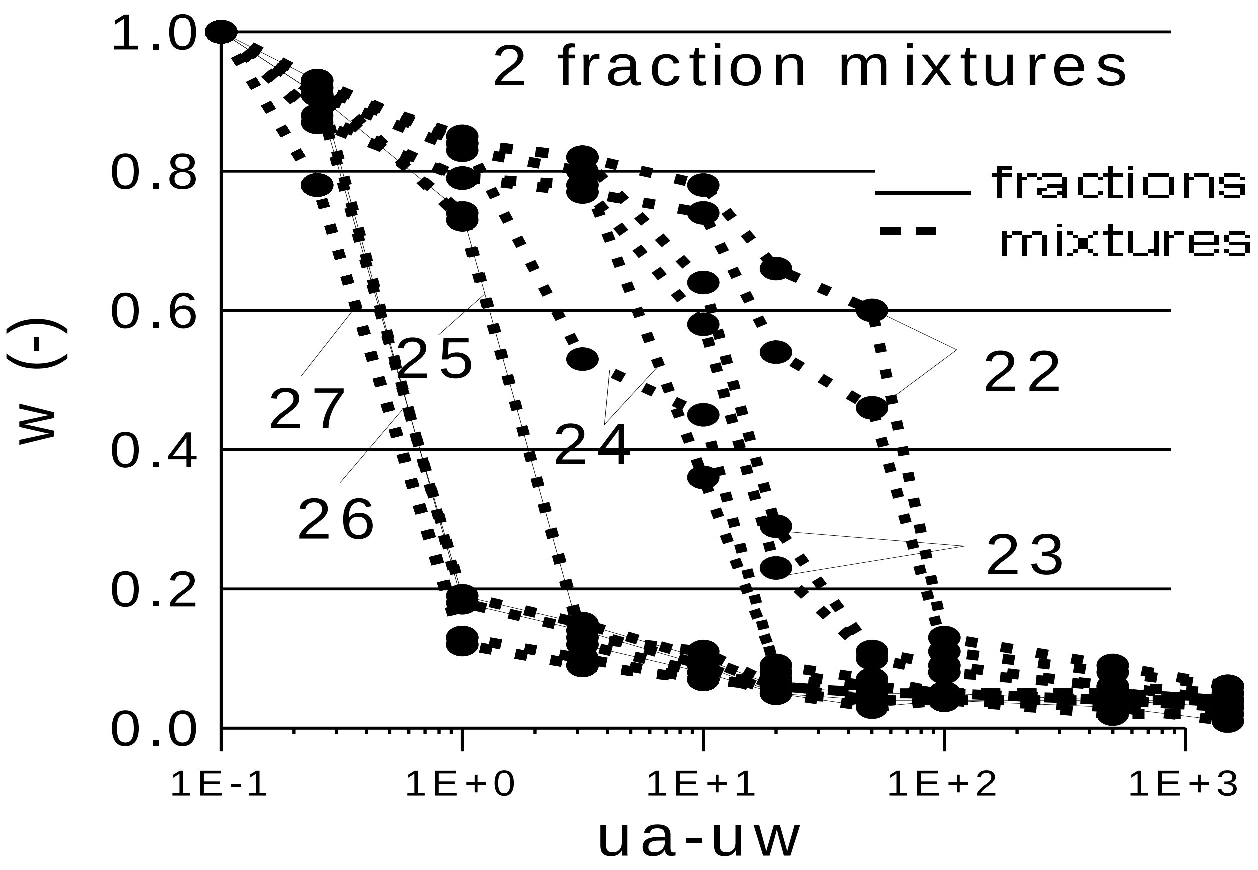
- The follower merit function can be used to facilitate optimization of the real-life (noisy) merit function and to formulate reliability criteria. If it is convex, the minimisation can be done by bracketing.
- The reliability criteria can be tested through the use of “minimal sections” of the follower merit function, where all parameters are eliminated except one.

- Using this method, the five parameters of the Van Genuchten water retention curve equation [w_r,, w_s-w_r ,a, n, m] were identified and the standard deviation of these were computed for the measured water retention curves. The two „linear” parameters eliminated and computed by SVD algorithm: w_r,, w_s-w_r .

