A perfectly insulated, rigid tank with a volume of 0.2 m^3 contains a perfect gas which has a molar mass of 18 kg/mol and a ration of specific heats of 1.45. Initially the pressure and temperature in the tank are 9 bar and 320 K respectively. A fan inside the tank is spun at 3600 rev/min for 20 seconds. The torque required to turn the fan is 30 Nm. Calculate the following:

(a) The R, c_p and c_v values of the gas and the mass of gas in the tank.

(b) The work input to the gas from the fan.

(c) The final temperature of the gas.

(d) The increase in entropy of the gas.

$$R = \frac{\tilde{R}}{\tilde{m}} = \frac{8314.5}{18} = 462\ \frac{\mathrm{J}}{\mathrm{kgK}} \qquad \gamma = \frac{c_p}{c_v} = \frac{c_v + R}{c_v} = 1 + \frac{R}{c_v} \qquad \frac{R}{c_v} = \gamma - 1$$

$$c_v = \frac{R}{\gamma - 1} = \frac{462}{0.45} = 1026.7\,\frac{\mathrm{J}}{\mathrm{kgK}}$$

Avogadro constant: $N_A = 6.022140857 \times 10^{23}\ \mathrm{mol}^{-1}$

$$c_p = \gamma c_v = 1.45 \times 1026.7 = 1488.7\,\frac{\mathrm{J}}{\mathrm{kgK}}$$

$$m = \frac{p_1 V_1}{R T_1} = \frac{9 \times 10^5 \times 0.2}{462 \times 320} = 1.22\ \mathrm{kg}$$

$$W_{fan} = T\theta = T \cdot n \cdot 20 \cdot 2\pi = 30 \times \frac{3600}{60} \times 20 \times 2 \times 3.14 = 226.2\ \mathrm{kJ}$$

$$U_2 - U_1 = Q + W \qquad U_2 - U_1 = W_{fan} \qquad U_2 - U_1 = m c_v (T_2 - T_1) = W_{fan}$$

$$T_2 = \frac{W_{fan}}{m c_v} + T_1 = \frac{226.2 \times 10^3}{1.22 \times 1026.7} + 320 = 180.6 + 320 = 500.6\ \mathrm{K}$$

$$S_2 - S_1 = mR\ln\frac{V_2}{V_1} + m c_v \ln\frac{T_2}{T_1} = 0 + 1.22 \times 1026.7 \times ln\frac{500.6}{320} = 560.5\ \frac{\mathrm{J}}{\mathrm{K}}$$