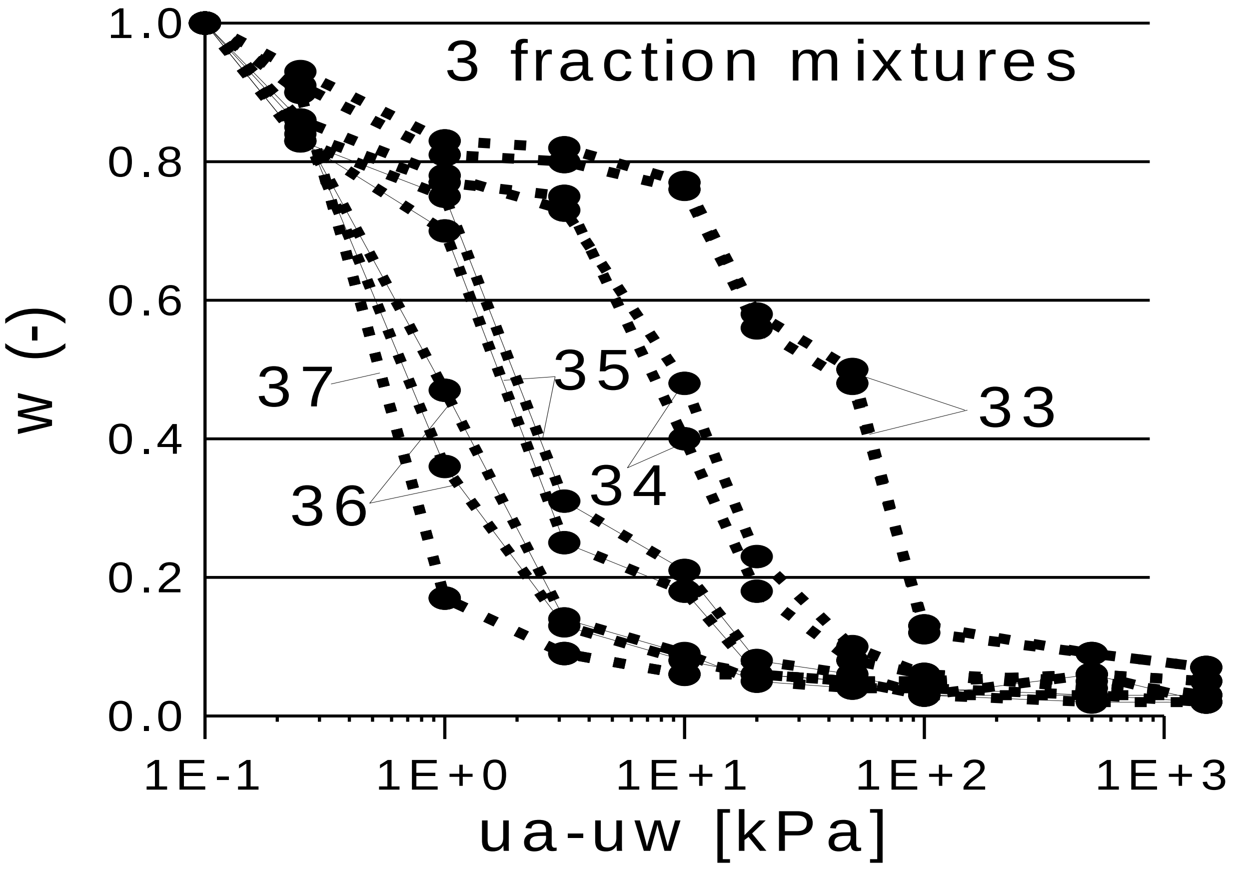
Results



Measured results on duplicate samples



Measured results on duplicate samples



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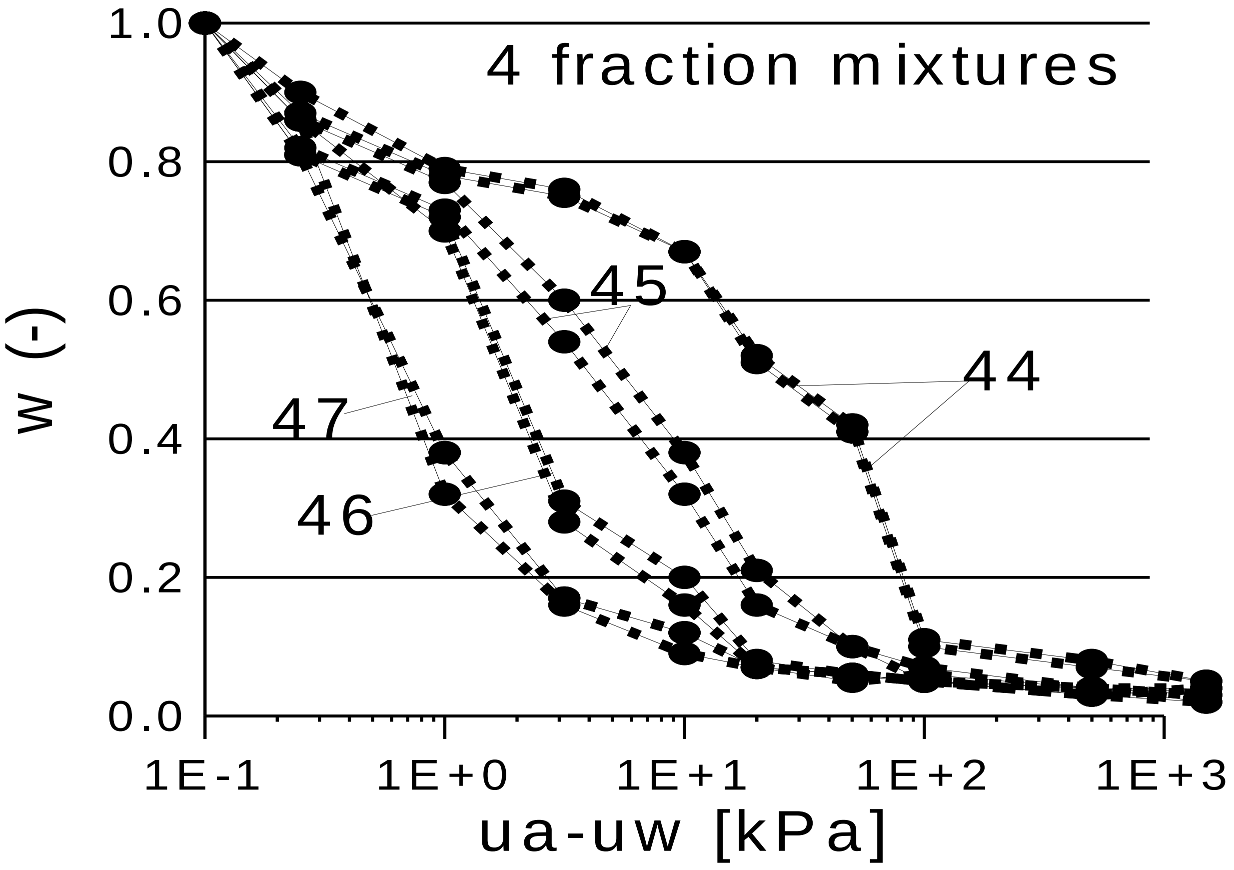
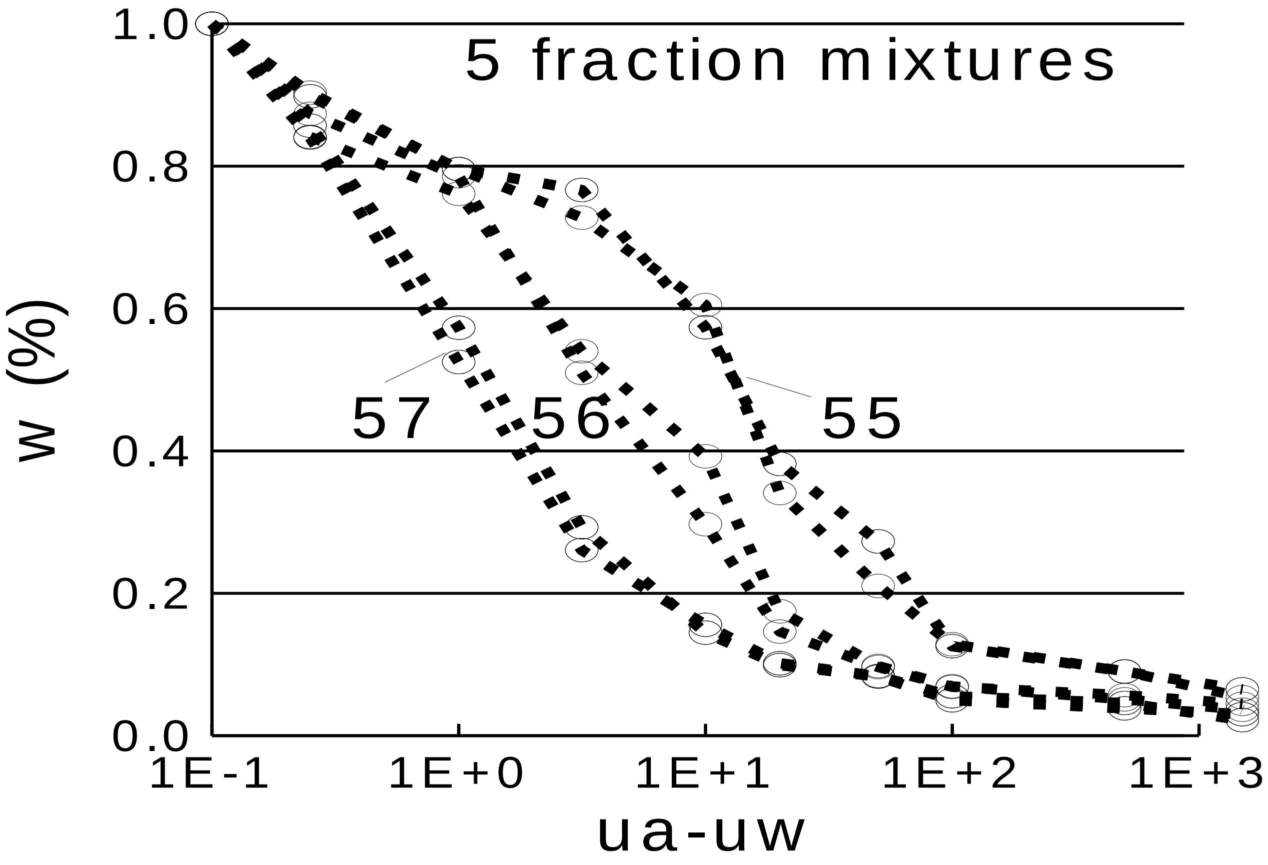
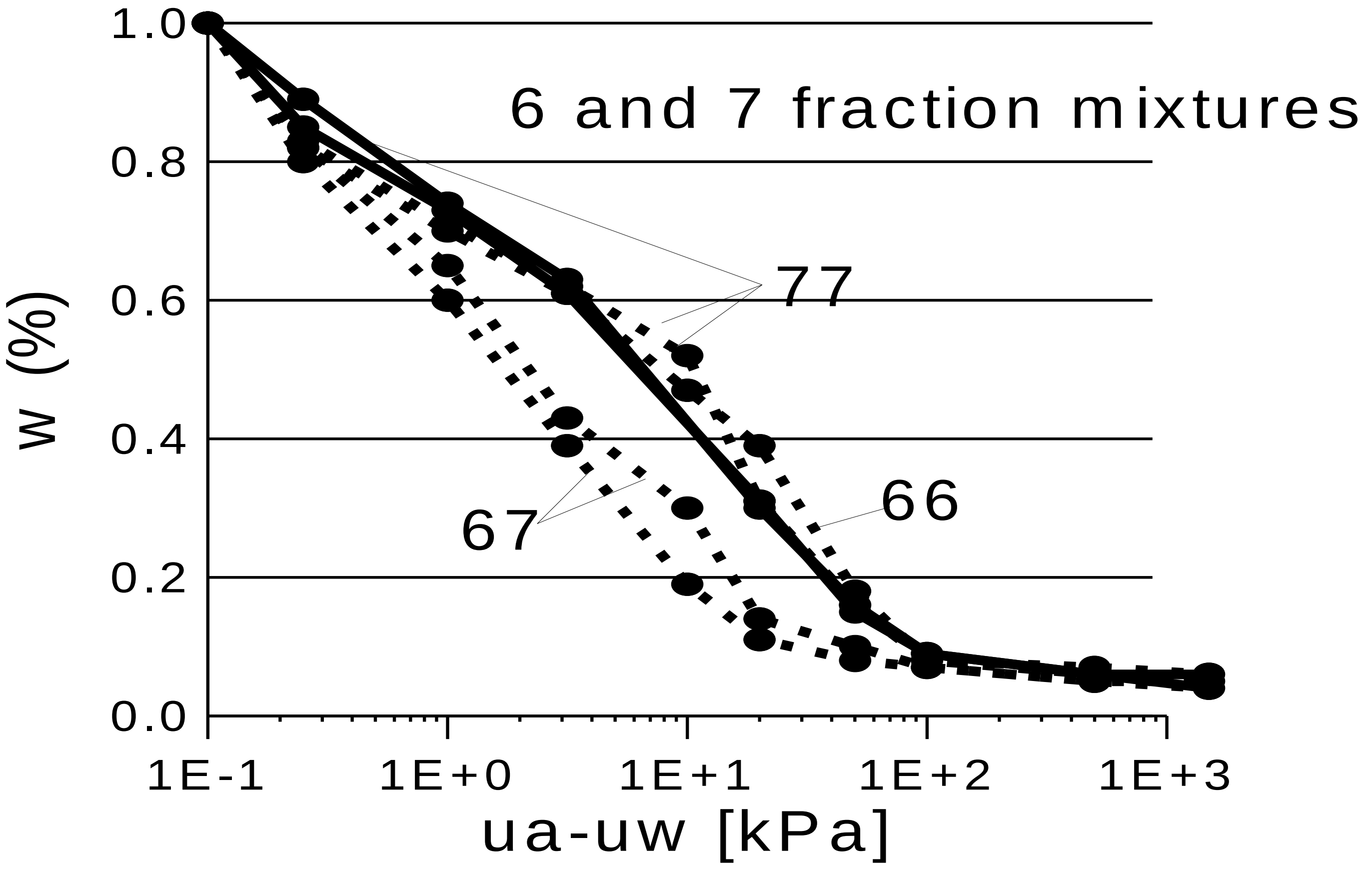


Measured results on duplicate samples



Measured results on duplicate samples



Parameter estimation results-difference of the identified „non-linear" parameters in sample pairs

The five parameters of the Van Genuchten water retention curve equation [w_r,, w_s-w_r ,a, n, m] and the standard deviation of these were identified for the measured water retention curves.

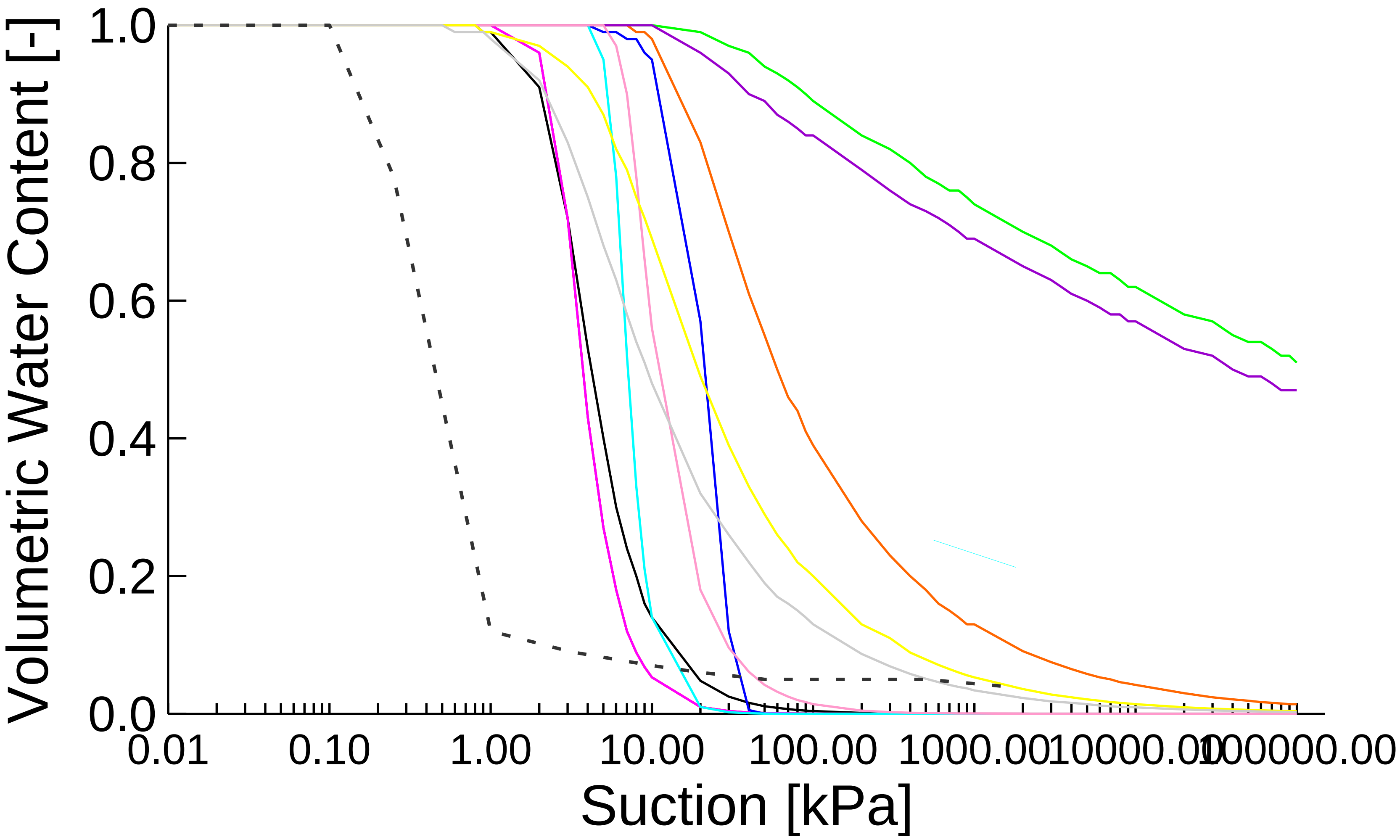
The difference of the identified parameters in sample pairs

Parameter Sample code	a	n	m	Reliability
67	7	4	-0.9	Non acceptable
66	0	0	0	
77	0	0	0	
57	0	0	0	
56	0	0	0	
55	0	0	0	
47	-1	-0.3	0	Non acceptable

The linear coefficient of variation of the identified parameters, statistics

Notation	Standard deviation of parameter/parameter					Fitting error [%]
	w_r [%]	w_s [%]	c=(a)	a(=n)	b(=m)	
mean	3,09	0,14	0,35	0,26	0,35	3,73
sd	2,30	0,06	0,27	0,11	0,13	1,50

Comparing the tested sand samples (dashed line) with literature data on sand to clay



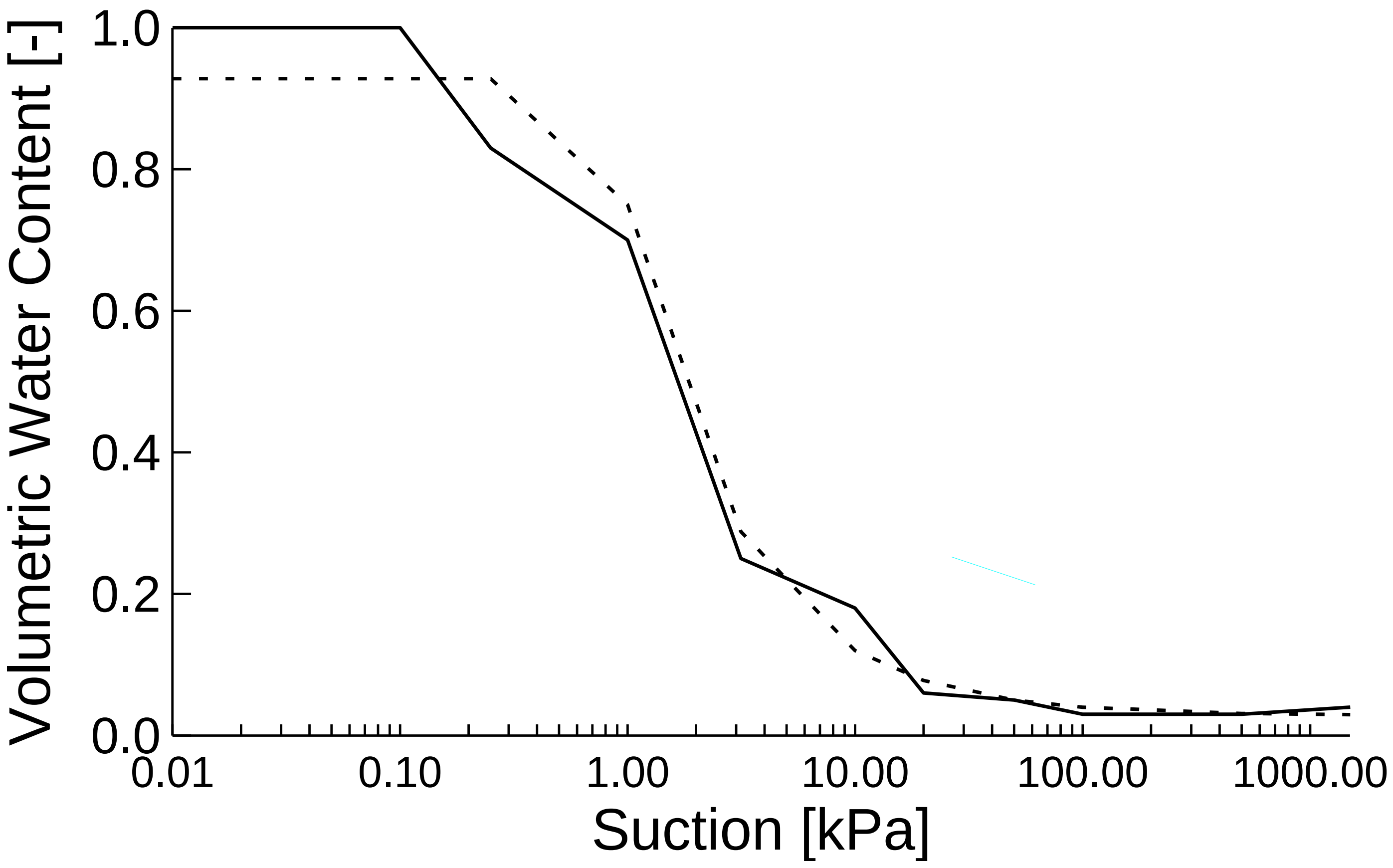
The parameter error

The linear estimation of the coefficient of variation of the nonlinear parameters were high, between 0.2 and 0.4.

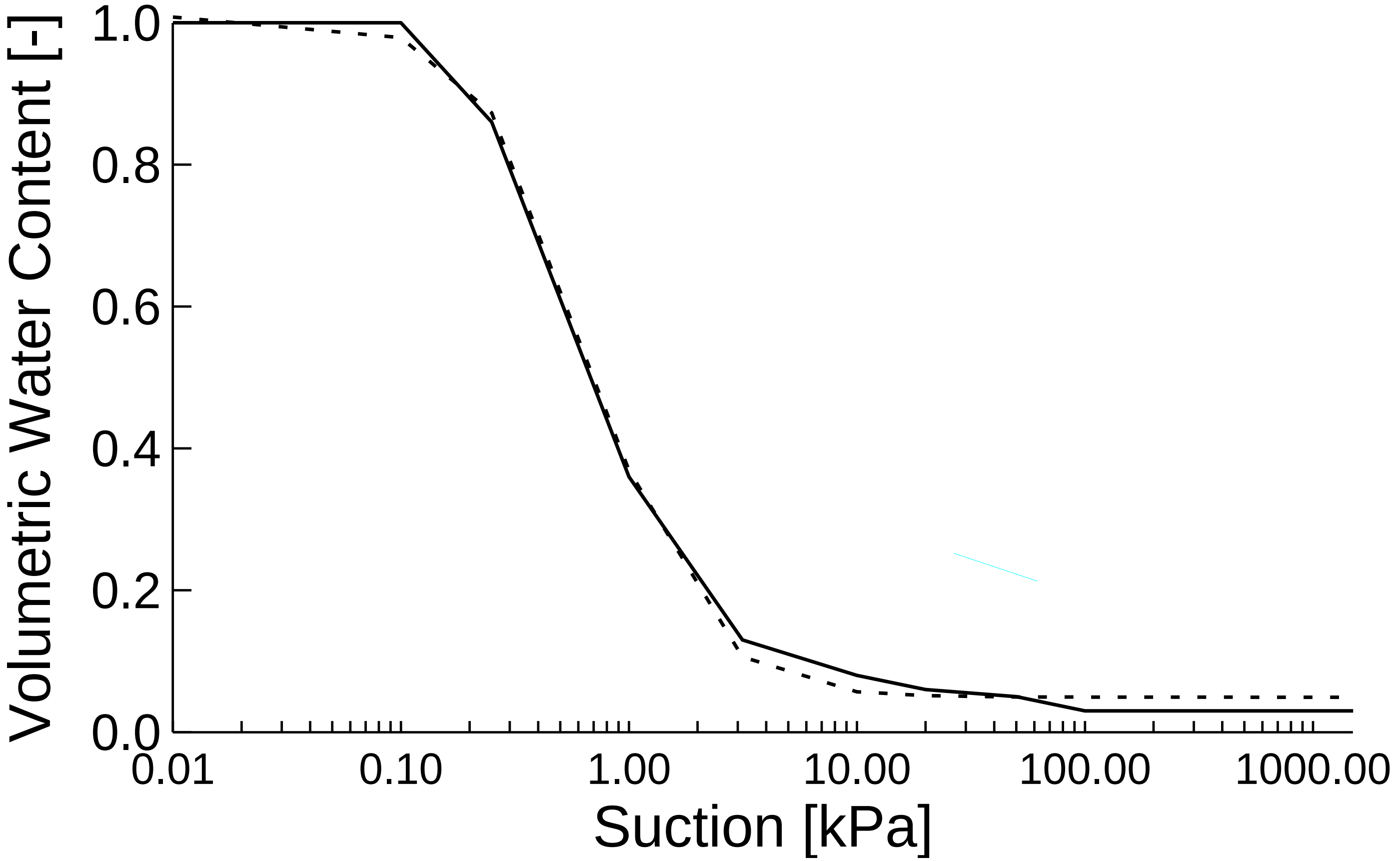
The nonlinear error was asymmetric and significant in some cases.

It was found that the range of the bracketing parameter set of the nonlinear parameters must be redefined in some cases, when the minimum was on the boundary or, the geometric parameter error domain was not possible to be assessed due to the huge non-linear parameter error.

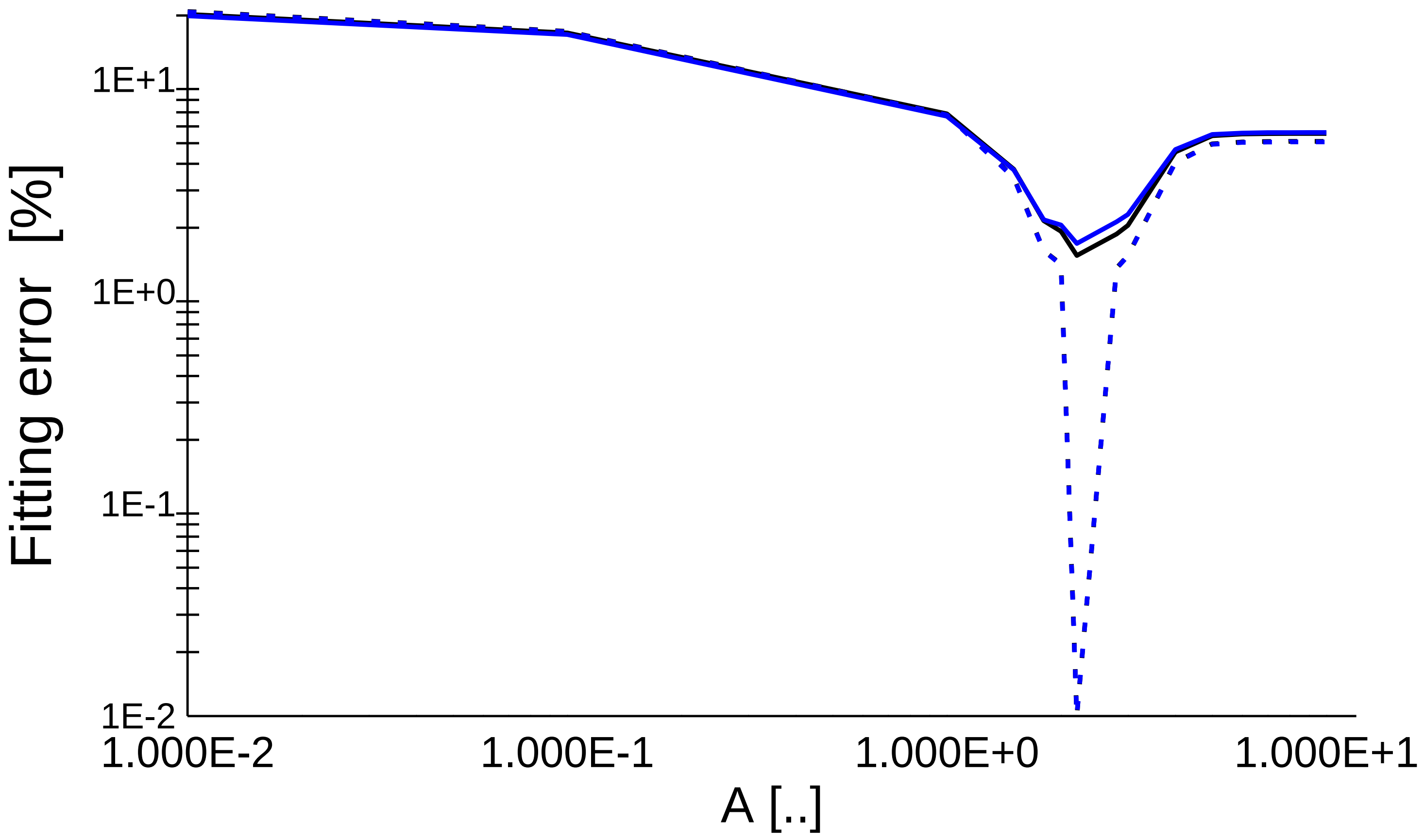
Measured and the fitted data, large fitting error at small suctions



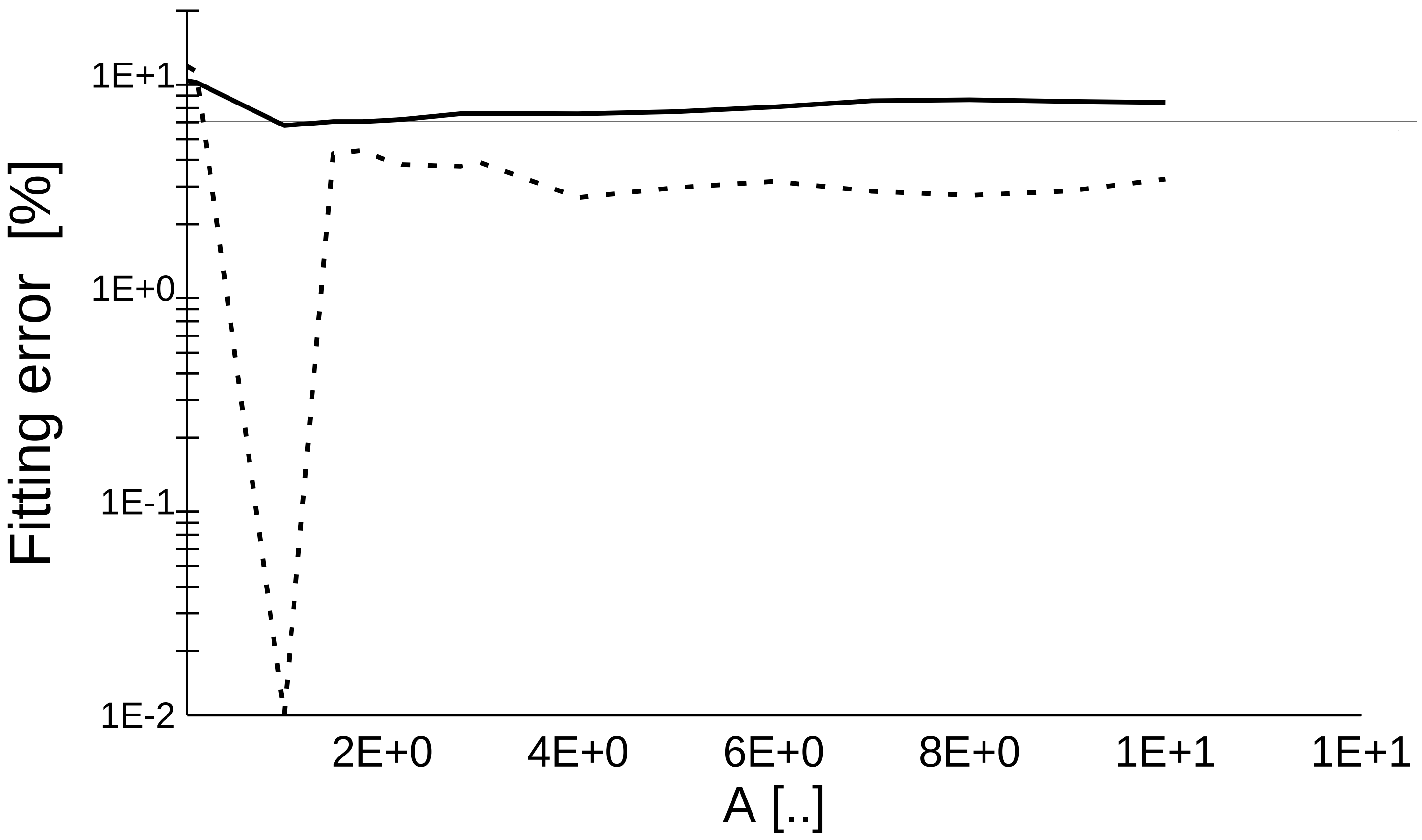
Measured and the fitted data, small fitting error



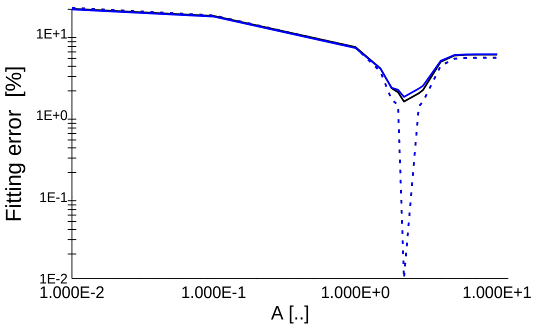
The minimal section of parameter *a*, small error



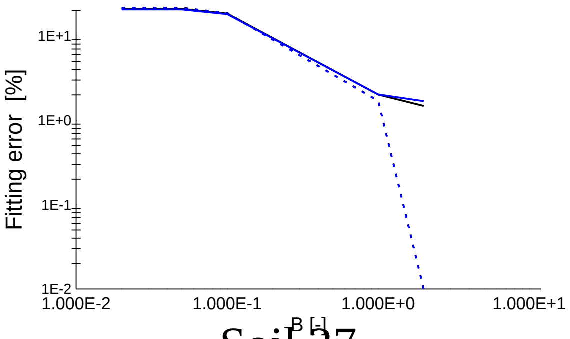
The minimal section of parameter *a*, large error



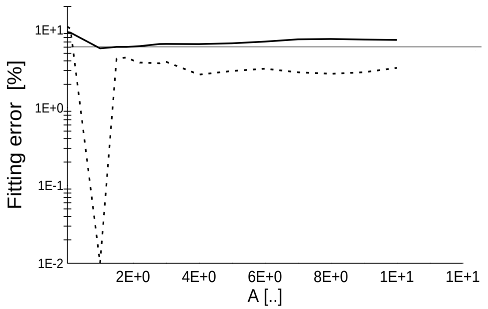
Summary



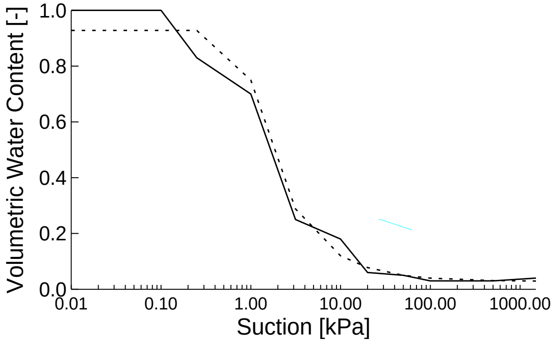
Soil 27



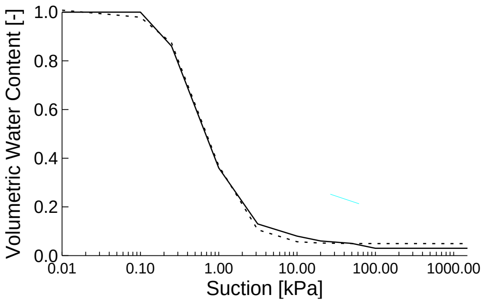
Soil 27



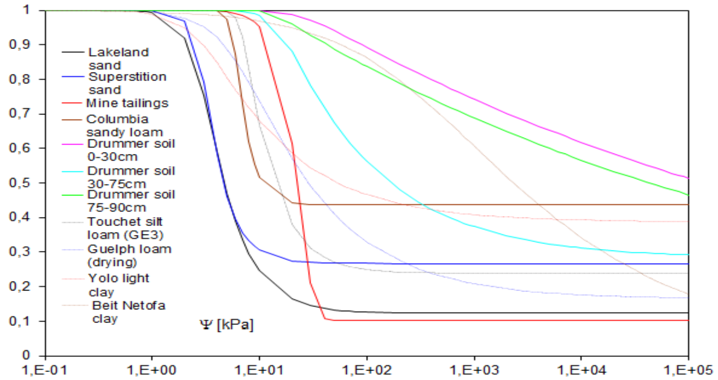
Soil 25



Measured and the fitted data mixture – large error



Measured and the fitted data mixture – small error



The Van Genuchten model, simulations

Conclusion

- In conclusion, first results indicated that the suggested evaluation method can be used for the automatised check of the reliability of the SWCC experiments made on duplicate samples.
- The coarse sand is unusual soil, and additional load stesp were needed and the drying out error at initial part of SWCC may occured randomly, sometimes yes, sometimes not. It is important to make an output with the representation of the measured-fitted curves.
- The coarse sand is an unusual soil - needs special testing technique - or at it is not possible to accept the model fitting result without reliability test. Large parameter error was found in about the 70% of the samples.
- After the first step of parameter identification, it turned out that the identification of linear parameters is unnecessary, since for the saturated water content is equal to 1, and for the tested sands, the residual water content was almost always the same.
- The automatic inverse problem solution will be repeated by not identifying the trivial parameters and by increasing the bracketing parameter values. The measurements are planned to be repeated by increasing the number of load steps.

CONCLUSION DETAILS

- The fitting error and the parameter error is deeply related. According to the first results, the mean fitting error of the mixtures was 4,18% with standard deviation of 1,59%. Only the 30% of the samples had a fitting error less 3%, where the parameter errors seemed to be acceptable. The large fitting error can probably be attributed to two facts. (i) The saturated and residual water contents were identified with large error. (ii) The values and number of suction loads was not originally designed for sand samples.
- Considering the linear parameter error, the identified residual water content had the largest linear parameter error in terms of coefficient of variation. Its identification may have influenced the preciseness for the other parameters. In addition, the SVD algorithm became numerically unstable several times, which hindered the extension of the bracketing parameter set. Since the measured residual water content was almost always the same for the tested sands, and the normalized saturated water content is equal to 1, the identification of these parameters can be omitted.
- Considering the non-linear parameters, the identified linear and non-linear errors were comparable and were significant if the normalized fitting error was larger than about 3 %.
- Concerning the bracketing process, in the first stage of the evaluation, it was found that the range of the nonlinear parameters must be redefined in some cases since the global minimum was not within the compact parameter domain. Therefore, the bracketing process should be modified to be iterative in this way.

Conclusion details

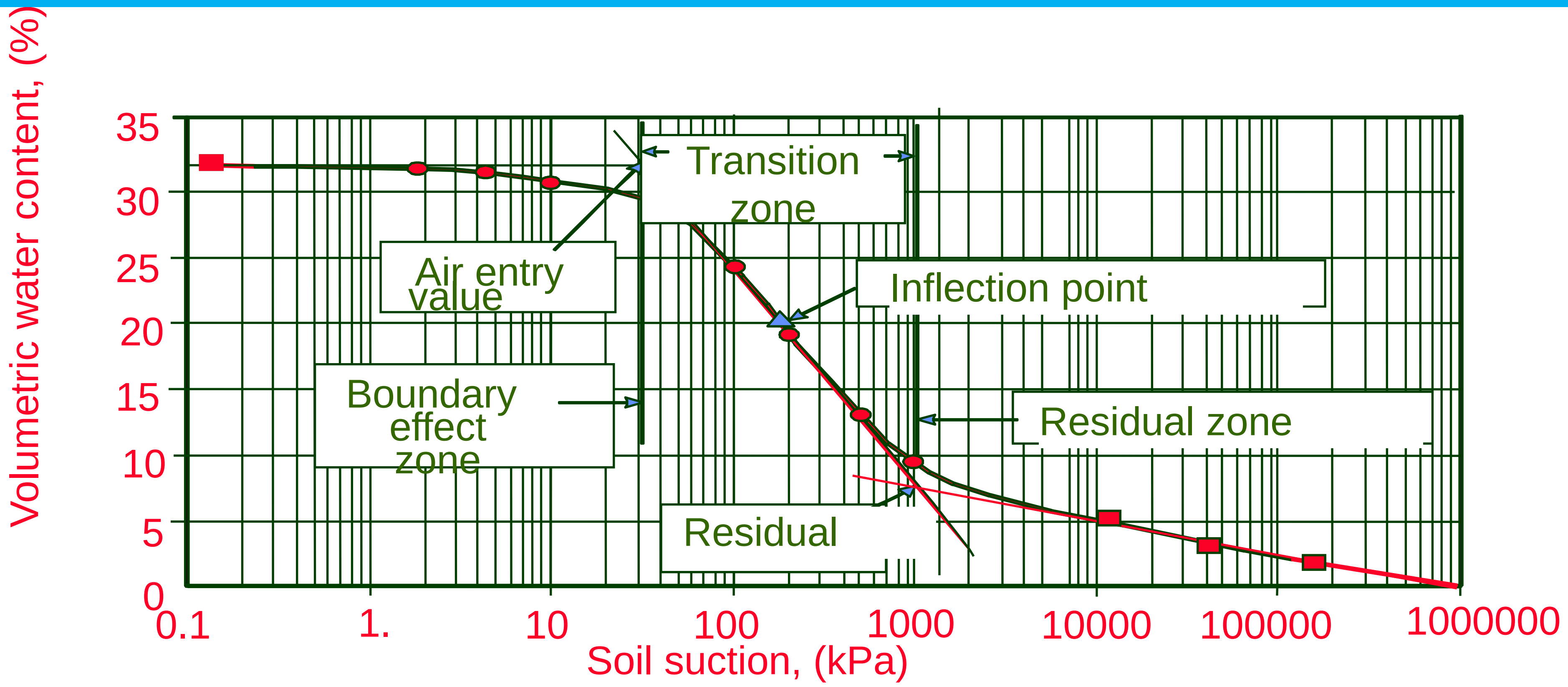
The drying out error at initial part of SWCC may occur randomly, sometimes yes, sometimes not. It is important to make an output with the representation of the measured-fitted curves.The coarse sand is an unusual soil - needs special testing technique - or at it is not possible to accept the model fitting result without reliability test. In further research, the analysis is suggested to be repeated taking into account the foregoing.

Since the identification of the linearly dependent parameters is not giving new information in case of the tested sands, only the non-linearly parameters is needed to be identified. It will be clarified if there is a smaller parameter error after reducing the parameter number, not identifying the linear parameters. The bracketing process should be modified to be iterative to ensure that the global minimum is within the compact parameter domain.

Further research is suggested on the convexity of the merit function, and in general, about then geometrical uncertainty domain and on the connection of the probability and the contour values of the noise-free merit function in the case of various random noises. Moreover, the effect of the deterministic errors is suggested to be studied.

The geometric parameter error can either be determined by numerical integration or approximately determined for the non-linearly dependent parameters, using a linear estimation for both sides and a fixed percentage of the fitting error, and the mean value can be computed.

Model



Volumetric water content, (%)
35
30
25
20
15
10
5
0
Transition zone
Air entry value
Inflection point
Boundary effect zone
Residual zone
Residual
0.1
1.
10
100
1000
10000
100000
1000000
Soil suction, (kPa)

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THANK YOU VERY MUCH FOR YOUR ATTENTION